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A General Educator's Instructional Adaptation for Students With Mathematics Disability in Standards-Based Mathematics Instruction

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A General Educator's Instructional Adaptation for Students With Mathematics Disability in Standards-Based Mathematics Instruction

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Dedication

To the God and people in Him;

Especially to My mother, Sun-Hee Hwang, and my husband, Hyungju David Chung,

for their endless love and support

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A General Educator’s Instructional Adaptation for Students With Mathematics Disability in Standards-Based Mathematics Instruction

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The Individuals With Disabilities Education Act (IDEA), implemented in 1997 and updated in 2004, requires all students, including students with disabilities, to participate in and make progress in the general education curriculum. Under IDEA, students with disabilities, including students with mathematics disability (MD), are entitled to be provided with adapted instruction using empirically validated instructional approaches to teaching mathematics, which can help them succeed in general education classrooms. However, there is limited knowledge about whether and in what ways instruction is adapted for students with MD and the degree to which students with MD have access to the standards-based mathematics general education curriculum adopted by today’s mathematics education. Thus, the purpose of this case study was to examine (a) a fourth-grade teacher’s instructional adaptations for 3 students with MD in a standards-based mathematics, general education classroom and (b) the mathematics learning of 6 fourth-grade students with differing levels of ability

(3 students identified MD, 2 students struggling with mathematics, and 1 student without a disability) in a standards-based mathematics, general education classroom.

An embedded, single case study design (Yin, 2003) was employed to provide exploratory and instrumental information about the research topics of this study. Data were collected through case study methods including direct observations, interviews, survey, and document reviews for 12 weeks, December 2005 through March 2006. Analyses of data involved a descriptive statistics as well as a qualitative case analysis using data display matrices to drive emergent themes (Miles & Huberman, 1994; Strauss & Corbin, 1997; Yin, 2003).

Seven themes emerged from the findings of this study: Four on the fourth-grade teacher's instructional adaptations for her students with MD in the standards-based mathematics, general education classroom and three on the learning of students with differing abilities in this environment. The findings of this study indicated that the teacher endeavored to adapt her mathematics instruction for 3 students with MD using diverse components of effective mathematics instruction in standards-based mathematics curriculum and instruction, but that her instructional adaptations were implemented very restrictively in terms of the number of students with MD whose difficulties were addressed and the types of difficulties addressed by the adaptations. Possible factors inhibiting the teacher's instructional adaptations included the number of students who were struggling with mathematics in her class, including 3 students with MD.

On the other hand, the findings of this study indicated that the quality and the quantity of learning of mathematics knowledge and skills were different across students with differing ability in the standards-based mathematics, general education classroom in terms of prerequisite skills, problem-solving accuracy, concept or procedures for problem solutions, and transfer of knowledge and skills. All the students with differing ability benefited to some degree from standards-based mathematics instruction, but the benefits of students with MD from this instructional environment were marginal in comparison to the benefits of their peers without disabilities. Alternative instructional methods should continue to be explored to maximize the benefits of students with MD in standards-based mathematics, general education classrooms, including more frequent integration of varied types of components of effective mathematics instruction into standards-based mathematics instruction and considering the cognitive, behavioral characteristics of students with MD. Limitations of this study and implications of this study for practices and future research were discussed.

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CHAPTER 1:

INTRODUCTION

Evidence (Beatty, 1997; National Center for Education Statistics [NCES], 2000) of U.S. student mathematics performance has raised concern about mathematics education (Woodward & Montague, 2002). Results from the TIMSS-Repeat (NCES, 2000), the Program for International Assessment in Education (PISA, 2003), and the National Assessment of Education Progress (NAEP; Braswell, Daane, & Grigg, 2003; Perie, Grigg, & Dion, 2005) have revealed that U.S. students are not performing as well as students in other developed countries. Moreover, compared to other developed countries, a higher percentage of U.S. students are struggling with mathematics, particularly advanced mathematics skills (PISA, 2004).

As a whole, at the fourth-grade level, students begin to receive mathematics instruction emphasizing more advanced mathematics knowledge and skills (e.g., algebra), on which international and national assessments focus (NAEP, 2003). However, results from national assessments (e.g., NAEP, 2003) have shown that many fourth-grade students had difficulties in understanding and acquiring mathematics knowledge and skills that they were expected to attain at the grade level. For example, results from the NAEP (Perie et al., 2005) have shown that 64% of fourth-grade elementary students did not reach the proficient level of mathematics achievement, which represents competency over challenging subject matter. Of the nonproficient student group, 20% did not attain the basic level of mathematics achievement, indicating that they did not achieve even partial mastery of prerequisite knowledge and skills. In Texas, 23% of students in the fourth grade (16% for English-speaking students and 30% for Spanish-speaking students) did not achieve grade-level skills in mathematics on the Texas Assessment of Knowledge and Skills (TAKS) of 2006 (Texas Education Agency [TEA], 2006). For students with disabilities, the performance results are even less promising. In 2006, 32.5% of fourth-grade students who received special education services in Texas did not achieve grade-level skills in mathematics on the TAKS (TEA, 2006), whereas 29% of fourth-grade special education students failed to show the grade-level mathematics skills on the TAKS of 2004 (TEA, 2004a, 2004b).

Studies in the field of special education consistently have documented that 6–8% of school-age children have significant mathematics deficits (Badian, 1983; Geary, 2004). Unfortunately, research has shown that the mathematics performance of students identified as having mathematics disabilities (MD) tends not to show significant improvement over time. According to Cawley and Miller (1989), the mathematical abilities of students with MD are developmentally delayed across years, implying that students with MD lag behind their typically achieving peers in mathematics performances in their later school years. For example, 8- and 9-year-old students with MD performed at about the first-grade level on calculations and applications, whereas 16- and 17-year-old students with MD performed at about the fifth-grade level. Similarly, Cawley, Parmar, Yan, and Miller (1998) found that students with MD ages 9–14 not only performed at lower levels than typically achieving students in the same age range, but also did not show a significant progress from one age to another.

Increasingly, students with learning disabilities (LD), including students with MD, are receiving their mathematics instruction in the general education setting (U.S. Department of Education, 2002). Compared to 21% of students with LD in 1992, approximately 45% of students with LD spent more than 80% of their instructional time in general education in 2002 (U.S. Department of Education, 2002).

In today's classrooms, mathematics education has been influenced by efforts to reform mathematics instruction from a more basic skill and practice approach to a problem-solving, inquiry-based approach. Mathematics education has adopted standards-based mathematics curricula and instruction developed by national organizations such as the National Council of Teachers of Mathematics (NCTM), the National Research Council (NRC), and the NAEP (Apthorp et al., 2001; Lappan, 2000). These standards include instructional expectations for teachers, many of which are characterized as socioconstructivist instruction (i.e., student-centered, inquiry-based, and contextualized problem-solving instruction), as well as performance expectations for students (Forman & Steen, 2000), although the degree of adoption of a socioconstructivist view about learning and instruction varies across standards documents. Thus, many students with MD, particularly at the elementary level, are likely to receive mathematics education by general education teachers who are implementing standards-based mathematics curricula and instruction involving socioconstructivist practices.

Of the standards documents developed by several professional organizations, most states have adopted the NCTM (2000) standards to establish their state-level standards for school mathematics (Lappan, 2000). However, researchers (Gersten & Baker, 1998; Harris & Graham, 1996; Woodward & Montague, 2002) in the field of MD have expressed concerns about mathematics learning of this group of students in standards-based mathematics instruction. Based on the findings of studies (Kroesbergen & Van Luit, 2003; Swanson, Hoskyn, & Lee, 1999) on effective mathematics instruction for this group of students, direct, explicit instruction remains most promising for teaching those struggling students. Importantly, a balanced approach (i.e., direct and inquiry-based) may be necessary to assist those struggling students in learning mathematics in standards-based mathematics, general education classrooms.

In education, there is a long-standing belief that instructional adaptations to address students' individual needs make a crucial contribution to student learning (Fuchs, Fuchs, & Bishop, 1992). Moreover, teachers in today's general education classrooms are required to adapt their instruction for students with disabilities to maximize their access to the general education core curriculum (Individuals With Disabilities Education Act [IDEA], 2004). Given the findings on effective mathematics instruction for students with MD and the requirement by law, general education teachers, implementing standards-based mathematics curricula and instruction, should incorporate direct, explicit instructional features into standards-based mathematics core instruction to help students achieve learning expectations in these environments.

In the fields of mathematics education and learning disabilities, how general education teachers are adapting standards-based mathematics instruction for students with MD and how successfully students with MD are able to access the curriculum and learn the curriculum content in these instructional environments have not successfully attracted researchers' attention. Thus, this study proposed to investigate how a fourth-grade, general education teacher adapted standards-based mathematics instruction for 3 students with MD in her classroom and how 5 fourth-grade students with differing levels of ability (3 identified MD, 2 struggling, and 1 without disability) learned grade-level mathematics knowledge and skills in a standards-based mathematics classroom.

To ground and inform this study, this chapter describes (a) the socioconstructivist approach, (b) standards-based mathematics curricula and instruction, (c) standards-based mathematics curriculum programs, (d) characteristics of students with MD, and (e) instructional differentiations (adaptations) for students with MD in standards-based mathematics classrooms. The next section describes socioconstructivist's beliefs about learning and instruction and how these beliefs are embedded in the NCTM (2000) standards, which influence today's mathematics education.

Socioconstructivist Theory and Mathematics Education

Proponents of the socioconstructivist view believe that student learning is a social product attained by interactions with social and cultural environment (Wells & Claxton, 2002). They assume that learning involves a community of learners rather than an isolated individual learner and is supported and extended by social functions including language, tools, and social interactions (Clements & Battista, 1990).

Vygotsky (1978) maintained that cooperation and collaboration with other people generates higher levels of intellectual functioning. When a student solves a problem independently, the student's responses reflect learning that already has matured (Bottge, 2001). When students interact with each other or adults, the students' responses reflect their learning potential, which Vygotsky called the "zone of proximal development." Social interactions extend the students' zone of proximal development, which is conceptualized as the distance between the actual developmental level as determined by independent problem solving and the level of potential development, as determined through problem solving under adult guidance or in collaboration with more capable peers. As a result, students can accomplish increased types and quantity of tasks. Thus, the socioconstructivist classroom is seen as a culture in which students are involved in not only discovery and invention, but also a social discourse involving explanation, negotiation, sharing, and evaluation (Clements & Battista, 1990).

Regarding students with LD, proponents of the constructivist approach resist embracing a deficit view of low-achieving student that focuses on the inability of at-risk students to complete tasks (Barley et al., 2002). They oppose decomposing complex tasks into simpler tasks and presenting mathematics skills sequenced from a lower level (e.g., counting skills) to a higher level (e.g., problem-solving skills) for at-risk students (Barley et al., 2002).

The socioconstructivist view of learning and instruction currently influences contemporary mathematics education. The NCTM (2000) mathematics principles and standards include six principles that reflect a socioconstructivist view of learning and instruction: (a) equity, (b) curriculum, (c) teaching, (d) learning, (e) assessment, and (f) technology. The constructivist views are embedded in those six principles. For example, the equity principle implies high expectations and strong instructional support for all students. Equity can be achieved by “accommodating differences to help everyone learn mathematics” (NCTM, 2000, p. 13), through “tasks which are constructed so that learners, regardless their backgrounds in mathematics, are able to locate themselves and negotiate the difficulty of the tasks they set for themselves” (Davis & Maher, 1996, p.96). The learning principle emphasizes that “students must learn mathematics with understanding, actively building new knowledge from experience and prior knowledge” (NCTM, 2000, p. 20). The following section describes the *Principles and Standards for School Mathematics* (NCTM, 2000) in more detail.

Standards-Based Mathematics Curriculum and Instruction

Curriculum

Mathematics underachievement has been addressed primarily either by designing new mathematics curricula or developing innovative interventions (Ginsburg-Block & Fantuzzo, 1998). In response to the mathematics performance of U.S. students on mathematics test results (Beatty, 1997; NCES, 2000), national tests (Braswell et al., 2003), and the influence of developments in cognitive science on mathematics education, the NCTM (2000) published the *Principles and Standards for School Mathematics*, which spurred reform in the curriculum and delivery of instruction.

The NCTM (2000) included standards for content and process, respectively. Content standards are defined as standards that describe knowledge that students are expected to know in specific areas of mathematics such as number or algebra theory (Apthorp et al., 2001). Five content standards are included in the *Principles and Standards for School Mathematics*: (a) number and operation, (b) algebra, (c) geometry, (d) measurement, and (e) data analysis and probability.

Process standards refer to the mathematics skills and processes needed to use the content to solve problems in school and real-world settings (Apthorp et al., 2001). Five process standards are (a) problem solving, (b) reasoning and proof, (c) communication, (d) connection, and (e) representation. Problem solving refers to the ability to formulate problems in a mathematical way, represent problems in mathematical terms, and solve these problems (Kilpatrick, Swafford, & Findell, 2001). More specifically, problem solving includes the ability to formulate problems in mathematics terms by identifying assumptions, identifying what is known, and determining what sort of answer is needed. In the NCTM standards, the flexible use of multiple representations is treated as a separate skill, whereas some other standards documents describe it under problem-solving strand (Kilpatrick et al., 2001).

In addition, the NCTM (2000) standards specify expectations for learning content and processes within four grade-level bands: (a) prekindergarten through second grade, (b) Grades 3–5, (c) Grades 6–8, and (d) Grades 9–12. For example, for learning in geometry, students in primary grades are expected to be familiar with and reason about two- and three-dimensional (2-D and 3-D) geometry shapes and their properties. In the upper elementary grades, students are expected to understand and use transformations, symmetry, and geometric modeling to solve problems.

Instruction

The NCTM (2000) standards for mathematics instruction were built on beliefs that all students can attain better understanding of mathematics concepts or skills through student-centered, inquiry-based, and discourse-driven instruction in contextualized problems (Foreman & Steen, 2000; Woodward & Montague, 2002). Student-centered instruction uses problems that students see as relevant and interesting, that help students learn to work with others, and that strengthen students' technical communication skills (Forman & Steen, 2000). Inquiry-based instruction encourages students to explore and discover a variety of strategies for problem solutions and to investigate available data for problem solving (Forman & Steen, 2000). Discourse-driven instruction encourages students to learn mathematics concepts or procedures by engaging in the construction of shared mathematics knowledge in their classrooms, which is usually accomplished by verbal interactions between a teacher and students or among students (Baxter, Woodward, & Olson, 2001). In addition, student-centered, inquiry-based, and discourse-driven instruction occurs as contextual instruction. Contextual instruction involves students engaging with problems first in context, then with mathematical formality, and encourages students to see connections of mathematics to work and life (Forman & Steen, 2000).

In standards-based mathematics classrooms, all students, including those with MD, are expected to explore and develop multiple problem-solving solution strategies for specific contextualized problems, which are “authentic” and significant to the students, while the teacher probes students to develop greater conceptual understanding (NCTM, 2000). Even though students continue to learn basic computational skills in these environments, they are expected to spend more of their time and efforts in solving open-ended, challenging problems using multiple strategies (Baxter et al., 2001).

In these environments, students are expected to “construct personally meaningful understanding of mathematics concepts or skills” through interactions with their teacher or peers (Cobb, Wood, Yackel, & McNeal, 1992). Furthermore, they are expected to actively contribute to the shared knowledge of mathematics concepts in the classrooms by explaining their mathematical reasoning to others and following the explanations of their peers (Baxter et al., 2001).

To learn mathematics well in the classrooms envisioned in the NCTM (2000) principles and standards, students should have prerequisite skills or abilities to communicate with their teachers or peers using mathematical language or representations, to invent or understand problem-solving strategies, and to build their own knowledge about mathematical concepts through classroom interactions. In other words, standards-based mathematics instruction may be effective mostly for students who possess the necessary prerequisite skills or abilities. However, research findings support that many students with MD who are included in standards-based mathematics, general education classrooms lack the necessary skills to participate in standards-based mathematics instruction and master mathematics skills within those environments (Baxter et al., 2001). Nevertheless, currently, 49 out of 50 states have adopted the reformed standards-based framework to establish their state-level standards for mathematics curriculum and evaluation of student performances (Lappan, 2000). Additionally, the influences of standards-based instruction are pervasive in today's mathematics programs.

Standards-Based Mathematics Programs

Professional organizations funded by national funding foundations (e.g., National Science Foundation) have published mathematics programs, which are based on the NCTM standards or standards from the other organizations. For example, Investigations (Technical Education Research Center [TERC], 2004) is one program that incorporates the NCTM standards. However, the findings of analyses of the NCTM standards-based programs (Carnine & Jitendra, 1997; Jitendra, Carnine, & Silbert, 1996; Jitendra, Salmento, & Haydt, 1999) have not been optimistic about the degree to which these programs contain instructional features that benefit students with MD.

Analyses were done to examine qualitatively whether basal mathematics textbooks adequately addressed the pedagogical issues and research findings on instruction that has been shown to be effective for students with MD or who are at risk for mathematics failure. Each analysis compared lessons of a specific mathematics topic (e.g., subtraction across zeros, division, and fractions) from each basal, with instructional design criteria established from the research findings on effective teaching practice (Dixon, 1992). The findings of these analyses indicated that mathematics basal programs were inadequate for meeting the needs of most students, especially students with learning problems in general education classrooms, even though they were incorporating the recent reform curricular and instructional recommendations.

Considering that textbooks or curricular materials determine most of the instruction implemented in classrooms (Garner, 1992; Porter, 1989), it is unlikely that students with MD in standards-based mathematics classrooms receive appropriate instruction that responds to their difficulties or needs, without general educators' instructional adaptations. It behooves general education teachers to provide instructional adaptations that address the limitations of mathematics basal programs and that respond to the learning characteristics of students with MD. The following section describes the cognitive and performance features of students with MD, which have been identified through previous research.

Students With MD

Researchers have expressed concerns about the learning of students with MD in standards-based mathematics classrooms (Woodward & Montague, 2002). Concerns about including students with MD in standards-based mathematics, general education classrooms come mainly from the fact that the features of standards-based mathematics curriculum and instruction may not be desirable to open the lock of learning of students with MD, due to their cognitive characteristics. For example, students in these classrooms are expected to solve a problem using a variety of problem solutions, which may include varied procedures. To explore or develop diverse solutions involving multistep procedures, students must be at a mastery level of basic math facts or calculation and be able to learn and apply grade-appropriate procedures or strategies for problem solutions. However, research on students with MD has shown that this group of students demonstrates difficulties with arithmetic combinations and calculations (Bryant, Bryant, & Hammill, 2000; Geary, 2004; Gersten & Chard, 1999; Robinson, Menchetti, & Torgesen, 2002) and has difficulties in learning and applying grade-level-appropriate, problem-solving procedures or strategies (Gross-Tsur, Manor, & Shalev, 1996; Hitch & McAuley, 1991; Jordan, Hanich, & Uberti, 2003). The following sections describe weaknesses on facts retrieval, procedural knowledge, and spatial representations of numeral information that have been consistently documented in the literature on the characteristics of students with MD (e.g., Geary, 2004; Swanson & Jerman, 2006).

Fact Retrieval of Students With MD

The most consistent finding in the literature is that students with MD are different from their typically achieving peers in the ability to retrieve arithmetic combinations from long-term memory (Bryant et al., 2000; Geary, 2004; Jordan, Hanich, & Kaplan, 2003a, 2003b). Moreover, the ability to retrieve arithmetic combinations does not substantially improve across the elementary grades for most students with MD, without regards to their reading ability (Geary, 2004). These students commit many more errors and often show error patterns that are sometimes found with students with neurological problems (Ashcraft, Yamashita, & Aram, 1992). Literature on the representations of arithmetic combinations in long-term memory has indicated that difficulties in fact retrieval may be related to the ability to use working memory resources to temporarily store numbers while the student is attempting to produce an answer for an arithmetic facts or combination (Geary, Hoard, Byrd-Craven, & DeSoto, 2004; Swanson & Jerman, 2006).

Procedural Ability of Students With MD

Research findings on the characteristics of students with MD have documented difficulties in executing multistep procedures or developmentally mature procedures for solving a problem as a characteristic of students with MD (Geary, 2004; Swanson & Jerman, 2006). Students with MD often commit miscounting or lose track of the counting process (Geary, Hoard, & Hamson, 1999). For example, Geary and his colleagues (Geary et al. 1999; Geary et al., 2004) found that first-grade students with MD and reading disability (RD) and students with MD committed more counting procedural errors than students with RD and their typically achieving peers. According to Hitch and McAuley (1991), these problems may result from their difficulties in retaining and monitoring the counting process in working memory.

On the other hand, research on the use of strategies of students with MD has shown that students with MD have problems in executing age-appropriate, mature strategies to solve problems.

For example, students with MD, without regard to their reading ability, use the same types of strategies as typically achieving students when solving simple arithmetic problems or simple word problems (Hanich, Jordan, Kaplan, & Dick, 2001). However, their ability to mix strategies and the pattern of developmental change in the strategy mix are different from those of their typically achieving students (Geary et al., 1999; Hanich et al., 2001). As noted, Geary et al. (1999) found that first-grade students with MD and RD and students with MD used developmentally immature counting strategies more frequently than did the RD and typically achieving peers. In the same line, Jordan and Montani (1997) found that students with specific MD performed worse than the typically achieving group on both story and number-facts problems in timed conditions but not in untimed conditions. This result suggests that students with MD relied more on back-up strategies (immature procedures) than did typically achieving students.

Geary et al. (1999) also found that development of strategy use of students with MD was different from that of students with RD and their typically achieving peers. Many students with MD do not show a shift from procedural-based problem solving to memory-based problem solving (Ostad, 1997). Students with MD, without regard to their reading ability, continued to use the finger-counting strategy in the second grade, whereas their typically achieving students and students with RD moved from heavy reliance on the immature finger-counting strategy to verbal counting and retrieval. According to a more recent study by Geary et al. (2004), difficulties in the strategy shift were found to be associated with working memory deficits and poor counting knowledge.

Visual and Spatial Representation of Students With MD

Students with MD also show difficulties in visual and spatial representations of numerical or mathematical information (Geary, 2004). For example, students with MD have difficulties in aligning numbers in multidigit, arithmetic calculation problems, often write numbers in reverse, or show confusions between some numbers, such as 6 and 9. They also show difficulties in areas requiring spatial ability, such as geometry and place values (Swanson & Jerman, 2006). Although the underlying cognitive mechanisms of this difficulty are still under investigation, as with those of the other types of difficulties of students with MD, current research suggests the relatedness of this difficulty to self-regulation (Swanson & Jerman, 2006). This difficulty may be related to the weaknesses of students with MD on monitoring their performances, detecting errors, and correcting errors by themselves.

In summary, cognitive analyses of mathematics skills of students with MD suggest that the students with MD are characterized by difficulties (a) in retrieval of arithmetic facts from long-term memory, (b) in learning and execution of mature or multistep problem-solving procedures or strategies, and (c) in visual or spatial representations of mathematical information. Researchers have attempted to explain these difficulties with underlying mechanisms, including (a) developmental delay in the application of computations or problem-solving procedures or strategies, (b) deficits in working memory, and (c) weakness on self-regulatory skills.

Based on research findings on the characteristics of students with MD, it is likely that many students with MD do not possess sufficient prerequisite skills for participating and learning in standards-based mathematics classrooms. Thus, it is important that alternative instructional methods be considered and implemented for students with MD, who receive their mathematics instruction in standards-based mathematics, general education classrooms but do not possess the prerequisite skills or abilities for mathematics learning in these instructional environments. Also, it is necessary that the alternative instruction involve appropriate evidence-based instructional practices, including the critical features of mathematics interventions for students with MD, which can complement the standards-based mathematics instruction. The next section describes instructional adaptations for students with MD.

Instructional Adaptations for Students With MD

The reauthorization of IDEA in 2004 requires that students with disabilities have access to the general education curriculum in the regular classroom to the maximum extent possible. For attaining the goal of maximum access of students with disabilities to the general education curriculum, it is suggested that teachers implement scientifically based instructional practices as part of their core instruction (IDEA, 2004).

The NRC (Kilpatrick et al., 2001) recommended that general educators provide focused, explicit instruction *along with* problem-solving approaches advocated in recent reform efforts by the NCTM (2000) for successful inclusion of students with MD. However, in today's elementary school classroom, the foci of mathematics instruction tend to be placed on the investigation and development of multiple problem-solving strategies for authentic problems (NCTM, 2000). However, students with MD are not likely to be equipped with the prerequisite knowledge and skills for learning mathematics in these instructional environments, such as communication skills using mathematics language and representations and skills for invention and use of problem-solving strategies (Bryant et al., 2000; Bryant, Kim, Hartman, & Bryant, 2006; Geary, 2004; Jordan, Hanich, & Kaplan, 2003a, 2003b). Thus, general education teachers should adapt their mathematics instruction by integrating direct, explicit instruction into their core instruction in order to address the needs of students with MD and give them successful access to standards-based mathematics, general education curriculum.

According to Fuchs and Fuchs's (2001) framework for the prevention and intervention of mathematics difficulties, the prevention and intervention of mathematics difficulties can be implemented in three levels: (a) primary, (b) secondary, and (c) tertiary. At the primary level, mathematics difficulties are addressed by instruction that is designed to benefit all students, including those with learning problems. Secondary level efforts involve adaptations that are feasible to implement, nondisruptive to the targeted child, and nonintrusive for the rest of the class (Bryant, 2005). Finally, tertiary efforts include intensive and individualized prevention and intervention in the special education.

Based on Fuchs and Fuchs's (2001) framework, general education teachers can contribute to the prevention and intervention of mathematics difficulties at both the primary and secondary levels. Prevention and intervention at the primary level of mathematics difficulties in general education classrooms involves teachers' using scientifically based mathematics practices with their current mathematics curriculum. Prevention and intervention at the secondary level may involve instructional adaptations for those students initially identified as struggling with mathematics based on their individual needs. The following sections provide information about categories of instructional adaptations that can be made in general education classrooms.

Categories of Instructional Adaptations

Adaptations and *differentiations* are terms interchangeably used to mean making curriculum and instruction appropriate for students' learning needs (Bryant & Bryant, 2001; Fuchs et al., 1992; Glaser, 1977). More specifically, the Vaughn Gross Center for Reading and Language Arts (VGCRLA, 2001) described adaptations as appropriate adjustments, accommodations, and modifications to instruction or supports that allow students to meet academic requirements and conditions of the curriculum, most often the general education curriculum. With instructional adaptations, teachers assess individual students' success of the previous lessons and adapt subsequent instructional strategies, materials, or goals to meet the abilities and needs of all students, especially struggling learners within a classroom (Glaser, 1977; Gunter, Denny, & Venn, 2000). Adapted instruction occurs based on on-going assessments (e.g., progress monitoring).

Adaptations occur in four different categories: (a) content, (b) activities, (c) delivery, and (d) technology or materials to help students master instructional objectives (Bryant & Bryant, 1998). Within Bryant and Bryant's (2001) adaptations framework (AF), the first adaptation category is instructional content, which consists of the instructional objective, the state's curriculum, and the school district curriculum. The AF's second adaptation category is instructional activity, which is the procedure, intervention, or strategy used to teach the content. The AF's third adaptation category is instructional delivery, which consists of how the activity is taught, such as grouping practices, the instructional steps, and the instructional language. The AF's last adaptation category is instructional materials and technology, including textbooks, manipulatives, and instructional and assistive technology devices. Besides adapting instruction according to the instructional adaptation categories, teachers can change their instruction in terms of intensity (e.g., the increased amount of time spent with a struggling student) to help students master instructional objectives.

The effects of instructional adaptations depend on how instruction is modified in response to individual students' needs (Fuchs et al., 1992). Research in special education has identified critical features of instruction that produce positive results and large effect sizes when teaching students with learning disabilities. These features of instruction that have been demonstrated as effective for students with MD are described in the following section.

Critical Features of Effective Instruction for Students With MD

Meta-analyses of mathematics intervention research have shown that direct, explicit instruction combined with strategy instruction produce large effects on student's mathematics learning (S. Baker, Gersten, & Lee, 2002; Kroesbergen & Van Luit, 2003; Miller, Butler, & Lee, 1998; Swanson et al., 1999; Xin & Jitendra, 1999). For example, Swanson et al. (1999) found that a combined direct instruction and strategy instruction model was an effective procedure for remediating LD students when compared to other models. The findings of their meta-analysis also included instructional components such as grouping or sequencing instruction, which were critical to treatment outcomes across academic areas. Even though their study did not separate the findings on mathematics instruction from the findings on other academic instructions, the significance of the critical features of effective instruction identified by Swanson et al. (1999) in teaching mathematics to students with MD have been supported through the findings of other syntheses or meta-analyses (S. Baker et al., 2002; Kroesbergen & Van Luit, 2003; Miller et al., 1998; Xin & Jitendra, 1999) conducted on research findings of mathematics intervention studies. Definitions of the critical components of effective instruction and links between the critical components identified by

Swanson et al. and the findings from other meta-analyses of mathematics intervention are shown in the glossary.

However, the standards-based mathematics classrooms where many students with MD are likely to receive mathematics instruction today emphasize student-centered, inquiry-based, and contextualized problem-solving instruction like those of their typically achieving peers, rather than direct instruction paired with strategy instruction suggested by literature in special education. Thus, it is necessary for general education teachers to incorporate the critical features of effective mathematics instruction into standards-based mathematics instruction in order to assist students with MD in attaining better understanding of mathematics concepts or skills in these environments. The critical features of effective mathematics instruction can be incorporated into standards-based instruction, which is grounded in socioconstructivist understanding of how people learn.

Statement of the Problem

Despite increased efforts to reform U.S. mathematics education over the last two decades (e.g., NCTM, 2000), recent national and international assessment data indicate the need for additional efforts to improve the performance of U.S. students on mathematics. For example, according to the latest international comparison of mathematics performances of fourth graders (Braswell et al., 2003; PISA, 2003), U.S. fourth graders do not perform as well as their peers in other developed countries. Further, the standing of U.S. fourth graders relative to their peers in 14 other countries has degraded over time (NCES, 2004). For example, in 1995, U.S. fourth graders were outperformed in mathematics by fourth graders in 4 of 14 countries, but in 2003 on average, U.S. fourth graders were outperformed by fourth graders in 7 of these countries. Further, the results from national assessment data indicate that a significant proportion of U.S. students do not attain the proficient level of understanding of the mathematics concepts and procedures at a given grade, and the percentage of struggling students gets larger as they grow older (Perie et al., 2005). For example, approximately 70% of students in eighth grade did not show the proficient level of understanding of mathematics procedures and concepts on the nationally representative and continuing mathematics assessment of 2005, but 64% of students in fourth grade did (Perie et al., 2005). Considering that increased numbers of students will struggle as they get older, it is important to search for ways to help students be successful before they reach the higher grades, where they will be expected to learn more complicated, advanced mathematics knowledge and skills.

In today's classrooms, struggling students, including those with MD, receive most portion of daily mathematics instruction in their general education classrooms (U.S. Department of Education, 2002). Contemporary mathematics curriculum and instruction is under the influence of the recent reforms in mathematics education (e.g., NCTM, 2000), which emphasize inquiry-based, discourse-driven, and contextualized problem-solving instruction. By 2000, 49 out of 50 states had adopted a reformed standards-based framework to establish their state-level standards for mathematics curriculum and evaluation of student performances, such as the Texas Essential Knowledge and Skills (TEKS), as noted by Lappan (2000). In Texas, the TEKS for mathematics has evolved to reflect the NCTM reform efforts and has provided the standards for school mathematics since September1998 (TEA, 2006). Consequently, it is expected that many U.S. students with MD are receiving a significant portion of their daily mathematics instruction in standards-based mathematics, general education classrooms.

However, professionals in the field of MD and related fields have expressed concerns about placing students with MD in standards-based mathematics, general education classrooms for mathematics instruction without supplemental assistances to address individual needs of students with MD. Most of these concerns are based on the findings of intervention studies or meta-analyses of the studies on effective mathematics interventions conducted with students with MD and studies on cognitive and behavioral characteristics of students with MD (e.g., Swanson et al., 1999).

First, syntheses or meta-analyses of mathematics intervention research conducted with students with MD have shown that direct, explicit instruction combined with strategy instruction produced large effects on mathematics learning of students with MD (S. Baker et al., 2002; Kroesbergen & Van Luit, 2003; Miller et al., 1998; Swanson et al., 1999; Xin & Jitendra, 1999). Specifically, interventions demonstrated as effective for students with MD involved instructional features aligned with a direct, explicit instructional approach or strategy instructional approach, such as explicit modeling, sufficient number of teaching examples, direct questioning, corrective feedback, and explicit teaching of cognitive or metacognitive strategies. Accordingly, concerns can be raised about placing students with MD without complementary assistances in standards-based mathematics classrooms where indirect, implicit instruction are pervasive.

Second, the findings of research on cognitive and behavioral characteristics of students with MD have indicated that students with MD are not likely to learn mathematics knowledge and skills as well as students without disability, unless they receive supplemental assistance to address their difficulties in background knowledge and skills. To benefit from standards-based mathematics instruction where students are expected to learn mathematics skills through activities based on their own inquiry or class discourse, students should be equipped with the prerequisite skills for experimenting or exploring their own ideas in a systematic way and should be able to understand and apply at least basic arithmetic knowledge and procedures included their experimentations or explorations. Also, students should have the ability to understand their peers or their teacher's ideas and get the gist of issues at discussion.

However, the findings of research on cognitive and behavioral characteristics of students with MD have indicated that this student population has deficits in the skills of retrieving arithmetic facts, using multistep strategies or procedures to solve problems, and spatially representing mathematical information (e.g., Geary, 2004; Swanson & Jerman, 2006). These difficulties are related to the deficits in the abilities of keeping information in working memory or monitoring their working processes. Accordingly, it is not likely that students with MD bring prerequisite skills or minimum skills required for learning mathematics in standards-based mathematics curriculum and instruction. In fact, studies using students with MD in standards-based mathematics classrooms have suggested that this student population struggles with task demands of standards-based mathematics instruction and tends to be disengaged in standards-based classroom discussions focusing on the sharing of problem-solution strategies (Baxter, Woodward, Voorhies, & Wong, 2002). These studies have suggested that students with MD will not learn mathematics as well as their typically achieving peers in a standards-based mathematics classroom without complementing that instruction with direct, explicit instruction paired with strategy instruction.

Based on the recent revision of IDEA (2004), general education teachers are required to adapt their instruction using evidence-based instructional features in order to assist students with disabilities in having access to the general education core curriculum to the maximum extent. Therefore, general education teachers should make adaptations of their standards-based mathematics instruction using instructional features demonstrated as effective for teaching students with MD to help these students access the general education curriculum.

Given the concerns about placing students with MD in standards-based, general education classrooms for their daily mathematics instruction without complementary instruction as well as the legislative mandates requiring instructional adaptations for students with MD using evidence-based instructional features in the instructional setting, it was important to examine how general education teachers were actually complementing standards-based mathematics instruction for students with MD and to investigate what the access of students with MD to the mathematics general education curriculum and instruction looked like in standards-based mathematics classrooms. Because those two research topics were inseparable, interwoven within an integrated, bounded system of the standards-based mathematics, general education classroom (Stake, 1995), it was desirable to investigate both research questions within a study. Therefore, the purpose of this study was twofold: First, this study would investigate how a fourth-grade general education teacher would adapt standards-based mathematics instruction for 3 students with MD. Second, this study would examine how fourth-grade students with differing levels of ability (3 identified as MD, 2 struggling, and 1 without disability) would learn grade-level mathematics knowledge and skills in a standards-based mathematics classroom. In this study, mathematics knowledge and skills are defined as a construct consisting of mathematics concepts and procedures including key concepts, strategies, representations, and algorithms (Ginsburg & Pappas, 2004).

Significance of the Problem

Although standards-based mathematics instruction has been recommended as a promising instructional practice for enhancing mathematics knowledge and skills of all students, including students with MD, at issue is mathematics learning of students with MD placed in these instructional environments. For this group of students, general education teachers are required to modify their instruction using evidence-based instructional features so that they can serve instructional needs of students with MD by the law (IDEA, 2004).

However, little research has examined whether and how general education teachers adapt their instruction for students with MD within standards-based mathematics classrooms and what the mathematics learning of students with MD look like within the instructional environments. To have better understanding of promising mathematics instruction for students with MD in general education setting, it is important to know how they are taught in today's standards-based mathematics, general education classrooms and how they make progress in this instructional environment in comparison to their peers. Research on these topics may provide implications for further understanding of why and how students with MD struggle in standards-based general education classrooms and how general education teachers can help these students' learning.

Purposes of the Research

The purpose of this study was to investigate (a) how a fourth-grade general education teacher adapted her mathematics instruction within a standards-based mathematics curriculum for 3 students with MD and who received mathematics instruction in general education classroom, and (b) what the mathematics learning of students with MD in standards-based mathematics, general education classroom looked like compared to their peers with differing levels of ability. Thus, the investigation was guided by the following two research questions:

Research Question 1: How does a fourth-grade general education teacher adapt mathematics instruction within a standards-based mathematics curriculum for students with MD who have an Individualized Education Program (IEP) in mathematics and who receive mathematics instruction in the general education classroom?

Research Question 2: How do fourth-grade students with different abilities (3 identified as MD, 2 struggling, and 1 without disability) who receive mathematics instruction in a standards-based mathematics, general education classroom use mathematics knowledge and skills taught in class to solve curriculum-based problems after they have received classroom instruction?

CHAPTER 2:

LITERATURE REVIEW

According to the recent effort for improving IDEA (2004), students with disabilities, including students with MD are entitled to have access to general education curriculum to the maximum extent possible and are expected to make progress in the general education curriculum. By the law, general education teachers are expected to help students with disabilities access the general education curriculum by implementing instructional adaptations in which evidence-based instructional practices are integrated into the core curriculum (IDEA, 2004).

In today's mathematics education, standards-based mathematics curriculum and instruction have gained in popularity and implementation as promising for teaching all students, including students with disabilities (Lappan, 2000; NCTM, 2000). However, in the professional community related to students with MD, concerns have been raised about whether standards-based mathematics curriculum and instruction is within easy access of students with MD who suffer from the lack of some significant cognitive skills necessary for learning grade-appropriate mathematics knowledge and skills, and whether relying on standards-based mathematics instruction alone is sufficient to teach students with MD (e.g., Baxter et al., 2001; Rivera, 1993; Woodward & Baxter, 1997; Woodward & Montague, 2002). Rather, it has been suggested that general education teachers should adapt their core instruction for students with MD by incorporating evidence-based instructional components into standards-based mathematics instruction to help them meet the learning expectations of standards (Maccini & Gagnon, 2000).

However, the extent to which instructional adaptation for students with MD in inclusive general education environments is occurring or how adaptations are being implemented and what the mathematics learning of students with MD looks like within standards-based mathematics curriculum and instruction in comparison to their typically achieving peers have received little attention in the literature. Thus, this study investigated (a) how a fourth-grade general education teacher adapted mathematics instruction for her 3 students with MD in standards-based mathematics classroom, and (b) what mathematical knowledge and skills of students with differing levels of ability looked like in the standards-based mathematics general education classroom.

This literature review was conducted to inform and support this study. To inform the first research questions, previous research on (a) the necessity or importance of instructional adaptations for students with MD in standards-based mathematics general education classroom, and (b) the instructional adaptations supported by evidence in the literature were reviewed. To inform the second research question, this study reviewed studies on (c) development of students' geometrical thinking, and (d) the transfer of mathematics knowledge and skills to new problems. First, the necessity of instructional adaptations for students with MD in standards-based mathematics general education classroom was grounded by three bodies of studies on (a) the features of standards-based mathematics curriculum and instruction, (b) the characteristics of students with MD, and (c) the implications from analyses of standards-based mathematics curriculum programs. Second, instructional practices for instructional adaptations were found in two bodies of literature including (a) studies on the aspects of instruction that can be changed or modified, and (b) studies on the components of instruction that can be integrated into the core curriculum to promote the learning of students. Third, this study reviewed a model of students' geometrical thinking to inform this study. Finally, literature on transfer of mathematics knowledge and skills across problems having different similarity was reviewed in this study.

The Necessity of Instructional Adaptations for Students With MD in the Standards-Based Mathematics General Education Classroom

In this section, three bodies of literature were reviewed to support the necessity of instructional adaptations for students with MD in standards-based mathematics general education classroom. The literature included studies on (a) the features of standards-based mathematics instruction, (b) the characteristics of students with MD, and (c) analyses of standards-based mathematics curriculum programs.

The Features of Standards-Based Mathematics Instruction

In mathematics education, there have been long-standing efforts to reform school mathematics (Woodward & Montague, 2002). The efforts resulted in standards documents for school mathematics which were published by national, professional organizations including; (a) *Principles and Standards for School Mathematics* (NCTM, 2000), (b) the NAEP (1996) *Mathematics Framework for the 1996 and 2000 National Assessment of Educational Progress*, (c) the American Association for the Advancement of Science (1989) *Mathematics: Report of the Project 2061 Phase I Mathematics Panel*, and (d) the National Research Council's *Adding it up: Helping Children Learn Mathematics* (Kilpatrick, Swafford, & Findell, 2001).

Among those standards documents, the latest version of NCTM (2000) standards reflected a decade of these reform efforts in mathematics education (Woodward & Montague, 2002) and have been a framework for establishing standards for mathematics education across states, including the mathematics portion of TEKS (Lappan, 2000). This section described the features of instruction envisioned by the NCTM standards and principles for school mathematics, which for this study was supposed to be met by the criteria set for the mathematics portion of TEKS.

Even though the degree of emphasis on socio-constructivistic instructional features varied across standards documents, standards documents for mathematics instruction, including the NCTM (2000) standards, were built on the beliefs that mathematics instruction should be based on student inquiry-based mathematics problem-solving activities. For example, mathematics instruction envisioned in the recent reform (NCTM, 2000) was described as student-centered, inquiry-based, and discourse-driven instruction in contexualized problem-solving instruction (Forman & Steen, 2000; Woodward & Montague, 2002). Student-centered instruction uses problems that students see as relevant and interesting that help students learn to work with others and that strengthen students' technical communication skills (Forman & Steen, 2000). Inquiry-based instruction encourages students to explore and discover a variety of strategies for problem-solutions, and to investigate available data for problem solving (Forman & Steen, 2000). Discourse-driven instruction encourages students to learn mathematics concepts or procedures by engaging in the construction of shared mathematics knowledge in their classrooms, which is usually accomplished by verbal interactions between a teacher and students or among students (Baxter et al., 2001).

In addition, student-centered, inquiry-based, and discourse-driven instruction occurs as contextualizd instruction. Contextualized instruction involves students engaging with problems first in context, then with mathematical formality, and encourages students to see the connections of mathematics to work and life (Forman & Steen, 2000). In these classrooms, tasks are more likely to resemble those found in everyday life or in the workplace than those found in school textbooks. Students need to think about each problem, without the clues provided by a specific textbook chapter. Rather than just being asked to solve an equation or calculate an answer as in traditional classrooms, students in the standards-based mathematics classrooms are asked to design, plan, evaluate, recommend, review, define, critique and explain, all of which will be needed to do their future job (Forman & Steen, 2000). Tasks are often non-routine and open-ended, with solutions taking from minutes to days, and requiring diverse forms of presentation (e.g., oral, written, video, or computer) (Forman & Steen, 2000).

This form of instruction may be effective for students who have intuition or abilities for independently developing problem-solutions, or skills for higher levels of psychological processing (e.g., ability to evaluate solutions). However, students with MD have been shown to be less likely to have adequate ability to benefit from these instructional environments (Bryant et al., 2000; Geary, 2004). Thus, it should be considered that to ensure learning and access to the curriculum, instructional adaptations need to be implemented for students with MD in standards-based mathematics instruction to address the specific difficulties of students with MD in these environments.

Cognitive Characteristics of Students With MD

Even though all students, including students with MD, are expected to attain the same learning goals in standards-based mathematics instruction, it has been revealed in literature on students with MD that students with MD are different from their peers who don't have disabilities in significant information processing abilities such as working memory. For example, research has shown that this group of students demonstrates difficulties in basic mathematics skills including arithmetic combinations and calculations (Bryant et al., 2000; Geary, 2003; Gersten & Chard, 1999; Robinson, Menchetti, & Torgesen, 2002) and have difficulties in learning and applying grade-level appropriate problem-solving procedures or strategies (Gross-Tsur, Manor, & Shalev, 1996; Hitch & McAuley, 1991; Jordan, Hanich, & Uberti, 2003), which are required for learning mathematics in standards-based mathematics instruction. For example, Bryant and her colleagues (2000) examined the specific mathematics behaviors of students with LD who have teacher-identified math weaknesses, in order to identify the characteristics that discriminate students with potential MD. This study identified nine behaviors as most predictive of the classification of mathematics weakness for students with MD. The behaviors included (a) having difficulty with multi-step problems, (b) making "borrowing"(e.g., regrouping) errors, (c) being unable to recall number facts automatically, (d) misspelling number words (e.g., threeteen, twoty), (e) reaching "unreasonable" answers, (f) poorly calculating when the order of digit presentation is altered, (g) failing to copy numbers accurately, (h) ordering and spacing numbers inaccurately in multiplication and division, and (i) failing to remember number words or digits.

From a constructivist developmental point of view of mathematical learning, some difficulties of students with MD are described as developmental lags (Ginsburg, 1997). Whereas, from the viewpoint of information processing process theory on student learning, some difficulties, particularly weaknesses in fact retrieval are thought of as skill deficits (Jordan, Hanich, & Uberti, 2003). Research has shown that students with MD experience developmental lags in application of calculation strategies or procedures and deficits in fact retrieval. For example, Russell and Ginsburg (1984) found that fourth grade students with MD performed qualitatively similar to third-grade students without MD in terms of strategies for metal calculation, the ability to solve simple story problem, and the ability to solve written calculation. In other words, procedures that students with MD use to solve mathematics problems were very similar to procedures that typically achieving younger students use. On the other hand, weaknesses in fact retrieval are found among students with MD throughout their elementary school grades (Ostads, 1998). Without regard to their other abilities (e.g., reading ability), students with MD have difficulties in retrieving math facts from their long-term memory (Jordan & Hanich, 2003; Jordan, Hanich, & Kaplan, 2003).

Whether a developmental lag or skill deficit, it is evident that students with MD are different from their typically achieving peers in dealing with mathematics information. In the following sections, the difficulties of students with MD were described in three aspects (e.g., Geary, 2004; Swanson & Jerman, 2006) that have consistently been documented: (a) fact retrieval, (b) procedural knowledge, and (c) spatial representation of numerical information.

Fact Retrieval of Students With MD

The most consistent finding in the literature is that students with MD are different from their typically achieving peers in the ability to retrieve arithmetic combinations from long-term memory (Bryant et al., 2000; Geary, 2004; Jordan et al., 2003a; Jordan et al., 2003b). Moreover, the ability to retrieve arithmetic combinations does not substantially improve across the elementary grades for most students with MD without regard to their reading ability (Geary, 2004). These students commit many more errors and often show errors patterns that are similar to those found in some students with neurological problems (Ashcraft, Yamashita, & Aram, 1992). Literature on the representation of arithmetic combinations in long-term memory indicates that difficulties in fact retrieval may be related to the ability to use working memory resources to temporarily store numbers while the student is attempting to produce an answer for an arithmetic fact or combination (Geary, Hoard, Byrd-Craven, & DeSoto, 2004; Swanson & Jerman, 2006).

Procedural Ability of Students With MD

Research findings on the characteristics of students with MD document difficulties in executing multi-step procedures and/or developmentally mature procedures for solving a problem as a characteristic featuring the students with MD (Geary, 2004; Swanson & Jerman, 2006). Students with MD often commit counting errors or lose track of the counting process (Geary, Hoard, & Hamson, 1999). For example, Geary and his colleagues (1999) found that first-grade students with mathematics disabilities (MD)/reading disability (RD) and students with MD committed more counting procedural errors in counting than students with RD and their typically achieving peers (Geary et al., 1999; Geary et al., 2004). According to Hitch and McAuley (1991), these problems may result from their difficulties in retaining and monitoring the counting process in working memory.

On the other hand, research on the use of strategies of students with MD has shown that students with MD have problems in executing age-appropriate, developmentally mature strategies to solve problems. For example, students with MD, without regard to their reading ability, use the same types of strategies as typically achieving students during solving simple arithmetic problems or simple word problems (Hanich, Jordan, Kaplan, & Dick, 2001). However, their ability to mix strategies and the pattern of developmental change in the strategy mix are different from those of their typically achieving peers (Geary, Hoard, & Hamson, 1999; Hanich et al., 2001). For example, Geary and his colleagues (1999) found that first-grade students with mathematics disabilities(MD)/reading disability (RD) and students with MD used developmentally immature counting strategies more frequently than did the RD and typically achieving peers. Along the same lines, Jordan and Montani (1997) found that students with specific MD performed worse than the typically achieving group on both story and number-facts problems in timed conditions but not in untimed conditions. This result suggests that students with MD relied more on back-up strategies (immature procedures) than typically achieving students.

Geary and his colleagues (1999) also found that the development of strategy use of students with MD was different from that of students with RD and their typically achieving peers. Many students with MD do not show a shift from procedural-based problem solving to memory-based problem solving (Ostad, 1997). Students with MD, without regard to their reading ability, continued to use finger-counting strategy in the second grade, while their typically achieving peers and students with RD progressed from a heavy reliance on an immature finger-counting strategy to verbal counting and retrieval. According to a recent study by Geary and his colleagues (2004), difficulties in this strategy shift were found to be associated with working memory deficits and poor counting knowledge.

Visual and Spatial Representation of Students With MD

Students with MD also show difficulties in visual/spatial representations of numerical or mathematical information (Geary, 2004). For example, students with MD have difficulties in aligning numbers in multi-digit arithmetic calculation problems, and often write numbers in reverse or show confusions between some numbers such as 6 and 9. They also show difficulties in areas requiring spatial ability such as geometry and place values (Swanson & Jerman, 2006). Although the underlying cognitive mechanisms of this difficulty are still under investigations as are those with other types of difficulties common to students with MD, current research suggests the relatedness of this difficulty to self-regulation (Swanson & Jerman, 2006). This difficulty may be related to the weaknesses of students with MD on monitoring their performances, detecting errors, and correcting errors by themselves.

In summary, cognitive analyses of the mathematics skills of students with MD suggest that the students with MD are characterized by difficulties (a) in retrieval of arithmetic facts from long-term memory, (b) in learning and execution of mature or multi-step problem-solving procedures or strategies, and (c) in visual or spatial representations of mathematical information. Researchers have attempted to explain these difficulties with underlying mechanisms including (a) developmental delay in the application of computations or problem-solving procedures or strategies, and (b) deficits in working memory.

Based on research findings on the characteristics of students with MD, it is highly likely that many students with MD do not possess sufficient prerequisite skills for participating and learning in standards-based mathematics classrooms. Accordingly, it is expected that standards-based mathematics basal programs which account for the majority of teachers' instruction will include instruction to address the difficulties of students with MD in prerequisite skills for learning mathematics in standards-based mathematics classrooms.

Standards-Based Mathematics Curriculum Programs

Textbooks serve as critical tools for knowledge acquisition in school and are primary sources of information (Garner, 1992). Specifically, mathematics curricular materials account for about 75% of what is taught in mathematics instruction (Porter, 1989). It is possible that many students struggle with mathematics just because mathematics textbooks do not meet their instructional needs (Gickling & Thompson, 1985; Jitendra et al., 1996). This study reviewed four analytic studies of standards-based mathematics basal programs to provide information about how the difficulties of students with MD were addressed in the mathematics basal programs.

Four descriptive analyses of mathematics basal programs identified for this review were conducted by Jitendra and her colleagues (Carnine & Jitendra, 1997; Jitendra et al., 1996; Jitendra et al., 1999; Jitendra et al., 2005). Although there were differences in mathematics topics and grades analyzed among those studies (i.e., teaching division in the fifth grade, teaching fractions in the fifth-grade, teaching subtraction across zeros in the fourth-grade, and teaching problem-solving in third grade), these studies, generally, purported to qualitatively examine whether incorporating the standards was sufficient to meet the instructional needs of low performing students and students with disabilities.

Instructional design criteria or variables, serving as instructional criteria for the four comparative analyses of mathematics basal programs, were established from the literature on effective teaching practices (Dixon, 1992). The two earlier studies (Carnine & Jitendra, 1997; Jitendra et al., 1996) employed similar criteria for basal comparisons, while the latter two analyses (Jitendra et al., 1999; Jitendra et al., 2006) used additional criteria combining criteria from the two previous studies. Criteria for examining the adequacy of basal mathematics programs for struggling students included: (a) specification of objectives, (b) number of additional concepts taught, (c) prerequisite skills knowledge, (d) explicit explanation, (e) instructional efficiency (i.e., time for manipulative activities), (f) appropriateness and adequacy of teaching examples, (g) adequacy of practice, (h) appropriateness of review, and (i) effective feedback.

Findings of the four basal analyses indicated that basal mathematics textbooks did not adequately address the pedagogical issues and research findings on students with MD and students who are at-risk for failure in mathematics. Incorporating the standards in mathematics education may not be sufficient to meet the instructional needs of low performing students and students with disabilities. For example, the findings of Jitendra and her colleagues (1996) showed that the 1990s' editions had pedagogical deficits similar to their 1980 editions including: (a) prior knowledge was rarely addressed, (b) content was introduced too rapidly, (c) coherence of information taught was lacking, (d) teaching demonstrations were often not explicit and straightforward, and (e) practice and review were often inadequate. Likewise, Carnine and Jitendra (1997) showed that the fifth-grade basal programs published after the NCTM standards did not satisfy effective instructional design criteria for teaching mathematics (e.g., insufficient identification and integration of big ideas, rapid introduction of content, teaching demonstrations that were not clear and explicit, open-ended and ineffective manipulative activities, and inadequate review). In the study of Jitendra and her colleagues(1999), it was found that only three of seven mathematics basal programs satisfied most instructional design criteria (i.e., 7 or 8 of 9 criteria) for effective teaching practices. Of the 9 criteria, some person decided or research evidence showed that 4 variables were far more important for the success of students with MD and those at risk. The following four instructional design features were identified as the instructional components that should be modified for diverse learners across all basal mathematics programs. The instructional features include (a) clarity of objective, (b) explicit teaching explanations, (c) sufficient and appropriate teaching examples, and (d) effective feedback. Similarly, Jitendra et al. (2005) found that only two textbooks met most instructional design criteria. Instructional design criteria of (a) clarity of objectives, (b) sufficient teaching examples, and (c) nonexamples were not met in more than half of the textbooks analyzed in the study.

In summary, the findings of these analyses indicated that mathematics basal programs were inadequate for satisfying the needs of most students, especially students with learning problems in general education classrooms. For successful inclusion, general education teachers should make adaptations in instructional methods and materials to address not only skill deficits of students with learning problems but also the limitations of mathematics basal programs. In fact, studies on students with mathematics difficulties indicated that without additional supports, struggling students may not make progress in standards-based mathematics general education classrooms as expected by the standards. Thus, it is important that alternative instructional methods be considered and implemented for students with MD, who receive their mathematics instruction in standards-based mathematics general education classrooms but who do not possess the prerequisite skills or abilities for mathematics learning in these instructional environments. Also, it is necessary that the alternative instruction involve appropriate evidence-based instructional practices including the critical features of mathematics interventions for students with MD, which can complement the standards-based mathematics instruction. The next section will describe instructional adaptations for students with MD which had been recommended from literature.

Suggestions for Instructional Adaptations

Instructional differentiation (or adaptation) is described as teachers' assessing individual students' success of previous lessons and adapting subsequent instructional strategies, materials, or goals to meet the abilities and needs of all students, especially struggling learners within a classroom (Glaser, 1977; Gunter, Denny, & Venn, 2000). Instructional adaptation includes instructional modification to meet individual students' needs based on-going assessments of student progress. When teachers systematically modify instruction in response to individual students and objective, on-going assessment data, struggling learners learn reliably and more readily (Fuchs, Fuchs, Hamlett, Phillips, & Bentz, 1994; Fuchs, Fuchs, Hamlett, & Stecker, 1991; Jones & Krouse, 1988). Although the topic of instructional adaptations has not been studied sufficiently for its importance in education, some studies have been conducted on the tendency of general education teachers to make instructional adaptations (e.g., J. M. Baker & Zigmond, 1990; Fuchs et al., 1992; Fuchs et al., 1995; McIntosh et al., 1993), on the instructional aspects or categories in which instructional adaptations would be implemented (e.g., Bryant & Bryant, 2001), and on the instructional components or features that can be integrated in the core instruction for teaching students with MD (e.g., Swanson, Hoskyn, & Lee, 1999; Maccini & Gagnon, 2000). This section will provide reviews of (a) the findings of studies on general education teachers' instructional adaptations, (b) the categories of instructional adaptations, and (c) evidence-based effective instructional components that can be integrated into core instruction.

Findings on General Educators' Instructional Adaptations

Research on instructional adaptations provides information about the patterns or the features of general educators' instructional adaptations. Previous findings from this area of research suggested that general educators did not naturally and sufficiently adapt their instruction for struggling students. For example, J. M. Baker and Zigmond (1990) found that elementary teachers did not change their planned instruction significantly for struggling students. The study included teacher interviews and observations of reading and mathematics classes in one elementary school to examine teachers' use of routine adaptations. The results of this study showed that the teachers taught in single, large groups and did not adapt their instruction for individual students with special needs.

Related to the finding of the above study, Fuchs et al. (1992) also found that instructional adaptations for low-achieving students occurred insufficiently in general education settings. For example, among 110 elementary or middle school general education teachers, each of whom taught reading or mathematics to at least one student with a learning disability, only one in four made a revision in their six week instructional plans for their students with LD. A similar finding was found in a study by Fuchs et al. (1995). Even though general educators were specifically prompted and supported to engage in specialized adaptation, only 17% of the adaptations targeted students with LD exclusively.

In addition, general educators' instructional adaptations are not based on the needs of struggling students. For example, Fuchs et al (1992) found that the types of mathematics teachers' instructional adaptations fell at disappointingly lower levels compared to reading modifications (i.e., 77% of the instructional adaptations that occurred during mathematics instruction were categorized as lowering expectations, rather than attempting to improve their programs). Moreover, the extent to which mathematics teachers established their on-going routines to accommodate adaptations (i.e., use of more than one instructional group, varied goals, and diverse materials) was related more to teachers' skills in managing students than to students' learning problems. Fuchs et al.(1995) conducted research to examine 40 general educators' specialized adaptation in mathematics for students with LD. Random assignment procedure was employed to assign teachers to two treatment groups: routine adaptation (use of curriculum-based measurement and peer-mediated instruction) and routine plus specialized adaptation (prompting and special support to implement adjustments in response to individual student difficulty). One of their findings showed that the quality of teachers' specialized adaptation was not associated with enhanced learning of students with LD.

Categories of Instructional Adaptations

There have been some studies whose findings suggested categories of instructional adaptations that general educations teachers should implement. First, Glaser (1977) categorized teachers' instructional adaptations according to the degree to which a method was adaptive. Within Glaser's (1977) framework of adaptive education, the degree to which a method could be considered as adaptive depended on the number of alternative teaching actions available and the degree to which alternative actions were chosen to fit the learner's readiness to profit from them. In this framework, repeating the lesson and increasing practice opportunities, with which teachers just extended the planned instructional routine instead of adapting instruction for individual struggling students, were regarded as the minimal level of adaptation. Examples of mid-level (i.e., third level) of instructional adaptations, which required alternative instructional routes to common goals, included change of instructional materials and revision of instructional strategies. With these two types of adaptations, teachers could reformulate and adapt their instruction for individual students. The modification of instructional grouping was another example of mid-level of instructional adaptation. Changing grouping did not require reformulation of instruction, but was relatively intrusive to classroom organization and provided alternative routes to common goals. Changing goals may be a relatively high level of adaptation within Glaser's framework, because it resulted in changes in other pedagogical variables such as grouping assignment, instructional technique, and practice time.

Second, Fuchs et al.(1992) categorized instructional adaptations made by general educators into two groups: routine adaptation and specialized adaptation. Routine adaptation referred to the extent to which teachers established their initial routines to facilitate ongoing adaptation or varied goals. This category of adaptations included varied types or levels of adaptations in materials, grouping arrangements, and goals, which teachers established at the beginning of the year within their standard routines because they anticipated a need for adapted instruction to accommodate ability differences among students (Fuchs, Fuchs, Hamlett, Phillips, & Karns, 1995). Peer-mediated instruction was an example of a routine adaptation. Whereas, specialized adaptation referred to how teachers modified planned instruction beyond their routine adaptation in light of specific student difficulty. When a student with LD responded poorly to the planned instruction, a general educator adjusted instructional components such as goals, materials, teaching activities, grouping, time, and motivational strategies to satisfy the student' needs.

Third, Cole, Horvath, Chapman, Deschenes, Ebeling, and Sprague (2000) identified nine types of instructional adaptations that could be utilized to assist students including students with MD in successfully gaining knowledge within general education classrooms. Specific categories of instructional adaptations included (a) input, (b) output, (c) size, (d) time, (e) difficulty, (f) level of support, (g) degree of participation, (h) modified goals, and (i) substitute curriculum. Input adaptations involved changes in instructional strategies used to facilitate student learning. This type of adaptations may include modifications such as computer-assisted instruction, media-anchored instruction, and manipulative or learning aid to support active learning. Output adaptation was related to altering the way that learners demonstrated understanding and knowledge. For example, students may write a song, tell a story, or perform an experiment rather than write their knowledge with pencil. Size adaptations included modification of the length or portion of an assignment, or performance that learners are expected to complete. Time adaptations were related to providing the flexible time needed for students' learning. With this type of adaptation, teachers changed the pacing of instruction or provided an individualized timeline for project completion. Difficulty of a task may be modified by adapting the skill levels, the problem type, or the conceptual levels and processes involved in learning. Allowing a struggling student to use a calculator was an example of difficulty adaptations. Adapting the level of support referred to modify the amount of social, personal, material, or physical assistance to the learner. Co-operative group activities are one way to modify the level of support provided by whole-group instruction. Along with the above types of adaptations, teachers may modify degree of participation, learning goals, or curriculum for individual students with special needs. For example, a teacher may have a struggling student play a part that has more physical action rather than numerous lines to memorize (degree of participation), change an outcome expectations of the student within the context of a general education curriculum (modified goals), or substitute curriculum and instructional materials to meet the learner’s identified goals.

Fourth, Bryant and Bryant (2001) developed the Adaptations Framework (AF), which is composed of four components: setting-specific demands, student-specific characteristics, instructional adaptations, and evaluation. In this framework, instructional adaptations are proposed through comparison of setting-specific demands and students-specific characteristics. When a student is unable to successfully meet the demands of the setting (e.g., abilities to be needed for completion of a task), this framework suggested instructional adaptations, which are individualized, relevant, and effective. Within this framework, instructional adaptations can occur in at least one of four categories including; (a) instructional content, (b) instructional activity, (c) delivery of instruction, and (d) materials and technology. Instructional content means skills and concepts that are the focus of teaching and learning. The instructional content is related to the instructional objective and the state's curriculum. Instructional activity refers to the procedure, lessons, or strategy to teach the content. Instructional delivery consists of how the activity is taught, such as instructional grouping, instructional routines, and instructional language. Finally, materials include textbooks or other manipulatives used for mathematical representation, and technology includes computer software for drill and practice on basic math facts, calculators, or internet facility used for mathematics activities, for example. Table 2.1 summarized the criteria for categorizing instructional adaptations used by these researchers. This table shows that instructional adaptation or modification can be made in multiple dimensions of the construct such as the extent to which new instruction deviates from typical instruction to meet struggling students' needs as well as a variety of instructional variables.

Table 2.1

The Criteria for Categorizing Instructional Adaptations

Authors	Criteria
Glaser (1977)	The degree to which a method is adaptive. The degree of adaptiveness depends on the number of alternative teaching actions available and the degree to which alternative actions are chosen to fit the learners' readiness to profit from them.
Fuchs et al. (1995)	The extent to which teachers establish their initial routines to facilitate ongoing adaptation or varied goals (routine adaptation) and how teachers modify planned instruction beyond their routine adaptation in light of specific student difficulty (specialized adaptation).
Cole et al. (2000)	Instructional variables or components adapted or modified.
Bryant & Bryant (2001)	Instructional variables or components adapted or modified.

Evidence-Based Mathematics Instructional Components for Teaching Students With MD

Under IDEA of 2004, students with disabilities, including students with MD, are entitled to have maximum access to the general education curriculum and instruction and make progress in the general education core curriculum. Correspondingly, the law (IDEA, 2004) also mandates that teachers should implement instructional adaptations for students with disabilities, incorporating evidence-based instructional components into the core curriculum. In the field of mathematics disability, syntheses or meta-analyses of research findings on mathematics instruction for students who struggled with mathematics, including students with MD, have revealed evidence-based instructional components that can be integrated into the core instruction (S. Baker et al., 2002;

Kroesbergen & Van Luit, 2003; Miller, Butler, & Lee, 1998; Swanson, Hoskyn, & Lee, 1999; Xin & Jitendra, 1999). Research showed that students with MD were able to effectively learn basic math skills (i.e., basic math facts and computation) or word problem-solving skills when they are taught using systematic, explicit instruction paired with strategy instruction (S. Baker et al., 2002; Kroesbergen & Van Luit, 2003; Miller, Butler, & Lee, 1998; Swanson, Hoskyn, & Lee, 1999; Xin & Jitendra, 1999). In this section, the components of effective mathematics instruction emerging from the findings of four recent syntheses or meta-analyses are summarized.

Miller et al.'s (1998) synthesis of mathematics intervention research identified validated mathematics instruction for students with MD from research published between 1988 and 1997. This synthesis identified self-regulation, strategy instruction, and the use of manipulative devices and drawings as effective for teaching both computation and word-problem solving to students with MD. Particularly, the use of manipulative devices and drawings was identified as an effective strategy for teaching computation to primary grade students with MD who were mainstreamed into general education classrooms (Funkhouser, 1995; Harris, Miller, & Mercer, 1995). The findings of this study showed that students with MD benefited from cognitive/meta-cognitive strategy instruction with step-by-step processes that guided their thinking and performance when solving mathematics problems. Additionally, direct instruction (e.g., scripted lessons, fast-pace, choral responding, hand signals, much repetition) and direct instructional formats (e.g., demonstration, modeling, guided practice, independent practice, feedback) were also beneficial to these students' learning.

Xin and Jitendra (1999) conducted a synthesis of word-problem-solving intervention research, including studies which were conducted between 1986 and 1996 with samples of students with learning problems. Their study identified computer-assisted instruction, representation techniques, and strategy training as significantly effective interventions for teaching word-problem-solving to students with MD. This analysis also identified instructional features associated with positive treatment outcomes including treatment length, instructional grouping, and task difficulty. For example, long-term (more than one month) intervention effects were significantly higher than short-term or intermediate-term intervention effects. Individually provided instruction was more effective than group instruction. Additionally, interventions involving simple one-step problems yielded larger effect sizes than multi-step word problems or mixed problem types.

Swanson and his colleagues (1999) conducted a meta-analysis of intervention research across academic areas, published between 1964 and 1997, to identify effective instructional models (i.e., direct instruction, strategy instruction) that yielded high effect sizes, as well as the components that comprised those models. Even though this study did not separate findings on mathematics instruction from findings in the other areas, this study identified important findings regarding effective instructional models and instructional components for students with LD. The findings of this study indicated that a combined model of direct instruction and strategy instruction was the most effective procedure for remediating students with LD than other competing instructional models. The important instructional components involved in this model were (a) attention to sequencing, (b) drill-repetition-practice, (c) segmentation of information into parts or units for later synthesis, (d) control of task difficulty through prompts and cues, (e) use of technology, (f) systematic modeling of problem-solving steps, and (g) use of small interactive groups. Additionally, ten instructional components were critically related to treatment outcomes without regard to the general models of instruction. The critical components of effective instruction across academic areas included (a) sequencing, (b) drill-repetition and practice-review, (c) segmentation, (d) direct questioning and responses, (e) control of difficulty or processing demands of a task, (f) technology, (g) modeling of problem-solving steps by teacher, (h) small group instruction, (i) a supplement to teacher and peer involvement (i.e., homework, assistance from parents), and (j) strategy cues (i.e., reminder to use strategies, use of think-aloud models).

In a recent review of mathematics instruction, Kroesbergen and Van Luit (2003) conducted a meta-analysis of 58 empirical studies on mathematics interventions for elementary students with special needs, published between 1985 and 2000. This analysis revealed that the majority of interventions examined the effect of an intervention in the domains of basic math skills, most of which produced a large effect size. Regarding the instructional components of interventions, the findings of this study found that direct instruction was most effective for teaching basic math skills while self-instruction was most effective across mathematics skills. In addition, teacher instruction was more effective than computer-assisted instruction and peer-tutoring.

In summary, the findings of recent syntheses or meta-analyses of mathematics intervention research for students with learning problems indicated that struggling students were able to learn math skills more effectively when their teachers repeatedly provided direct, systematic instruction paired with cognitive/meta-cognitive strategy instruction in one-to-one or small group settings. However, standards-based mathematics curriculum and instruction emphasizes student-centered, inquiry-based, and discourse-driven learning for all students, rather than these instructional features. Thus, it is important for general educators to implement instructional adaptations in standards-based mathematics instruction by integrating the components or effective features of instruction for teaching students with MD included in these environments.

Summary

After reviewing the features of standards-based mathematics curriculum and instruction, characteristics of students with MD, and instructional adaptations suggested in the literature, it was apparent that standards-based mathematics curriculum and instruction would not be sufficient to address the cognitive and behavioral characteristics of students with MD. This raised questions about the mathematics learning of students with MD in standards-based mathematics instruction and calls for instructional adaptations for those students as part of the standards-based mathematics instruction. However, little research has been conducted to examine mathematics learning of students with MD in standards-based mathematics general education classrooms and the extent to which instructional adaptations for this group of students in standards-based mathematics classrooms is occurring or how instructional adaptation is implemented. Therefore, this study investigated how and if four general education teachers adapted mathematics instruction within standards-based mathematics curriculum for students with MD in their classrooms. The following two sections provide reviews of the development of students' geometrical thinking and transfer of mathematics knowledge and skills to inform the second research question.

Development of Geometrical Thinking

The second research question of this study was to explore students' learning of mathematics knowledge and skills in standards-based mathematics general education classroom. The students' learning was examined particularly in terms of generalizations or transfer of geometry and spatial reasoning knowledge and skills and probability and spatial reasoning knowledge and skills, which had been taught in class to solve new problems. This study intended to review studies of both geometry and spatial reasoning and on probability and statistics, but the researcher was unable to locate applicable studies on probability and statistics during her search of the literature. Therefore, only research on geometry and spatial reasoning was reviewed in the following section. In addition, this study reviewed studies on transfer of mathematics knowledge and skills to new problems having different similarity with the base problem.

Geometry and spatial reasoning skills are essential mathematics skills in that they help students to develop a systematic representation of their world (NCTM, 1989). A model by Van Hiele (1986) of the development of geometrical thinking has provided a framework for teaching Geometry and Spatial Reasoning in standards-based mathematics curriculum and instruction (Choi-Koh, 1999). The model focused on levels of geometrical thinking and the role of instruction in promoting students transfer from one level to next level (Fuys, Geddes, Lovett, & Tischler, 1988). This section provided a review of the Van Hiele's model on the development of student geometrical thinking to inform the second research question of this study.

According to the Van Hiele model, the development of students' geometrical thinking takes place in terms of five different levels of understanding which are identifiable, and instruction addressing specific levels of students' understanding is the most desirable for teaching geometry and spatial reasoning (Fuys et al., 1988). Instruction produces larger effects on students' progress from one level to the next level than students' biological maturation.

With appropriate instruction, students pass through the five levels of geometrical understanding, from Level 0 through Level 4. At Level 0, students can identify, name, compare, and operate on geometric figures according to their appearance. At Level 1, students show the skills of analyzing figures in terms of their components and relationships among components and discovering properties/rules of a class of shapes empirically. At Level 2, students show understanding of the skills of logically interrelating previously discovered properties/rules by giving or following informal arguments. At Level 3, students show skills of proving theorems deductively and establishing interrelationships among networks of theorems. At Level 4, students show the skills in establishing theorems in different postulational systems and analyzing/comparing these systems.

In the Van Hiele (1986) model, teaching geometry language (vocabulary) is essential, because language plays a critical role in promoting students to move to higher levels of understanding in the model. This model notes that use of inappropriate levels of language which students cannot understand may prevent students' progress in geometrical thinking and movement through the Van Hiele levels, from concrete (Level 0), through visual (Levels 1–2), to abstract (Levels 3–4).

In summary, this study reviewed the Van Hiele model on the development of students' geometrical thinking, which included five different levels, from concrete level (Level 0), through visual levels (Levels 1–2), to abstract levels (Levels 3–4). This model hypothesizes that instruction has more impacts on students' moving from one level of understanding to the next level of understanding than biological maturation alone. To improve students' geometrical thinking levels, instructions should involve appropriate levels of language and should be directed toward students' current levels of geometrical thinking.

Transfer of Mathematics Knowledge and Skills

In education and cognitive science, transfer is described as the application of knowledge acquired in one situation to a different situation (Singley & Anderson, 1989). Especially in mathematics education, transfer has been defined as the skills of applying a particular mathematics formula or principles across isomorphic problems, which have different surface features but have the same conceptual structure (e.g., Bassok, 1997; 2001; Bassok & Holyoak, 1983; Reed, 1987; Ross & Kennedy, 1990) and non-isomorphic problems, which have different conceptual structure so that students need to modify their solution to a known problem to solve the problem (Reed, 1993). For example, Bassok (1997) conducted a study on high school and college students' transfer of mathematics world problem-solving skills across isomorphic problems, which could be solved by the same equation or formula. The word problems included distance problems, salary problems, and other problems varied in problem context or storylines, which could be solved by the same equation or formula. In this study, once students mastered the solution of a distance problem, they received other types of problems and asked to solve them.

On the other hand, Reed (1993) conducted a study on transfer of word problem-solving skills to new problems that were not isomorphic. In his study, students received instruction on problem solving (e.g., typing speed problem) and then were asked to solve non-isomorphic problems which differed in their conceptual structures (e.g., schema or mathematics equation) as well as their surface features (e.g., context). It was possible for students to solve the problems by first modifying their solutions to the original problem/solution. The findings of this study revealed that the more transformations were required, the less students used the original solution.

Based on these studies, the ability to transfer skills may indicate that a student can recognize the mathematical, conceptual structures that are in common in contextually diverse situations of mathematical problems (Wagner, 2003), that a student can generalize knowledge acquired in one situation to other situations or problems (Wagner, 2003), and that a student can modify knowledge acquired in one situation to solve new problems having different structures. In addition, research on transfer showed that students tended to more readily recognize problems that were similar in surface features (e.g., context, storyline, and numbers) than problems that were similar in the conceptual structures (e.g., schema, and problem type) (Gentner, Rattermann, & Forbus, 1993). Novices were especially apt to categorize problems based on more the surface features than the conceptual structures (Chi, Feltovich, & Glaser, 1981). According to this line of research, it is possible that students with MD may fail in transfer of mathematics skills to new problems not only because they did not attain understanding of the skills required at mastery level, but also because they may not recognize the conceptual structure common in the new problems and the original problem learned.

Summary

This review of the literature examined studies on the features of standards-based mathematics curriculum and instruction, characteristics of students with MD, and instructional adaptations suggested by the literature to inform and support the first research question regarding what were the general education teacher's instructional adaptations for students with MD in standards based mathematics instruction. To inform the second research question, the learning of students with different ability in standards-based mathematics general education classroom, this study reviewed a model of the development of geometrical thinking, the Van Hiele model, and studies on transfer of mathematical knowledge and skills across new problems having isomorphic and non-isomorphic patterns in surface and/or conceptual features.

Based on the findings of the literature review, it was evident that standards-based mathematics curriculum and instruction alone would not be sufficient to address the cognitive and behavioral characteristics of students with MD. This raised questions about the mathematics learning of students with MD in standards-based mathematics instruction and calls for instructional adaptations for those students as part of the standards-based mathematics instruction. However, little research has been conducted to examine mathematics learning of students with MD in standards-based mathematics general education classrooms and the extent to which instructional adaptations for this group of students in standards-based mathematics classrooms is occurring or how instructional adaptation is implemented.

CHAPTER 3:

METHOD

Under the recent revisions of IDEA (2004), students with disabilities, including students with MD, are entitled to have access to and are expected to make progress in the general education curriculum to the maximum extent possible. In order to attain these goals, teachers are required to adapt their general education instruction for students with disabilities by incorporating evidence-based instructional practices into the core curriculum according to the needs of individual students (IDEA, 2004).

In today's mathematics education, there is a growing tendency for students with LD, including students with MD, to be taught within general education classrooms using NCTM standards-based mathematics curriculum and instruction (Lappan, 2000; NCTM, 2000). However, research in standards-based mathematics instructional settings has indicated that the features of NCTM standards-based mathematics instruction are challenging for struggling students, including students with MD (Woodward & Montague, 2002). Given the mandates by law and the challenges that students with MD may encounter in standards-based mathematics instruction, general education teachers using the standards-based mathematics, general education curriculum should adapt their typical instruction using evidence-based, explicit, systematic instruction or interventions according to the needs of students with MD. Such instruction may allow students with MD to achieve mathematics learning approaching general education expectations in standards-based mathematics, general education classrooms.

The purpose of this study was twofold. First, this study investigated how a fourth-grade general education teacher adapted standards-based mathematics instruction for students with MD in terms of (a) frequency and settings of instructional adaptations, (b) categories of instructional adaptations, (c) use of evidence-based instructional components in instructional adaptations, and (d) consistency of instructional adaptations with the individual students' difficulties. Second, this study examined what progress in the general education curriculum looks like for students with differing abilities (3 students who have been identified as having MD, 2 non-MD struggling students, and 1 typically achieving student) in standards-based mathematics classrooms where instructional adaptations occurred. In other words, this study investigated how comparable mathematics knowledge and skills were acquired across students with differing abilities in standards-based mathematics classrooms in which the teacher participant adapted her mathematics instruction for students with MD. In this study, *mathematical knowledge and skills* are defined as key concepts, strategies, representations, and algorithms that a student uses to solve problems in mathematics (Ginsburg & Pappas, 2004). This chapter describes the methodology for this study, including (a) research design, (b) participants, (c) measures, (d) data collection procedures, (d) data analysis procedures, and (e) credibility of the research.

Research Design

An embedded, single-case study design (Yin, 2003) was employed to provide exploratory and instrumental information about the topics of this study: (a) instructional adaptations for students with MD within standards-based mathematics, general education classrooms to assist them in having access to general education curriculum, and (b) the learning of general education curriculum content (mathematics knowledge and skills) of students with different ability in standards-based mathematics, general education classrooms. A case study was selected because of the nature of the inquiry of this study. This study was designed to understand (a) one general education teacher's instructional adaptations for her students with MD within standards-based mathematics curriculum and instruction and (b) the mathematics learning of students with MD in the standards-based mathematics, general education classroom in comparison to their peers with different levels of ability.

A large number of variables could influence the teacher's instructional adaptations and the MD students' learning in the general education mathematics classroom (e.g., curriculum, the teacher's instructional philosophy, the teacher's professional development, student characteristics such as IQ and self-efficacy, peer collaboration, acceptance from peers, family support, and administrator support). It was difficult to ignore the variables or contexts that influenced the teacher's instructional adaptations for students with MD and their learning of mathematics general education curriculum content. The use of a case study is suitable when variables of interest are intertwined with their context (Yin, 2003) and when a study aims to understand the features or the patterns of a phenomenon within an integrated, bounded system (Stake, 1995). Particularly, case studies are the preferred strategy when "how" or "why" questions are being posed, when the investigator has little control over events, and when the focus is on a contemporary phenomenon within some real-life context (Yin, 2003).

In addition, even though this case study was about the teacher participant's standards-based mathematics class, attention was also given to subunits (Yin, 2003): instructional adaptations for each individual student with MD for Research Question 1 and three groups of students with different levels of ability for Research Question 2. The subunits embedded in the case were expected to "add significant opportunities for extensive analysis, enhancing the insights into the single case" (Yin, 2003, p. 46). Thus, this study employed an embedded, single-case study methodology to examine (a) how a general education teacher adapted mathematics instruction for her students who had an IEP in mathematics and who received mathematics instruction in the standards-based mathematics, general education classroom, and (b) how students with different levels of ability, including students with MD, learned mathematics in the standards-based mathematics, general education class.

This study was instrumental and exploratory in nature (Stake, 1995; Yin, 2003). An instrumental case study examines a typical case to increase understanding of an issue or phenomenon or to refine theory (Stake, 1995). In an instrumental case study, the case is of secondary interest. The case is selected because it is expected to advance understanding of the topic of interest. This study was an instrumental case study in that it was not focused on the specific cases themselves, but on the topics of interest. This single case was investigated to provide insight into mathematics instruction for students with MD in standards-based general education classrooms to ensure their access to the general education curriculum and progress of students with differing ability in the instructional environment (Stake, 1995).

Exploratory case study is conducted to develop pertinent hypotheses and propositions for further inquiry (Yin, 2003). This study was designed to examine what was happening in standards-based mathematics, general education classrooms related to the access of students with MD and their progress in the general education curriculum in comparison to their peers, which is a topic requiring more systematic investigation. This study did not purport to provide findings generalized across situations or cases. Rather, this study aimed to describe what happened in standards-based mathematics classrooms to ensure the access of students with MD to the curriculum and instruction and what mathematics learning of students with MD looked like in the instructional environment. Findings could induce further experimental research (Yin, 2003).

Setting and Participants

The participants in this study were 1 fourth-grade general education teacher who employed standards-based mathematics curriculum and practices for her mathematics instruction, and 6 students with different levels of ability (3 students who were identified as having MD, 2 non-MD students who were struggling with mathematics, and 1 typically achieving student). This section described the setting for this study and the selection criteria and demographics of the participants.

Setting

This study took place in one elementary school in a school district in central Texas. A suburban school district in central Texas was contacted to obtain permission for conducting this study in the school district. Once this study was approved by the school district, a mathematics curriculum coordinator of the school district was contacted to identify schools for possible inclusion in this study. Through a face-to-face informal conversation, the school district mathematics curriculum coordinator was asked to nominate as many schools as possible that employed standards-based mathematics curriculum and instruction and that were likely to participate in this study. The mathematics curriculum coordinator nominated three schools in the school district. Then, the researcher investigated the demographic information of the three schools nominated by the mathematics curriculum coordinator and the schools' ranks in the school district on the Texas Assessment of Knowledge and Skills (TAKS) to get additional information for possible inclusion of the schools in this study.

Based on the nomination of the mathematics curriculum coordinator of the school district, the demographics of the schools, and the schools' ranks on the TAKS, the researcher identified two schools that met the school selection criteria for further procedures. Criteria for selecting schools included the following: (a) The schools used mathematics basal programs such as Investigations (TERC, 1998) that incorporated the recent NCTM (2000) standards; (b) the schools were similar in terms of environment and demographic data, including ethnicity and the percentage of students receiving free or reduced-price lunch; (c) the schools were ranked at or above the 50 percentile on the TAKS in the school district; (d) most students with LD in these campuses received their mathematics instruction in general education classrooms; and (e) English was the language of instruction.

Principals of the two nominated schools that met the selection criteria were contacted to determine inclusion of their schools in this study. Both school principals agreed to participate in this study. However, only one of the schools was selected for this study, because the other school did not have a fourth-grade class with complete participant members (a teacher, at least 1 student who had been identified as having MD, 2 non-MD struggling students, and 1 typically achieving student) available for this study. Table 3.1 shows the demographics for the school district in which this study occurred.

Table 3.1

District Demographics

Demographics	No.	%
Total enrollment	78,679	
Minority enrollment		
African American	10,710	13.6%
Hispanic	41,877	53.2%
White	23,743	30.2%
Asian	2,152	2.7%
Native American	207	0.3%
Other	—	—
Economically disadvantaged		50.1%

Teacher

Potential teacher participants were first identified based on nominations by the school district's mathematics curriculum coordinator and the two principals whose schools met the school selection criteria and who wished to participate in this study. The researcher asked the mathematics curriculum coordinator and the principals to nominate fourth-grade teachers who had been observed as effectively implementing standard-based curriculum and instruction in the schools. The researcher contacted 3 teachers who had been nominated by either the school district curriculum coordinator or their school principals in order to proceed to further selection procedures. Of those 3 teachers, only 1 teacher, Ashley Hamilton (pseudonym), had a complete set of student participants (at least 1 MD student, 2 struggling students, and 1 typically achieving student). Ashley was considered as the potential teacher participant of this study.

Before making a final decision on inclusion of the potential teacher participant in this study, with the teacher's permission, the researcher observed the potential teacher participant's mathematics instruction twice and conducted an informal interview to determine if the teacher was actually adapting mathematics instruction for her students with MD in her class. During the informal interview, the teacher's information about teacher selection criteria was also identified. The potential teacher participant was asked to participate in this study because she met the following criteria: (a) The teacher was engaged in instructional adaptations for a student with LD who had an IEP in mathematics at least once in both the preliminary observations; (b) the teacher had 1 or more years of experience in teaching TEKS-based mathematics curriculum; (c) the teacher had at least one student with LD who had IEP goals and objectives for mathematics; and (d) the teacher had been trained to implement NCTM standards-based or TEKS- based mathematics curriculum and instruction.

Ashley Hamilton is a fourth-grade general education teacher in a suburban school district in central Texas. She identified her ethnicity as European American and is in her late 20s. She majored in elementary education at a college in eastern Texas. She was certified in teaching reading in Grades 1–8. She had been teaching elementary schools for 4 years. Since she started her career as a teacher, she has always taught at the fourth-grade level.

The year of study was her 2nd year to use the Math Investigations (TERC, 1998) program for her mathematics instruction. Before starting to teach Math Investigations, she was trained to teach the program at a daylong workshop by the school district where she is employed. After starting to use the program for her mathematics instruction, she received ongoing, whole-day trainings at 6-week intervals about how to use the program to teach mathematics.

Students

The student participants of this study were selected from the class whose teacher met the teacher selection criteria and wished to participate in this study. Once the teacher participant agreed to volunteer for this study, the researcher asked the teacher to identify students with LD who had an IEP in mathematics, multiple students who were struggling with mathematics, and multiple students who were typically achieving students in their class. Six students with different abilities were selected from the class: 3 students who had been identified as having MD (identified as LD and having an IEP in mathematics), 2 students who had not been identified as having LD but were struggling with mathematics, and 1 student who was typically achieving.

Students With MD

For selection of students with MD or LD who have an IEP in mathematics, the teacher participant was asked to nominate and contact the parents of students who met the following criteria: (a) students who had been identified as having LD or MD by the school district, (b) students who had IEP in mathematics, and (c) students who received their mathematics instruction in the general education classroom. The teacher sent parental permission slips to 3 students who met the selection criteria of students with MD and obtained the parents' written permissions for all 3 students to volunteer for this study. Then the teacher obtained student assent from all 3 students. The 3 students, Lee, Kevin, and Tina (Pseudonyms), met the selection criteria, provided both parental and student consents, and thus were selected as MD participants for this study.

Struggling Students

For selection of struggling students and typically achieving students, the researcher asked the teacher participant to rank all her students according to their mathematics ability. Then, the researcher asked the teacher to identify students who met the following criteria for selecting struggling student participants: (a) students who had passed TAKS mathematics; (b) students who were ranked between 25 percentile and 45 percentile on mathematics ability, as perceived by the teacher; and (c) students who had not been identified as LD or MD but usually did not achieve the learning expectations of each mathematics lesson. The teacher identified 3 students as potential participants as struggling students, sent parental permission slips to the students' parents, and then obtained the parents' written permission from 2 students' parents. Student consents were also obtained from both the students. Two students, Laura and Jose, met the selection criteria, provided both parental and student consents, and thus were selected as struggling student participants for this study.

A Typically Achieving Student

For selection of a typically achieving student participant, the teacher was asked to nominate and contact the parents of students who met the following criteria: (a) students who were ranked between 70 and 80 percentile on mathematics ability perceived by the teacher, (b) students who usually achieved the learning expectations of each lesson, and (c) students who did not have difficulties or problems in any other area (e.g., reading difficulties, psychological problems, and behavioral problems). The teacher identified 2 potential students and obtained both the parents' written permission and student consent for 1 student. Amy was selected as a typically achieving student participant for this study.

Consent and Confidentiality

This study was conducted after The University of Texas at Austin and the District Institutional Review Boards provided approval. Consent to participate in this study was secured from the teacher, the parents or guardians of each student, and each student. A pseudonym was given to the teacher participant and each student participant to maintain anonymity. Accessibility to audiotapes, field notes, and transcripts was limited only to the observer, the teacher participant, the peer debriefer, and a supervising professor from The University of Texas at Austin.

Measures

In this study, multiple data collection methods were used to ensure the validity of findings of this case study, including qualitative data (Denzin, 1978; Lincoln & Guba, 1985). Observations and interviews, including clinical interviews, were main data collection methods that produced both quantitative (descriptive statistics) and qualitative data on the topics. Particularly, the observations of the teacher's instructional adaptations for her 3 students with MD yielded the frequency data of the teacher's instructional adaptations across students, student difficulties, categories of instructional adaptations, and use of evidence-based instructional components as well as qualitative descriptions of the teacher's instructional adaptations on those topics. The clinical interviews on each targeted skill yielded quantitative data on the accuracy of individual students in problem-solutions and in transfer of mathematics knowledge and skills as well as qualitative information about students' concepts or procedures used for problem-solutions.

A teacher survey and document reviews were also used to collect data on the teacher's instructional adaptations and the students' mathematics learning. A teacher survey was used to gather quantitative information about student participants' difficulties with prerequisite skills required to learn fourth-grade geometry and spatial reasoning, or probability and statistics within the standards. The teacher survey questionnaire was developed by the researcher of this study based on the knowledge and skills on TEKS (TEA, 2006). Document reviews were employed to collect data on the teacher's instructional adaptations (e.g., lesson plans, student IEPs, and mathematics basal program) and on the students' learning (e.g., student's permanent products) in the standards-based mathematics, general education classroom.

Likewise, using multiple sources of evidence is often considered as a strategy to establish correct operational measures for the concepts being studied (Yin, 2003). Therefore, this study also used two different data sources, field notes and audio-taped data, during class observations and interviews including clinical interviews. Table 3.2 summarizes the main research questions of this study and specific data collection methods and sources.

Table 3.2

Data Source Table

Research questions	Data collection method	Data sources
1. How does a fourth-grade general education teacher adapt mathematics instruction within a standards-based mathematics curriculum and instruction for students who have an Individualized Education Program (IEP) in mathematics and who receive mathematics instruction in the general education classroom?	Class observations	Field notes Audiotapes
	Teacher interviews	Field notes Audiotapes
	Teacher survey	Survey questionnaire
	Document reviews (e.g., IEP, lesson plan, textbook, TEKS for math)	Document review forms
2. How do fourth-grade students with differing ability (3 identified MD, 2 students struggling with mathematics, and 1 typically achieving student) who receive mathematics instruction in a standards-based mathematics, general education classroom use mathematics knowledge and skills taught in class to solve curriculum-based problems after they have received classroom instruction?	Clinical interviews	Field notes Audiotapes
	Document reviews (e.g., student products)	Document review forms

Observations

For collecting data on the general educator's instructional adaptations for students with MD in standards-based mathematics, general education classrooms, this study employed a direct observation technique, which involved the process of directly observing and meticulously recording what was seen (Gall, Borg, & Gall, 1996). The primary researcher of this study, a graduate student in the Department of Special Education at The University of Texas at Austin, served as an observer throughout the study. The primary researcher went to the place where instruction for students with MD occurred and conducted direct observations in the natural settings, being aware of herself as an observer and as an interpreter of the data. Field notes were taken during the observations.

The foci of observations were placed on the teacher participant's instructional adaptations for the 3 students with MD in her standards-based mathematics classroom. Instructional adaptations refer to appropriate adjustments, accommodations, and modifications to instruction or supports that allow students to meet academic requirements and conditions of the curriculum, most often the general education curriculum (VGCRLA, 2001). In this study, instructional adaptations were identified by the teacher's interactions adjusted for individual students with MD or small groups involving the students with MD. More specific procedures of determining instructional adaptations are described in the results section of this study.

The teacher's instructional adaptations were identified and documented in terms of (a) the frequency of instructional adaptations for individual students with MD and the situations where the instructional adaptations occurred, (b) categories of instructional adaptations, and (c) the use of evidence-based mathematics instructional components. The categories of instructional adaptations included instructional content (e.g., instructional objective), instructional activity (e.g., making equivalent fractions using pattern blocks), delivery of instruction (e.g., grouping, instructional sequencing, and directed questioning), or instructional materials and technology (e.g., use of manipulatives, drill-and-practice software) (Bryant & Bryant, 1998). The use of evidence-based effective instructional components (Swanson et al., 1999) was identified and documented by instructional components such as (a) corrective feedback, (b) control difficulty, (c) direct questioning, (d) review of prerequisite skills, (e) group instruction, (f) strategy instruction, (g) progress monitoring, (h) use of manipulatives, (i) practice opportunities, (j) teacher examples, (k) reteaching, and (l) vocabulary instruction. Research has demonstrated these components' effectiveness for teaching students with MD (e.g., Jitendra et al., 2005; Swanson et al., 1999).

Interviews

This study employed three different forms of interviews as the second data source to explore each research question of this study: (a) formal interviews, (b) conversational informal interviews, and (c) clinical interviews. First, this study used both formal interviews and informal conversational interviews to examine 1 general educator's instructional adaptations for students with MD within standards-based mathematics general education classroom. Second, this study used clinical interviews to examine the mathematics learning of 6 students with differing ability in a standards-based mathematics, general education classroom (Ginsburg & Pappas, 2004). Because the procedures of clinical interviews were able to be modified to fit a student's needs (e.g., level of mathematics competence, linguistic understanding, motivation, or interest), it was assumed that the clinical interview format would allow this study to have unique access to a student's highly individualized psychological processes or experiences, opening the way to accurate assessments and effective instruction (Ginsburg, 1997a, 1997b; Greenspan & Greenspan, 2003). The next sections describe (a) the formal interviews and conversational interviews with the teacher participant, (b) clinical interviews with student participants, and (c) interview protocols including formal interview protocols and clinical interview tasks.

Formal Interviews and Conversational Interviews

To answer the first research question, the researcher conducted two types of interviews with the teacher participant: (a) formal interviews and (b) informal conversational interviews (Patton, 2002). Formal interviews were presented in a semistructured format. Semistructured interviews are conducted with a fairly open framework, which allows for focused, conversational, two-way communication and can be used both to give and receive information (ERIC, 1997). The formal interviews of this study started with more general questions or topics (e.g., standards-based mathematics instruction for students with MD), which were constructed before the interviews with the teacher (ERIC, 1997). Topics relevant to main questions were asked of individuals as they arose (e.g., what does your feedback look like when a student fails to answer correctly in standards-based mathematics instruction? How often do you use group instruction?).

The informal conversational interviews occurred in daily life conversations (Patton, 2002). This type of interview allows researchers to have maximum flexibility to be responsive to individual differences and pursue topics or ideas that arise during observations. Thus, this study employed informal conversational interviews to capture emerging information during direct observations.

During both formal interviews and informal conversational interviews, the researcher asked main key questions to begin and guide the conversation, supplementary questions to clarify points or seek more detail, and follow-up questions to pursue the implications of answers to the questions (Rubin & Rubin, 1995). With the teacher's permission, most formal interviews and informal conversational interviews were audio-taped (Holstein & Gubrium, 1995) and transcribed for an analysis. Field notes were also taken during all interviews.

Clinical Interviews

To explore answers for the second research question of this study, mathematics knowledge and skills of 6 student participants with differing ability (3 students with MD, 2 students who struggled with mathematics, and 1 typically achieving student) were examined through clinical interviews prior to and after receiving mathematics instruction on each skill in the standards-based mathematics classroom. Clinical interviews with a student produce information about developmentally appropriate functioning as well as accurate descriptions of the target behaviors being observed (Greenspan & Greenspan, 2003). To provide information about grade-appropriate, developmentally appropriate functioning of students with MD in mathematical thinking, this study included students without disability as well as students with MD and struggling students as student participants.

Clinical interviews of this study involved three essential components: (a) testing, (b) observing, and (c) asking for the students' account of the method of solution and the verification of their answer to each problem (Ginsburg, 1997a, 1997b; Ginsburg & Pappas, 2004). First, testing refers to the administration of carefully selected tests or tasks designed to clarify hypotheses about the student's mathematical thinking (Ginsburg & Pappas, 2004). This researcher developed and administered four clinical interview tasks, designed to evaluate the mathematics knowledge and skills that had been taught in the students' class. These tasks were administered to 6 individual students with different levels of abilities. In addition, each clinical interview task included four problems on prerequisite skills related to the targeted skills being examined in each clinical interview task. This study assumed that students with MD were not equipped with prerequisite skills required to learn fourth-grade mathematics knowledge and skills, and their difficulties in mathematics might be partly explained by the students' difficulties in related prerequisite skills. For example, Clinical Interview Task 1 included four problems about determining the volumes of 3-D buildings as the problems tackling prerequisite skills for learning the targeted skills of how to build

3-D buildings based on 2-D drawings. Second, the researcher observed a student's overt behaviors such as the way the student used manipulatives and the procedures the student followed to answer each problem of the clinical interviews. The third component of clinical interviews was asking for the student to account for the solution method (e.g., "How did you know how to make a building?") and the method of verifying the answer (e.g., "How do you know if you made this building correctly?"). In relation to the solution method, the individual students were asked if they remembered how their teacher and their peers solved the problems. During clinical interviews, individual students were asked to use a think-aloud method as they solved problems.

For this study, four clinical interview tasks were developed, two for geometry and spatial reasoning and two for statistics and probability. Each clinical interview task included four different problems on a target skill along with four problems on a specific prerequisite skill related to the target skill. The next section describes interview protocols that were used for formal and clinical interviews.

Interview Protocols

Formal interview protocol. Prior to the commencement of formal interviews, the researcher constructed semistructured, open-ended interview questions to be used to begin and guide the interviews. For the semistructured interview protocols, three general key questions were developed based on the research questions of this study, and subsequent questions were created during the interviews. Key topics that were explored through interview questions were (a) standards-based mathematics instruction for all students and instruction for students with MD; (b) prerequisite skills for learning fourth-grade geometry and spatial reasoning, and statistics and probability; (c) strengths and challenges of students with MD in the standards-based mathematics classroom; and (d) assistance for students with MD in learning mathematics in standards-based mathematics classroom. Appendix A includes questions to guide the formal interviews.

Two colleagues of the primary researcher, two graduate students from the Department of Special Education at The University of Texas at Austin, reviewed each general key question to determine (a) relevance (whether the question directly related to the purpose of the study and had a good probability of yielding the kind of data desired), (b) selection of the proper respondents (whether the question was answerable by the people to whom it would be asked), and (c) ease of response (whether the question was relatively easy to answer and would not create embarrassment for or an undue burden on the interviewee?) (ERIC, 1997).

Clinical interview tasks. To answer the second main research question, during a clinical interview session, each student was given four clinical interview tasks. Each clinical interview task contained (a) four problems requiring the use of one targeted fourth-grade mathematics skill that had been taught in the mathematics class (e.g., building 3-D buildings based on 2-D drawings, finding shapes that would make specific silhouettes, drawing bar graphs, and comparing two bar graphs) and (b) four problems about a specific prerequisite skill for learning the target skill. See Appendix F and Appendix G for more information about clinical interview tasks.

Targeted skills. A total of four curriculum-based tasks for clinical interviews (two tasks on geometry and the other two on statistics) were developed based on the mathematics basal program that the teacher used for her instruction, Math Investigations (TERC, 1998). During the observational period of this study, the teacher provided instruction on two specific geometry and spatial reasoning skills and two probability and statistics skills. The two geometry and spatial reasoning skills were (a) identifying and building a 3-D object from 2-D representations of the object (Clinical Interview Task 1) and (b) identifying and drawing a 2-D representation of a 3-D object (Clinical Interview Task 2). The two probability and statistics skills were (a) organizing and displaying data in a bar graph (Clinical Interview Task 3) and (b) interpreting bar graphs and comparing two graphs (Clinical Interview Task 4). The four clinical interview tasks of this study were created to measure students' knowledge and skills in specific areas.

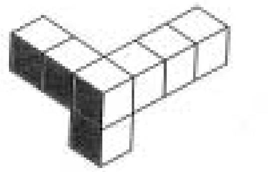
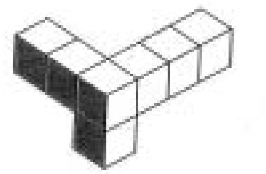
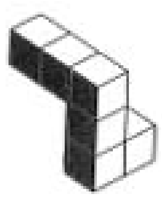
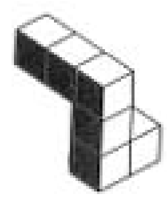
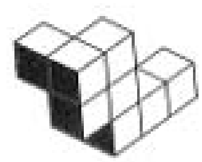
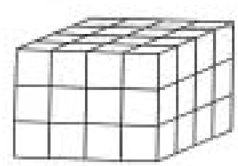
Specifically, on Clinical Interview Task 1, the students were asked to make a building with cubes based on the 2-D drawing on a card. On Clinical Interview Task 2, the students were asked to find and name 3-D solids that had a specific silhouette shown in the card. Each task consisted of four problems. On Clinical Interview Task 3, the students were asked to organize and display data shown in a T-chart using a bar graph. Finally, on Clinical Interview Task 4, the students were asked to compare two graphs.

Problem variation in each clinical interview task. The four problems included in each task were designed to measure the student participants' transfer of mathematics knowledge and skills taught in their mathematics class to new problems whose surface features or problem structures were different from the teacher example that the teacher used in the class instruction. Especially on Clinical Interview Task 1 (geometry) and Clinical Interview Task 3(statistics), three different types of problems were included to examine the students' knowledge transfer according to the similarity between the original problem taught in class and the new problems: one base problem, one near-transfer problem, and two far-transfer problems. The base problem was exactly same as the problem used to teach the skill in class. Near-transfer problems had the same structure (e.g., problem solutions) but surface features (e.g., context and the numbers shown on the problem) different from the original problems taught in class. Far-transfer problems were different from the original problems in both surface features and problem structures.

For example, in Clinical Interview Task 1, base problems (Problem 1 and Problem 2) were exactly the same as the original problem used for class instruction in terms of the problem structure (e.g., problem-schema and problem components influencing problem solutions, such as the number of layers of a building) and the surface features of problems (e.g., the number of cubes). Baseline problems were used to examine the students' mastery of the skills of making 3-D buildings shown in 2-D drawings, which they learned in their mathematics class. The other two types of problems (near-transfer and far-transfer problems) were target problems in which the students needed to apply the skills taught in class to complete. The near-transfer problem (Problem 3) differed from the original problem in terms of the surface features (e.g., the number of cubes or directions). The far-transfer problem (Problem 4) differed from the original problem used to teach the solving procedures in both the problem structure (e.g., the number of layers of a cube building) and the surface features (e.g., the number of cubes and directions of the sections of the building). To solve the far-transfer problem, the students needed to modify or transform the problem-solving procedures (the procedures related to making 3-D buildings) learned from their class. Clinical Interview Task 2 and Clinical Interview Task 4 included only base problems, which the teacher used in her class to teach the specific skills. The four problems included in Clinical Interview Task 1 are presented in Table 3.3 to illustrate problem variations in clinical interview tasks.

Table 3.3

Problem Variations Included in Clinical Interview Task 1

Targeted skills	Teacher example	Problem variations	Problem example
The skills of identifying and building a three-dimensional object from two-dimensional representations of the object	Problem 1 	Base problem	Problem 1 
	Problem 2 		Problem 2 
		Near-transfer problem	Problem 3 
		Far-transfer problem	Problem 4 

Likewise, in Clinical Interview Task 3, Problem 1 was the base problem, which was exactly the same as the original problem used for class instruction. The base problem on Clinical Interview Task 3 was used to examine the students' mastery of the skills of drawing a bar graph of data involving numerical variables shown in a table. The other two types of problems (near-transfer and far-transfer problems) were target problems in which the students needed to apply the skills taught in class to complete. The near-transfer problem (Problem 2) was the same as the original problems in terms of the problem structure but different from the original problem in terms of the surface features such as context (e.g., the number of brothers and sisters vs. student heights). The far-transfer problems (Problem 3 and 4) were different from the original problem used to teach the solving procedures in both the problem structure (the relationships or the types of variables, and solutions) and the surface features (context or physical features of graphs). To solve the far-transfer problem, the students needed to modify or transform the problem-solving procedures learned from their class. For example, on Clinical Interview Task 3, the students should count the number of students who chose a specific fruit instead of counting the number of students who had a specific number (heights or the number of brothers and sisters).

Prerequisite skills. In addition to the targeted knowledge and skills, one of prerequisite skills necessary for learning each targeted skill was examined through four problems in each clinical interview task. Two prerequisite skills examined with regard to the two targeted geometry and spatial reasoning skills were (a) the knowledge and skills for identifying the volume of a 3-D cube building (Clinical Interview Task 1, Prerequisite Skill 1:) and (b) the knowledge and skills for identifying numbers of 3-D solids and remembering their names (Clinical Interview Task 2, Prerequisite Skill 2). Two prerequisite skills included in clinical interview tasks on probability and statistics were (a) interpreting data shown in a T-chart (Clinical Interview Task 3, Prerequisite Skill 3) and (b) finding a typical number shown in a bar graph (Clinical Interview Task 4, Prerequisite Skill 4).

Presentation of problems. Before a clinical interview session, the researcher paired one prerequisite skill problem with one target skill problem, producing four sets of one prerequisite skill problem and one target skill problem in the same way across students. The researcher randomly sequenced the order of presenting the four sets of problems during a clinical interview with an individual student. After one pair of problems about prerequisite skills and the target skills was presented in sequence, the students were asked to explain how they solved the problem involving the target skills, how they knew they solved the problem correctly, and if they remembered their teacher's solution methods and their peers' solution methods. This same procedure was followed with another set of problems. During each clinical interview session, these cyclic procedures continued to be implemented until the student finished all four sets of one prerequisite skill problem and one targeted skill problem.

Each problem was presented to individual students in a similar way as had been taught in the mathematics lessons. For example, the researcher presented each problem to individual students using the same mathematics vocabulary that the teacher used during the mathematics lessons on the skills (e.g., *building, silhouette, 2-D,* and *3-D*).

A packet for each clinical interview task included a protocol consisting of directions, four sets of a prerequisite and a target skill problem, and problem presentational materials. The presentational materials were the following: (a) four figures of 3-D buildings and cubes (Clinical Interview Task 1); (b) wooden blocks and four laminated cards including silhouettes of 3-D shapes (Clinical Interview Task 2); (c) grid papers, pencils, and four cards including data in T-charts (Clinical Interview Task 3); and (d) four cards including two bar graphs to be compared (Clinical Interview Task 4).

Questionnaire Survey

To answer the first research question of this study, the researcher gathered information about the difficulties of the 4 student participants—3 students with MD participants and 1 typically achieving student—in prerequisite skills for learning the fourth-grade mathematics knowledge and skills. Prior to the commencement of observations of her mathematics class, the teacher was asked to rate the prerequisite skills of the 4 individual students according to the items, each indicating

Grade K–3 knowledge and skills shown in the TEKS strands of geometry and spatial reasoning as well as probability and statistics (TEA, 2006).

For the development of the questionnaire, the researcher created a list of geometry and spatial reasoning knowledge and skills as well as a list of probability and statistics knowledge and skills, which the students were expected to master on the TEKS by Grade 3. Eleven geometry and spatial reasoning skills and 10 probability and statistics skills were included in the questionnaire. The questionnaire included columns for 3-point rating scales (*not at all, sometimes,* and *always*) along with the 21 mathematics skills identified in the TEKS as potential prerequisite skills for learning fourth-grade geometry and spatial reasoning as well as probability and statistics. The questionnaire is shown in Appendix B.

Document Reviews

Through document reviews, this study identified student characteristics (e.g., IEP, student permanent products created during lessons or clinical interviews), prerequisite skills required for mathematics learning during each lesson (e.g., lessons in textbooks and TEKS), and plan and implementation of instruction (e.g., mathematics textbooks and teacher lesson plan), which were triangulated with information from other data sources. Data from these sources were used to provide a carefully documented description of MD students and a framework for observations and analysis (Bogdan & Biklen, 1982; Patton, 2002). For example, this study gathered data about the difficulties of students with MD in prerequisite skills using three different data sources: (a) teacher survey questionnaire, (b) teacher interviews, and (b) student IEP reviews. Individual students' IEPs provided a framework for understanding their difficulties and conducting interviews and observations.

This study also identified prerequisite skills required for learning the fourth-grade geometry and spatial reasoning standards as well as the probability and statistics standards by examining lessons in the mathematics textbook used by the teachers. The researcher compared and integrated information about prerequisite skills required for learning mathematics from this document review with the prerequisite skill difficulties of students with MD from the survey questionnaire and from the teacher interviews in order to produce information about how teachers should adapt instruction for the MD students to have access to the general education curriculum.

Lastly, this study collected documentation of teacher planning and implementation of lessons, such as lesson plans, mathematics textbooks, and the TEKS for mathematics. The primary researcher reviewed and summarized all documents. Appendixes C and D include document review forms.

Data Collection Procedures

To collect data, the researcher served as an observer, interviewer, survey administrator, and a document reviewer throughout the data collection period. From December 2005 through March

2006, data were collected in one fourth-grade general education classroom that employed a standards-based mathematics curriculum (Mathematics Investigations) and instruction and had 3 students with LD and IEP goals for mathematics. During this period, fourth-grade teachers in the school district where this study was conducted were expected to teach two mathematics strands in the TEKS standards, including geometry and spatial reasoning, and probability and statistics.

This study investigated inquiries about 1 general education teacher's instructional adaptations for her 3 students with MD and the mathematics learning of the students with MD in the standards-based mathematics, general education classrooms in comparison to their peers with different levels of ability. To do so, this researcher employed a survey and three qualitative data collection methods: (a) observations, (b) interviews (teacher interviews and student clinical interviews), and (c) document reviews.

Observations

The purpose of classroom observations for this study was to document mathematics instruction for students with MD within a standards-based, mathematics general education classroom. The teacher participant's classroom was observed from mid-February through the end of March 2006, two to three times a week, while geometry and spatial reasoning as well as probability and statistics were being taught. Class observations included two baseline observations (one for each mathematics content) and eight observations of adaptations, each lasting 60–90 minutes. Three observations of instructional adaptations (approximately 270 minutes) were conducted during the instructional period on geometry and spatial reasoning, and five observations of instructional adaptations (approximately 450 minutes) were conducted during the instructional period on probability and statistics. Those two mathematics topics were selected to be observed because little research had been conducted on these topics, even though they are essential skills for living and working in contemporary society.

Observer Training

To ensure reliable observations, observer training was conducted 2 weeks prior to the commencement of observations at a small office at The University of Texas at Austin. In this training, the primary researcher and a graduate student in education reviewed and discussed the definitions and the examples of the categories of instructional adaptations and evidence-based, effective, instructional components that were observed and recorded for this study (e.g., activity adaptation, adaptation of instructional delivery, task analysis, modeling steps, and sequencing instruction). The categories are described in the glossary of this study.

For the observer training, this study used the full transcripts of mathematics lessons of a fourth-grade teacher who implemented standards-based mathematics instruction based on the Math Investigations curriculum (Pearson Education Inc., 2004) and who was a teacher participant for the pilot study. The primary researcher and the second observer independently reviewed and coded interactions between a teacher and a student with MD in whole-group and small-group settings by categories of instructional adaptations or the components of evidence-based mathematics instruction. To calculate observational reliability, the observers' codes were compared with each other. Once the reliability was calculated with a lesson transcript, they discussed all disagreements. This process was repeated with two more lessons until interrater reliability reached 90%.

Baseline Observations

Prior to the commencement of observations of the teacher's instructional adaptations, the researcher visited the classrooms to become familiar with the students and observed the classroom twice to identify the instructor's typical standards-based mathematics instruction (baseline instruction). Another baseline observation was conducted a day before observing the teacher's instruction on probability and statistics. These baseline observations identified the standards-based mathematics instructional practices of the teacher for the whole class as well as interactions targeting students with MD.

During the first visit and observation, the teacher was teaching measurement (e.g., measurement units such as yard and inches, measuring self using a yard stick and a ruler). During the second observation, she provided a lesson on geometry and spatial reasoning (e.g., making a 3-D cube building) based on the standards-based mathematics curriculum, Math Investigations (TERC, 1998). Because instruction observed during the first visit was not related to instruction on geometry and spatial reasoning, the instruction was excluded in analysis of baseline observation on geometry and spatial reasoning. A baseline observation of standards-based instruction on probability and statistics was conducted a day before the commencement of observing the teacher's instructional adaptations for five consecutive lessons on the topic. The baseline lesson on probability and statistics was based on a lesson included in *Investigations in Number, Data, and Space* (TERC, 1998).

Observations of Instructional Adaptations

In this study, instructional adaptations were identified by the teacher's interactions adjusted for individual students with MD or small groups involving the students with MD. For eight mathematics lessons on either geometry and spatial reasoning or probability and statistics, the researcher observed the teacher's instructional adaptations during both whole-group and small-group instruction (or one-to-one instruction). During activities involving whole-class instruction, the researcher sat in the back of the classroom to observe whole-class interactions (both verbal and nonverbal) between the teacher and the individual students with MD. During instructional routines involving small-group instruction, the observer moved to a site near a group that included a MD student participant to better observe instruction provided to the particular group or the targeted student participant. When a teacher provided instruction in one-to-one format, the observer took a seat from which she was able to better observe the teacher's instruction to the targeted student participant.

Each lesson observed was also audio-taped. Field notes also were taken to record what was seen and heard and included reflections by the observer during observations.

Interviews

The researcher conducted two formal interviews with the teacher and six informal conversational interviews to examine the general educator's instructional adaptations for her students with MD within the standards-based mathematics, general education classroom. In addition, this study employed clinical interviews to examine the 6 student participants' mathematics knowledge and skills (Ginsburg & Pappas, 2004). The primary researcher also served as the interviewer throughout this study.

Formal Interviews With Teachers

Two formal interviews lasting approximately 60 minutes each were conducted with the teacher participant. One interview with the teacher was conducted prior to this study (early December 2005), and the second formal interview was conducted after the observational portion of the study was concluded (end of March 2006). The teacher was interviewed in her classroom at both interviews. During the interviews, the researcher asked the teacher to tell about (a) standards-based mathematics instructions for all students and for students with IEPs in mathematics, (b) prerequisite skills for learning geometry and spatial reasoning as well as probability and statistics, (c) the strengths and challenges of her students with IEPs in mathematics in the standards-based mathematics classroom, and (d) special assistance provided for students with IEPs in mathematics in her class. With the teacher's permission, the formal interviews was audio-taped (Holstein & Gubrium, 1995). Field notes were taken during these interviews.

Informal Conversational Interviews

Six ongoing informal conversational interviews were conducted with the teacher, while the researcher interacted naturally with the teacher during breaks between lessons or after school. The duration of conversational interviews varied, but they lasted approximately 15 minutes. During informal conversational interviews, the teacher was asked about inquiries that the researcher brought from her observations of her class or unanticipated issues emerging from observations (Patton, 2002). For example, when the researcher was not certain about if a portion of instruction was implemented as instructional adaptations for the students with MD, the researcher asked the teacher about why she provided the certain kind of instruction during her lesson. Most informal interviews (four out of six interviews) were audio-taped (Holstein & Gubrium, 1995). Field notes were also taken during these interviews.

Clinical Interviews

Clinical interviews with the student participants were conducted over 2 months, from January 2006 through March 2006, while they were receiving instruction on geometry and spatial reasoning, and probability and statistics. Four curriculum-based tasks for clinical interviews were developed (two tasks on geometry and two on statistics). Each task consisted of four alternative transfer problems on a mathematics topic taught in class. Especially on Clinical Interview Task 1 (geometry) and Clinical Interview Task 3(statistics), three different types of problems were included to examine the students' knowledge transfer according to the similarity between the original problem taught in class and the new problems: (a) one base problem, (b) one near-transfer problem, and (c) two far-transfer problems. Construction of the clinical interview tasks is described in the section of clinical interview protocols in more detail.

Forty-two interviews (eight for 5 individual students and six for 1 student who was absent during the third clinical interviews), each interview lasting approximately 20 minutes, were conducted with individual students a week before (baseline interviews) and a day after they were taught on the topic (postinstruction interviews). Performances at baseline clinical interviews served as baseline data to be used to provide a comparison for evaluating student performances at postinstruction interviews.

Implementation procedures. Prior to the beginning of this study, the researcher conducted clinical interviews with student participants (3 fourth-grade students with MD, 2 students who were struggling with mathematics, and 1 typically achieving student) to obtain baseline data for each student's skills in geometry and spatial reasoning and in probability and statistics. At both the baseline interviews and postinstruction interviews, the students received two clinical interview tasks on geometry and spatial reasoning (Clinical Interview Tasks 1 and 2) and two clinical interview tasks on probability and statistics (Clinical Interview Tasks 3 and 4).

Each session of clinical interviews lasted approximately 20 minutes. Each interview was conducted in the hallway near the classroom of the teacher participant and student participants. The clinical interviews were conducted usually from 8:00 to 10:00 in the morning, but the schedules changed according to the teacher or the school situations. Before starting interviews, the researcher put papers, a pencil, an eraser, and cubes on the desk placed in the hallway for this study. At the outset, the student was told that he or she would be asked to participate in some math activities and that he or she could use all the aids on the desk to solve problems.

The procedures of clinical interviews in this study were based on three essential techniques of clinical interviews: (a) testing, (b) observing, and (c) asking students to think aloud (Ginsburg, 1997; Ginsburg & Pappas, 2004). Individual student participants were asked to solve a series of tasks designed to assess the students' curriculum-based mathematics skills (e.g., the skill of creating

3-D buildings matching to 2-D drawings, the skill of finding shapes that would have a specific silhouette, the skill of drawing a bar graph based on the data shown on T-chart, and the skill of comparing two bar graphs) and prerequisite skills related to the target skill being taught, while thinking aloud their problem-solution procedures. Each student was given four sets of a prerequisite problem and a target skill problem in each clinical interview. The student was asked to think aloud while solving each problem. While each student was solving problems, the researcher observed and recorded the student's nonverbal and verbal behaviors relating to solving the problems. Then, once the student finished solving the problem provided, the researcher asked the student (a) how the student knew the method that he or she used for solving the problem (e.g., "How do you know how to find the solids that could make that silhouette?") and (b) how the student verified his or her answer (e.g., "How do you know if you found correct solids?"), (c) how the student's teacher and classmates solved the problem in their class, and (d) if the student knew another way to solve this problem.

Checking implementation fidelity of clinical interviews. To ensure consistency and reliability of clinical interview procedures across students and time, the researcher followed steps and directions that were explicitly scripted in clinical interview protocols, during all clinical interviews. To examine the researcher's adherence to the prescribed clinical interview method, the fidelity of clinical interview implementation (Cass, Cates, Smith, & Jackson, 2003; Fuchs, Fuchs, & Karns,

2001) was evaluated. A graduate student in education was given a checklist of the clinical interview steps and directions that the researcher was to follow when administering a clinical interview and was asked to check whether the researcher followed each step and direction in the same manner as prescribed. The implementation fidelity for implementing the 25% of clinical interview sessions (approximately 11 interview sessions) was 100%. The fidelity of a clinical interview was calculated by dividing the number of items exactly implemented by the number of all items listed in the checklist and multiplying by 100.

Survey Questionnaire

Prior to the commencement of the observations of the teacher's instructional adaptations and clinical interviews with students, the teacher was given a short survey questionnaire on each individual student participant's possession of prerequisite skills required for learning fourth-grade geometry and spatial reasoning, and probability and statistics. The teacher completed a questionnaire for each individual student participant and returned it in mid-December 2005, before the observations and clinical interviews began to be conducted.

On the questionnaire, the teacher was asked to rate her 4 students participating in this study, including 3 students with MD and 1 typically achieving student. The survey questionnaire included 21 items on the skills of geometry and spatial reasoning (11 items) and probability and statistics (10 items). Appendix A shows the actual items included on the survey questionnaire. Data from the survey were summarized in terms of individual students' weaknesses on prerequisite skills required for learning specific fourth-grade mathematics skills. Information from the survey questionnaire was compared and integrated with information from other sources (e.g., prerequisite skills for learning specific math skills identified from review of textbooks and interviews with the teachers) to produce information about what the teacher was supposed to do for the mathematics learning of students with MD in each lesson.

Document Reviews

This study included two types of documents: (a) student documents and (b) teacher documents. These documents, except the teacher's lesson plans, were collected prior to or at the beginning of the data collection period.

Student Documents

Student documents (IEPs and student permanent products) were reviewed to provide information about characteristics of MD participants. First, the researcher reviewed a student's IEP to determine if the teacher's instructional adaptations addressed the difficulties of the student with MD. Second, the researcher gathered student permanent products (e.g., student worksheets and response sheets) through clinical interviews and daily lessons observed for this study and reviewed them to obtain data about the learning of students with different ability in a standards-based mathematics, general education classroom. Student documents were summarized in terms of (a) source of the document (e.g., the IEP of Student A or worksheet of Student A on February 2, 2006), (b) brief summary of the document (e.g., difficulties, recommended adaptations in general education classrooms, procedures that Student A used to solve Problem 1, and time spent for solving Problem 1), and (c) reflective commentary. Student documents such as their permanent products from daily lessons and clinical interviews were photocopied for further analysis and reliability check. However, student IEPs were copied by hand, because photocopying them was not allowed. Appendix C includes a student document review form that was used in this study.

Teacher Documents

This researcher reviewed and summarized the teacher's documents, including the teacher's lesson plans, the mathematics basal curriculum used in the school, and TEKS for mathematics to determine student adequate yearly progress (AYP). First, the teacher's lesson plans were reviewed to provide a framework prior to observations and to gather information about mathematics instruction. Second, the researcher reviewed mathematics basal curriculum used in the school (Math Investigations) in order to have references for identifying instructional adaptations for the MD students and to identify prerequisite skills for the target skills being taught during the data collection period.

Teacher's lesson plans and lessons from the mathematics basal curriculum were summarized in terms of (a) topic or skill, (b) objectives, (c) prerequisite skills, (d) instructional routines with time assignment, (e) activities (both whole-class and small-group), (f) suggested (or planned) instructional adaptations if any, (g) the use of technology, and (h) assessment. Teacher lesson plans and textbook lesson descriptions to be taught were photocopied for further analysis and reliability check. The teacher document review form is attached in Appendix D.

Lastly, this researcher reviewed TEKS for mathematics (TEA, 1998, 2006). These data provided information to be triangulated with other data sources (e.g., observations, interviews, lesson plans, and mathematics textbooks) on instructional adaptations for a student with MD and to identify prerequisite skills for learning TEKS-based, fourth-grade geometry and spatial reasoning as well as probability and statistics.

Data Analysis Procedures

The primary unit of analysis was at the classroom level, Ashley's class, where she implemented instructional adaptations for her students with MD (Research Question 1) and where her 6 students with three different levels of abilities received standards-based mathematics instruction (Research Question 2). However, the attention of the analysis of this study was also given to the subunits of analysis for each research question (Yin, 2003). For the first research question, the teacher's instructional adaptations for each individual student with MD were analyzed as the subunit to enhance the understanding of the case. For the second research question, each group of students with differing ability (group of students with MD, group of students who were struggling with mathematics, and a typically achieving student) was analyzed to promote comparisons of student learning in a standards-based mathematics, general education classroom among groups of students with differing ability.

Analysis units at two different levels in a study allowed this researcher to analyze data from different sources, including survey, observations, interviews, and document reviews, from frequency or accuracy analysis to qualitative analysis through comparisons and syntheses of potential emerging themes (Strauss & Corbin, 1998; Yin, 2003). In addition, it allowed this study to integrate a cross-case analysis method including data display matrices into the case analysis to promote comparisons and integrations of findings across the subunits within the case (Miles & Huberman, 1994; Yin, 2003).

Quantitative Analysis

This study employed quantitative analysis as well. Quantitative data analysis included ratings on surveys, frequency of the teacher's instructional adaptations, and accuracy of student performances in problem-solutions of prerequisite skill problems and target skill problems.

Analysis of Survey Data

Difficulties of the 3 MD students with prerequisite skills for learning fourth-grade geometry and spatial reasoning, and probability and statistics were gathered through a one-time teacher survey about the difficulties of the 3 students with MD and the typically achieving student participant in the knowledge and skills that TEKS standards expected all students to attain by grade 3. Data from the survey were analyzed in terms of the levels of prerequisite skills of students with MD in comparison to the typically achieving student.

For this analysis, the teacher was asked to rate her typically achieving student and the 3 students with MD on the questionnaire. She rated her students from 1 to 3. When a student did not show the prerequisite skills shown on a survey item, the teacher rated the student as 1 on the prerequisite skills. When a student sometimes showed the prerequisite skills shown on a survey item, the teacher rated the student as 2 on the prerequisite skills. Finally, when a student always showed the prerequisite skills shown on a survey item, the teacher rated the student as 3.

The survey included 11 items on the prerequisite skills for learning fourth-grade geometry and spatial reasoning and 10 items on the prerequisite skills for learning fourth-grade probability and statistics. The teacher's responses for each student's prerequisite skills were divided by the mathematics topics. An average rating score was calculated for each student by the mathematics topics, and the average rating score for each student with MD was compared with that for the typically achieving student. In addition, the rating for individual students with MD on each item was compared with that of the typically achieving student. The comparative survey data were triangulated by interview data with the teacher and the prerequisite skills identified in the textbook.

Instructional Adaptations

Identification of instructional adaptations. The teacher's instructional adaptations were analyzed in terms of frequency across lessons, students with MD, categories of instructional adaptations, and evidence-based mathematics instructional components as well as qualitative aspects. In this study, instructional adaptations were identified by the teacher's interactions adjusted for individual students with MD and small groups involving the students with MD. To identify and determine instructional adaptations, the researcher first sorted out the teacher's interactions with any students with MD or small groups involving a student with MD from the pool transcripts of the teacher's instruction. Each interaction was compared with interactions in instruction for all students during a baseline observation within the same instructional routine to determine if it was "special" for students with MD or could occur in her ordinary class instruction within the specific routine. In addition, the teacher was interviewed about why she implemented the specific interactions with the individual students with MD after each observation was conducted. Adapted instruction for the individuals with MD was also determined by the teacher's written records, including notes for accommodations or adaptations in her lesson plans.

Coding and analysis. Once a portion of instruction was identified as an instance of instructional adaptation, the researcher coded the instance according to start list of codes (Appendix E) including (a) categories of instructional adaptations and (b) evidence-based mathematics instructional components. The coded data were clustered by lessons and by individual students with MD (*See Appendix I for examples of data codes*). Within each lesson, frequency of instructional adaptation was calculated across categories of instructional adaptation and evidence-based mathematics instructional components. Likewise, for each individual student with MD, frequency of instructional adaptation was calculated across categories of instructional adaptation and evidence-based mathematics instructional components.

Once frequencies were calculated for each lesson and for each student with MD across instructional adaptation categories and evidence-based mathematics instructional components, they were summed and averaged. This average represented frequency of the teacher's instructional adaptations shown on all three lessons on geometry and spatial reasoning (or all five lessons on probability and statistics) or frequency of the teacher's instructional adaptations using each evidence-based mathematics instructional component during geometry and spatial reasoning lessons or during probability and statistics lessons.

Mathematics Learning

This study analyzed individual students' baseline and postinstruction problem-solving performances on each clinical interview task in terms of accuracy of solving prerequisite skill problems, accuracy of solving target skill problems, and accuracy of transferring the solutions to new problems. The analysis also involved qualitative analysis of problem-solution procedures.

Prerequisite skills. Each student's performance on each prerequisite skill problem during baseline or postinstruction clinical interviews was scored for a correct answer (1 point) or an incorrect answer (0 point). The accuracy of individual student's baseline or postinstruction performances on prerequisite skill problems included in each clinical interview task was calculated by dividing the number of a student's correct answers by the total number of problems that the student tried to solve (4 problems on Prerequisite Skill 1, 10 problems on Prerequisite Skill 2, 4 problems on Prerequisite Skill 3, and 4 problems on Prerequisite Skill 4), and multiplying it by 100%. An average accuracy was calculated for each group of students with differing ability (MD, struggling, and typically achieving students).

Targeted skills. Each student's problem-solving performances on a problem in two geometry and spatial reasoning clinical interview tasks (Clinical Interview Tasks 1 and 2) were scored for a complete correct problem-solution (1.0 point), a partially correct problem-solution (0.5 point), or an incorrect problem-solution (0 point). A partial point (0.5) was given to student answers on the problems in Clinical Interview Task 2, which had more than one correct answer. When a student's answer included all possible correct answers, a complete credit (1.0 point) was given. When a student's answer included at least one correct answer, a partial credit (0.5) was given.

Similarly, on two probability and statistics clinical interview tasks (Clinical Interview Tasks 3 and 4), a score of 0, 0.5, and 1.0 points was given to each student's performance on each problem in each clinical interview task. On the problems in Clinical Interview Task 3, students got a full credit when they presented a complete correct bar graph representing the data shown in the table. Otherwise, their performances were scored as 0. On the problems in Clinical Interview Task 4, the student answer on each problem was scored as 1.0 only when it was correct and was based on comparisons using at least two of the following criteria: (a) range, (b) pattern, (c) spread and clumping, (d) typical values, (e) bar-by-bar comparison, and (f) relationships or variables embedded in the data set. When the student answer included comparisons based on only one criterion, a score for a partially correct problem-solution (0.5 point) was given. When the student answer did not include comparisons based on any of the above, the answer was scored as incorrect and received a score of 0. Comparisons based on physical features such as color or size were not counted as correct responses. The scores of four problems were summed and divided by 4 to produce the accuracy in each clinical interview.

Transfer of the solutions. Research on transfer of knowledge and skills has procedures to ensure students' acquisition of knowledge and skills in one situation before investigating transfer of the knowledge and skills to different situations (Bassok, 1997). Thus, for the analysis of transfer of mathematics knowledge and skills of the students with different ability, this study included only the students who correctly applied the problem-solving procedures taught in class to solve base problems in each clinical interview task (Problems 1 and 2 in Clinical Interview Task 1 and

Problem 1 in Clinical Interview Task 3). As a result, 3 students were selected for analyzing students' transfer of geometry and spatial reasoning skills taught in class. In probability and statistics, no students met the selection criteria. The 3 students selected for analysis of transfer of geometry and spatial reasoning skills were Tina (MD student), Jose (struggling student), and Amy (typically achieving student). These 3 students were compared in terms of their accuracy of problem-solutions for target skill problems according to problems with different similarity to the original problems.

Qualitative Analysis

A qualitative analysis of this study followed the analytic process of the cross-case analysis. The analytic process involved three interactive and cyclical flows of activities: (a) reducing data, (b) displaying data, and (c) drawing conclusions and verifying conclusions (Miles & Huberman, 1994).

Data Reduction

The process of reducing data involved reviewing sets of data line by line and coding them in a unique unit in a provisional start list of codes, which was predetermined by the researcher at the beginning of data analysis (Miles & Huberman, 1994) and evolved throughout data analysis to better account for the data. During this process, data were broken into discrete ideas or concepts and given a name of categories or meaning units in a start list of codes.

Start list of codes. For this study, a provisional start list of codes was developed based on the review of literature on mathematics instruction, student mathematics learning, and the open coding of an initial data set (Glaser & Strauss, 1967). A start list of codes for this study consisted of (a) different levels of categories, (b) codes representing the categories, and (c) the research questions of this study. The first column of the list showed a short descriptive label for general categories and individual codes, which stemmed from groundwork for this study. The second column had codes, and the third column linked each code to the research question from which it derived (Miles & Huberman, 1994).

Reviewing literature and open coding of initial data yielded three broad meaning units (categories) for the first research question and another three broad meaning units for the second research question. The three categories linked to the first research question were (a) categories of instructional adaptations, (b) evidence-based mathematics instructional components, and (c) instructional adaptations addressing the difficulties of students with MD in prerequisite skills. The three categories linked to the second research question were (a) prerequisite skills of students with different ability, (b) problem-solution of students with different ability, and (c) transfer of mathematics knowledge and skills.

Subcategories were embedded in each broad category. Some subcategories were key aspects or components constructing or representing the broad categories. Subcategories for the first research question included (a) the categories of instructional components (e.g., instructional content, activity, delivery of instruction, and material or technology), (b) the components of evidence-based mathematics instruction (e.g., corrective feedback, explicit explanation, and grouping), and (c) instructional adaptations to address the difficulties in prerequisite skills according to the identification source of the difficulties (e.g., teacher-identified difficulty, TEKS-based difficulty, and text-identified difficulty). Subcategories for the second research question were related to (a) the components of mathematics knowledge and skills (e.g., prerequisite, strategies, representations, and algorithm), (b) the application of mathematics knowledge and skills for solving new problems (e.g., use of knowledge and skills learned from class, exploration of multiple strategies, and verification of solution), and (c) mathematical communication skills (e.g., language, and symbols/notations).

During data analysis, when a new category or code was generated or a predetermined category or code did not fit the data, the start list of codes was revised to better account for the data. Categories or codes in the coding list were operationally defined and attached to the coding list for consistency of data analysis. Appendix E illustrates what the start list of codes for this study looked like.

Coding data. As soon as data collection began, the researcher reviewed the first sets of transcripts of field notes or audio-taped data line by line, broke down the data into discrete ideas, and categorized each unit of data set into one or more of the provisional start list of codes or generated names of categories that account for the unit of data set. The output of the first stage of data analysis, data reduction, was summaries of categories or codes of the teacher's instructional adaptations for individual student with MD and those of mathematics knowledge and skills that individual students used to solve problems. Appendix J shows one output from this stage.

Data Display

The second stage of data analysis for this study involved constructing cross-case data display matrices, one for each research question (Miles & Huberman, 1994). Although this study was not a multiple-case study but a single-case study, subcases (e.g., 3 different students with MD, 2 different struggling students) were embedded or nested in the case of this study (Ashley's class). Accordingly, this study used a cross-case study analysis method including the data display matrices. The matrices were used to explore patterns or potential themes that might represent the teacher's instruction for students with MD in the standards-based mathematics, general education classroom and the mathematics learning of her students with different ability in the instructional environment. In addition, the data display matrices could be used for data analysis of this study because this study used a priori ideas as bases for analysis (Miles & Huberman, 1994). For example, this study analyzed data on the teacher's instruction based on knowledge about evidence-based mathematics instruction for students with MD (e.g., Miller et al., 1998). In addition, this study analyzed data on student's mathematics learning based on age-appropriate mathematics knowledge and skills including concepts, representations, strategies, and algorithm shown in literature or documents (e.g., TEKS for mathematics, TEA, 1998) and information about the characteristics of students with MD (e.g., Geary, 2004; Swanson & Jerman, 2006).

In each matrix, nested cases appeared in the columns, and the variables of interest were shown in the rows. For example, the columns of the matrices showed individual student names (pseudonyms) and the rows of the matrices had variables related to the research questions of this study. For instance, the rows of the first matrix had variables related to the features of effective mathematics instruction, which appeared to be used for adapting mathematics instruction by the teacher (e.g., grouping, review of prerequisite skills, and use of manipulatives). Similarly, the columns of another matrix had variables including (a) prerequisite skills, (b) problem-solving, and (b) transfer of mathematics knowledge and skills of students with different ability in the standards-based mathematics, general education classroom.

To examine and summarize the teacher's instructional adaptations for individual students with MD and each student's mathematics knowledge and skills, the researcher synthesized categories from the coding process of observation data, interview data, and document reviews. The teacher's instructional adaptations for each student with MD and individual student's mathematics knowledge and skills were summarized in terms of specific variables related to instructional adaptations (e.g., use of features of effective mathematics instruction and modification of assessment) or student's mathematics knowledge and skills (e.g., prerequisite skills, mathematical problem-solving using mathematics knowledge and skills taught in class, and exploration of multiple strategies).

Once the summaries of instructional adaptations for each student had been developed, instructional adaptations across the students with MD were examined by comparing and synthesizing the individual students' summaries in order to find patterns or themes in the general education teacher's instructional adaptations within the standards-based mathematics classroom for students with MD (e.g., the teacher in standards-based mathematics classrooms adapted her mathematics instruction for students with an IEP in mathematics by using explicit, direct instruction).

Similarly, once individual students' summaries had been developed, individual MD students' summaries were compared to each other and synthesized to find patterns of mathematics learning of students with MD in the standards-based mathematics classroom. Then, the summary of mathematics learning of students with MD in the standards-based mathematics classroom was compared with those of other individual students with different ability within the class to produce information about the MD students' mathematic learning compared to their general education peers.

As themes or patterns on instructional adaptations for students with MD and MD students' mathematics knowledge and skills compared to their general education peers with different abilities began to appear, the matrices were reconfigured around these specific themes to draw further conclusions. For example, if use of the features of effective mathematics instruction was identified as a pattern of the teacher's instructional adaptations within the standards-based mathematics classroom, the researcher would examine which components or features were used most frequently in the teacher's instructional adaptations. When students with MD appeared to make progress between pre- and postinstruction clinical interviews, the researcher compared their performance with the typically achieving student's performance. Summaries of the findings for further questions were nested within the column of higher levels of categories on the matrices.

Drawing Conclusions and Verification

The last stage of data analysis was to draw conclusions about the teacher's instructional adaptations for students with MD within the standards-based mathematics curriculum and instruction, to draw conclusions about the MD students' learning of mathematics knowledge and skills compared to their general education peers with different abilities within these environments, and to verify these conclusions (Miles & Huberman, 1994). The researcher returned to examine the original data to confirm initial themes or concepts emerging from the data matrices and to integrate and refine them through explanatory statements of their relationships (Strauss & Corbin, 1998). Integration occurred over time, beginning with the first step of analysis and continuing until the final writing. To facilitate the integration process, diagrams representing the relationships between themes or concepts were created when they were helpful (Strauss & Corbin, 1998). The conclusions were verified by the processes of peer debriefing and member checking.

Credibility of the Research

A number of procedures, including redundancy of data gathering and procedural challenges to explanations, can be used to increase the probability that credible findings have been produced by qualitative research (Denzin, 1978; Lincoln & Guba, 1985). The same procedures can be used to maximize the validity and credibility of a qualitative case study (Yin, 2003). Thus, this study used triangulation, member checks, peer debriefing, and persistent observations to establish the credibility of the findings.

Triangulation

Triangulation is one of the procedures that qualitative case researchers commonly use to reduce the likelihood of misinterpretation of data (Stake, 1995). Triangulation is generally considered as a process of using multiple perceptions to clarify meaning, verifying the repeatability of an observation or interpretation (Stake, 1995). Two different modes of triangulation were used in this study (Denzin, 1978): (a) the use of multiple and different sources and (b) the use of different methods. First, this study employed different sources to obtain information about (a) the general education teacher's instruction for students with MD within standards-based mathematics curriculum and instruction and (b) mathematics learning of students with different levels of ability in these instructional environments. For example, the researcher took field notes and audio-taped interviews with the different general education teacher to increase the likelihood that credible findings were produced. Second, this study employed different data collection methods to verify the findings or clarify interpretations. For example, data for this study were collected by different methods, including observations, interviews, survey, and document reviews. The researcher compared data from observations with data from interviews and document reviews and integrated them to assure reliability of data and sound interpretations.

Persistent Observations

Persistent observations are used to identify the characteristics of the specific situations that are most relevant to the issues of interest and to focus on these characteristics in depth (Lincoln & Guba, 1985). This researcher continuously engaged in tentative concepts or patterns emerging from document review, observations, and interviews and then explored them in depth, to the point where either the initial patterns or themes produced inappropriate data, or they could be clearly understood. For example, when tentative themes or patterns (e.g., the teacher's instructional adaptations were restricted to 1 student whom the teacher recognized as struggling the most in each mathematics topic) emerged from original data on teacher's instructional adaptations, the researcher focused on the patterns or themes during the rest of the observations or throughout data analysis to explore them in depth. After finishing data collection, the researcher returned to the original data to determine if these initial themes or patterns represented the teacher's instructional adaptations for students with MD or the mathematics learning of students with differing ability in a standards-based mathematics general education classroom.

Peer Debriefing

Regular peer debriefing sessions were held twice a month during the period of data analysis to probe the researcher's possible biases, to explore meanings, and to clarify interpretations and findings. A doctoral student in education served as a peer debriefer. He was familiar with the research topic but was not directly related to this research (Lincoln & Guba, 1985).

Member Checks

To increase the overall credibility of the findings of this study, the teacher was asked to check data, analytic categories, interpretations, and findings during data analysis (Lincoln & Guba, 1985). Two formal member checks were conducted during the study, during and after the data collection period. Informal member checking occurred immediately when the researcher was unsure of the teacher's intention as indicated by her acting or telling specific things, when the researcher wanted to get additional information, or when the researcher desired to assess overall adequacy of data and to confirm individual data points. Informal member checks sometimes occurred during informal conversational interviews.

Researcher as Instrument

In a qualitative study, the researcher plays an important role during data collection and data interpretation. A qualitative researcher brings his or her personal biases and beliefs to the research that he or she is conducting, which influences the findings of the research (Stake, 1995). Thus, it is important to inform and describe the lenses through which a researcher filters multiple realities of specific phenomena (Scheurich, 1997).

The researcher's interest in MD developed from her former major, psychology, and her personal experiences as a clinical child psychologist. The researcher majored in cognitive psychology during her undergraduate and graduate programs. Through those programs, she was trained to understand disabilities, including MD, from an information-processing approach: A disability may be the result of skill deficits in a specific processing stage. As a cognitive psychologist, the researcher participated in several research projects to examine characteristics of students with MD or to develop screening or diagnostic tests for MD. Through these projects, she developed the belief that students with MD are different from their typically achieving (or developing) peers in terms of their cognitive and performance skills and should be treated differently from typically achieving students. In addition, as a clinical child psychologist, the researcher often observed that the performances of students with mathematics difficulties were improved by treatments or interventions based on their deficit skills or characteristics. Through these previous experiences, the researcher came to believe that students with LD should receive treatment or education based on their difficulties or needs.

For the 3 years prior to this research, the researcher studied LD in special education. During this period, she learned about educational features related to effective instruction for students with LD. She gained a sense of how teachers can help students with LD in education settings. Based on this learning in special education, the researcher came to believe that direct, systematic instruction should be implemented for students with LD in education settings. In addition, the researcher believes that general education teachers should provide mathematics instruction adapted according to student characteristics or difficulties.

As a research instrument, this researcher brought an advocating perspective for providing students with MD instructional adaptations based on individual student difficulties or needs, especially in standards-based mathematics classrooms to this study. Being aware of the threats to validity of research findings, thus, the researcher tried to make the influence of the threats minimal and gathered data best representing “reality” through persistent self-reflection and discussion with her academic advisor and peers during this study.

CHAPTER 4:

RESULTS

Despite increased efforts to reform mathematics education in the U. S. over the last two decades, such as the NCTM's (2000) *Principles and Standards for School Mathematics*, recent national assessment data indicate that a significant proportion of U.S. students do not attain the basic level of understanding of the mathematics concepts and procedures at any given grade (Perie et al., 2005). In addition, students continue to struggle with the basic level of mathematics concepts and procedures. For example, in eighth grade, approximately 31% of students scored at or below the basic level of understanding on a nationally representative and continuing mathematics assessment; these students failed to show adequate evidence of understanding the mathematical concepts and procedures, whereas 20% of fourth-grade students did not demonstrate the basic level of understanding at the fourth-grade level for mathematics concepts and procedures (Perie et al., 2005). Considering that increased numbers of students will struggle as they get older, it is important to search for ways to help students be successful before they reach the higher grades, where they will be expected to learn more complicated, advanced mathematics knowledge and skills.

In today's classrooms, struggling students, including students with MD, tend to receive their mathematics instruction in general mathematics classroom (The 24th Annual Report to Congress on

IDEA, 2002), which are guided by NCTM standards-based mathematics curriculum and instruction emphasizing a problem-solving, inquiry-based approach (Lappan, 2000; NCTM, 2000). However, the features of NCTM standards-based mathematics instruction have been questioned in terms of their effectiveness to teach struggling students, including students with MD (Woodward & Montague, 2002). The instructional features in today's mathematics classrooms are poorly suited to the individual learning needs of students with MD, who may be adversely affected by the synergy between their cognitive characteristics and the features of instruction.

In education, there is a long-standing belief that instructional adaptations to address student's individual needs make a crucial contribution to student learning (Fuchs et al., 1992). Teachers in today's general education classrooms are required to adapt their instruction for students with disabilities to maximize their access to the general education core curriculum (IDEA, 2004). By law, struggling students are expected to be able to achieve a similar level of understanding of mathematics knowledge and skills to that expected of their typically achieving peers.

Given the challenges that students with MD may encounter in standards-based mathematics classrooms and the mandates requiring instructional adaptations for the students with MD in the instructional setting, it is important to examine how general education teachers are adapting standards-based mathematics instruction for students with MD and how successfully students with MD are learning the curriculum in standards-based mathematics, general education classrooms.

Therefore, this study investigated how a fourth-grade general education teacher adapted standards-based mathematics instruction for 3 students with MD in her classroom and how 6 fourth-grade students with differing levels of ability (3 identified MD, 2 struggling, and 1 without disability) learned grade-level mathematics knowledge and skills in a standards-based mathematics classroom. The teacher's instructional adaptations were explored in terms of (a) frequency and settings associated with the occurrence of instructional adaptations, (b) categories of instructional adaptations, (c) use of evidence-based mathematics instructional components, and (d) instructional adaptations addressing student difficulties in prerequisite skills for learning each content (e.g., geometry and spatial reasoning as well as probability and statistics). The students' mathematics learning was explored in terms of application or transfer of knowledge and skills to new problems. Student mathematics knowledge and skills were probed prior to and after receiving mathematics instruction on the topics, compared, and described in terms of (a) prerequisite skills, (b) problem-solving performances, and (c) transfer of the knowledge and skills.

A single-case study design was employed to examine the teacher's instructional adaptations for students with MD and the mathematics learning of 6 students with differing levels of ability in a standards-based mathematics, general education classroom. This study explored answers for each research question by utilizing a case analysis involving data reduction, data display, conclusions, and verification of conclusions (Miles & Huberman, 1994).

This study was guided by the following two research questions:

1. How does a fourth-grade general education teacher adapt mathematics instruction within a standards-based mathematics curriculum for students who have an IEP in mathematics and who receive mathematics instruction in the general education classroom?

2. How do 6 fourth-grade students with differing ability (3 identified MD, 2 struggling, and 1 typically achieving student) who receive mathematics instruction in a standards-based mathematics, general education classroom use mathematics knowledge and skills taught in class to solve curriculum-based problems after they have received classroom instruction?

The following sections provide a description of the participants, the findings on instructional adaptations for the students with MD, and findings on mathematics learning of students with different ability in standards-based mathematics classrooms.

Participants

For the first research question, the researcher observed 10 mathematics lessons (two 90-minute lessons to establish baseline and eight 60- to 90-minute lessons during data collection, for a total observation time of approximately 680 minutes). These observations were conducted in the classroom of a fourth-grade general education teacher who had 3 students with an IEP in mathematics, during mathematics instruction that employed a standards-based mathematics curriculum. For the second research question, the researcher administered 25 clinical interviews with 6 students with differing levels of ability, for a total interview time of 750 minutes. The student participants were 3 students with IEP goals in mathematics (noted as students with MD), 2 students who were identified by their teacher as struggling with mathematics, and 1 typically achieving student from the teacher participant's class.

Below are descriptions of the 6 student participants and the single teacher participant of this study. The 6 student participants are described in terms of their demographic information, their disability or difficulty as identified by the teacher or the school district, and their strengths in the area of mathematics. The teacher participant is described in terms of her background, her school and class, and her beliefs and thoughts about standards-based mathematics curriculum and instruction.

Students

Lee Jordan

Lee Jordan was a fourth-grade student who had an IEP in mathematics and was receiving mathematics instruction in Ashley's class. Lee and her teacher (Ashley Hamilton) identified her ethnicity as African American. Lee's twin sister was in the same classroom; she was performing at the average level in all academic areas. Based on the researcher's observations on group work involving Lee, this student was very cooperative and actively involved in group work.

Lee had IEP goals in content mastery as well as in mathematics. Short-term objectives set on mathematics included improving skills of (a) telling time on an analog clock to the nearest 5-minute interval; (b) using concrete models to represent, compare, and order whole numbers through 999; (c) solving basic addition facts with sums to 18; (d) solving 2-digit subtraction problems without regrouping, (e) measuring lengths, using standard units such as inch, foot, or centimeter; and (f) counting a group of mixed coins in value up to $1.00.

The Admission, Review, and Dismal Process (ARD) committee recommended that Lee should be taught mathematics in general education classroom at least 30 minutes per day. Also, additional accommodations by coordination between the general education teacher and special education teacher were recommended for improving her mathematics skills. The state alternative assessment on mathematics was recommended.

Ashley recognized Lee's expressive verbal skills as her strength and facts memory and multistep problem solving as her greatest struggles. Lee had problems retrieving multiplication and division facts from her memory and solving a problem that involved the application of multiple steps to get an answer. Ashley also noted that Lee had difficulties in keeping her attention on task. Ashley noted,

> She is a very good talker. She is very good at explaining something, even though that may not be correct. She is good at explaining what she is thinking orally. She works very well with a group. She keeps her group on task. She does use her strategies that I have taught in class. She highlights important words something like that. She does use her strategies but I think she kind of rushes through her work a bit. Even in math, she struggles in spellings. So, whenever I ask her to write her answers or explain her answer. I have a hard time in understanding what she means on her worksheets. So, she starts to struggle actually when she starts to write her answers on her paper. She is better orally in explaining herself. Her multiplication facts, I mean, she has a hard time in remembering them throughout the entire

> year. Just memorizing processes like division, she struggles with steps of the process. Even the probability, I think that is hard to remember the process that she needs to add the total first, and you know, pull out one information out of them. But, another thing lately she has been really hard to keep her focus is that her attention span is pretty short. So, after a certain time, she is not really working any more, you know, so I have to have her quit her day.

Kevin Green

Kevin was another student who had an IEP in mathematics in Ashley's class. Kevin and his teacher identified his ethnicity as African American. He had IEP goals in language arts and content mastery as well as in mathematics. His IEP goals in mathematics included enabling him to achieve the skills of converting a fraction into a percent and the skills of solving two-step word problems. The ARD committee recommended that he be taught in his general education classroom at least 30 minutes per day for mathematics instruction. The committee also recommended that the general education teacher and special education teacher should make additional accommodations and modifications for his mathematics skills. Kevin also recommended to take state alternative assessments on mathematics, reading, and writing.

According to Ashley, Kevin was capable of retaining information from the class, using his strategies, and identifying certain important information in problems. However, he had difficulty with utilizing proper mathematics vocabulary, even though he could explain his idea using words. He also had problems in solving multiple-step word problems. The teacher thought that his low reading level had much to do with his struggles with fourth-grade mathematics. Ashley stated,

> Kevin, he, pretty much can retain information from the class. He is pretty good at memorizing things. He has a hard time with actually using proper vocabulary. He can explain using his words, but may have a hard time in remembering the actual math terms. Umm, he definitely uses his strategies, goes and highlights certain important information in a problem. He is pretty good at that. And, he struggles, I think, when it gets to be multiple step problem solving, more than one step. He has a hard time in reading them. His reading level is pretty low. So, I think it has a lot to do with his struggling with fourth grade mathematics.

Tina Jhaden

Tina was the 3rd student who had an IEP in mathematics in Ashley's class. Tina and her teacher identified her ethnicity as European American. She had IEP goals not only in mathematics but also in language arts and content mastery. Her IEP set a goal for improving mathematics skills of choosing the correct number sentence to solve a two-step word problem involving addition, subtraction, multiplication, or division. It was recommended that Tina be included in the general education classroom at least 60 minutes per day for the mathematics lesson. It was also recommended that the general education teacher and special education teacher should coordinate to make additional accommodations and modifications for her. Tina was not recommended for the state alternative test on mathematics (SDAA).

Ashley described Tina as one of her quieter students. Ashley recognized that Tina had a high level of reading skills, that Tina was capable of using strategies for mathematics problem solving (e.g., highlighting important information), and that Tina was capable of memorizing mathematics facts. She stated concerns about Tina's struggles with solving multistep word problems:

> Tina is, she is hard for judge. She is so quiet. I have several students like her in my class. It's hard to get anything out of them. She is very good at using her strategies. I don't have to remind her of going to important information. She is pretty good at recognizing code words that tell you how to solve the problem, which operations you choose. She struggles with multiple steps problems. She does not…She just wants one step instead of adding and then subtracting. She wants to solve a problem using one step. Her reading level is higher. So, she is able to read very well. Vocabulary, I don't think she has problems as much as steps. Well, she is pretty good at her multiplication facts and division problems. She is able to do that too. For some reasons, just when we got to do some problem solving, I don't know she just gets overwhelmed or confused by the length of problems when she sees these.

Laura Martinez and Jose Chavez

Laura and Jose participated in this study as struggling student participants who were identified by Ashley. These students were nominated and selected as participants as they met the criteria of selecting struggling students: They passed TAKS mathematics and had not been identified as having MD by their school district, but they usually did not meet the learning expectations of each mathematics lesson. They were also ranked between 25 percentile and 45 percentile on mathematics ability, as perceived by their teacher. Both students were described as Latin American by themselves and their teacher.

Amy Williams

Amy was participating in this study as a typically achieving student participant. She is European American. Her teacher, Ashley, nominated her as a typically achieving participant because (a) she perceived that Amy would be ranked between 70 percentile and 80 percentile on mathematics, (b) Amy usually satisfied the learning expectations of each lesson, and (c) Amy did not have any problems or difficulties in any areas. Ashley noted, “Well, she is one of my higher achieving students. She is trying to be hard. She can explain herself. She works well with other students. She works on time. I would rank her as third in my class.”

Teacher

Ashley Hamilton is a fourth-grade general education teacher in a suburban school district in central Texas. She self-identified as European American and is in her late 20s. She majored in elementary education at a college in eastern Texas. She was certified in teaching reading in Grades 1–8. She had been teaching elementary schools for 4 years. Since she started her career as a teacher, she has always taught at the fourth-grade level.

The year of this study was her 2nd year to use Math Investigations for her mathematics instruction. Before starting to teach Math Investigations, she was trained to teach the program at a daylong workshop by the school district where she is employed. After starting to use the program for her mathematics instruction, she received ongoing, whole-day training at 6-week intervals about how to use the program to teach mathematics.

In comparison to other schools in the state, the school where the teacher participant was employed had a smaller number of students from economically disadvantaged families and a larger number of students who passed the state-mandated assessment. Only 26% of students in the teacher participant’s school came from families that were classified to be economically disadvantaged, in contrast to 55% of students from this population across the state. In addition, 98% of all fourth-grade students in the school passed the mathematics part of the TAKS during 2004–2005 school year, compared to 82% of fourth-grade students state-wide (TEA, 2005).

During the observation period, 13 students were enrolled in Ashley's class. Her class students included 5 European Americans, 4 African Americans, 2 Asian Americans, and 2 Latin American (Hispanic). Out of 3 students with MD in Ashley's class, 2 students with IEP goals in mathematics also had IEP goals in other academic areas (e.g., reading and content mastery). She identified time management as her biggest problem in doing inclusion.

> My biggest struggle is the time. Because I have got students leave at certain times, basically, I have to stop my teaching at some points until they come back, because I don't want them to miss out one thing. Also, when they started an activity and leave, I struggle with making decision on whether I finish this activity before they come back and grade what they have done, or whether I have to wait for them to finish the work. So, it's hard to grade the students' work.

Ashley taught mathematics every day for approximately 90 minutes. She used the Math Investigation program for 3 days of each week. She also used another textbook for the remaining days of the week, usually to introduce the lesson or to assess students at the end. She recognized that hands-on learning, use of cooperative grouping, discussion, infrequent use of worksheets, and freedom for students to choose their own way of solving a problem as the distinctive features of standards-based mathematics instruction. She described her role during her instruction as a monitor of student learning rather than as a controller of student learning:

> Umm, I assume that a fourth-grade classroom, it would definitely involve some hands-on learning. You would see students using manipulatives, using, you know, even things like that. Today we used the clocks, things like they can actually put their hands on, building with cubes, building pattern blocks. Also, it involves partners, partner work, peer study, learning from peers. And, discussion would be, you know, the big part of a lesson. You wouldn't see students sitting on their desks to work on worksheets on their own. You would see group discussion, actually getting up and moving around, so some movements among the kids, maybe, having the movement from group to group or so. And then some, you will see students writing out their thoughts, explaining themselves, maybe discussing their explanations one another. And, it also would not involve one correct answer. It allows the kids to have some freedom to come up with their own answers as long as they can explain it. It's more freedom. It's a lot more student-centered, not teacher-centered. Kids can control over the lesson. I do not control their learning or lessons but monitor their learning and make it sure they're staying on tasks.

She did not perceive that teaching the Math Investigation program to high-achieving students and low-achieving students differed. Rather, she thought that some features of Math Investigation program, including hands-on activity and exploration of multiple strategies, were beneficial for both low- and high-achieving students.

> So, it gives me a lot of good information to help struggling students. It gives me several strategies instead of just one approach. You know, it gives the kids freedom to choose their own way of solving a problem. …I think it is beneficial to both high and low. The high kids might come up with a new way of solving the problem. Then, I have them show the class how they got it. And then, low kids are able to actually use and build things that they can figure it out from hands-on approach. So, I think it's good for both, high and low. …
>
> It [Math Investigations] really does not have differences in teaching for high and low kids. …It gives me techniques or something that I can go. It will say, you know, for those who have language barriers something, I have an ESL [English as a second language] student. Even it has helpful information to use for students struggling with language skills. As far as instruction goes, it does give you things or tips to use for your struggling students. Umm, but for the higher students, it does not really. You don't teach them differently, but they are allowed to have freedom to work with the problems in any way they want. They

> don’t have to work it in a certain way. As long as they are able to come up with an answer and explain how they got it, then the program accepts that. That’s what I like.

Differences she identified included helpful hints for teaching struggling students, especially for ESL students, more emphasis on the use of manipulatives and technology (e.g., calculator), and linkage of mathematics to real life.

> Yes, it pushes hands-on, building for the struggling students. They recommend that more than even for the higher students. …It does not focus a whole lot of technological things, computer things. They do allow for calculators. They like kids to check their work and learn how to use calculators. But main pressure is using hands-on and making it real to the kids. Like making them be able to compare to real world, finding examples in real world. So, each time we start a new unit, I send home a parent note so that parents can use some of these ideas at home.

Her favorite part of Math Investigations was that the program accepted a student answer as long as the student was able to come up with an answer and explain how he or she derived it. Her greatest struggle with Math Investigations was that it did not include assessments.

> I really like the program because for higher students, there is really no right or wrong way to work out the problems. They give them freedom to come up with their own way to work on the problems. And then, for the lower students, there are several, every Investigations unit has a page of helpful hints, approaches, and strategies. Even they sort of predict questions kids might ask ahead of time. So, they have that in my teacher’s manual that I can kind of plan ahead for the student questions. …
>
> Umm, Investigations program does not come with assessments or tests that we can give them. It just has different student sheets. So, I use the student sheets that they turn in to me that is related to that lesson for their assessment on that. But, I use tests from a chapter book. So you know, a math book, or textbook. …
>
> Assessment is one of my struggles this year. Investigations don’t have assessments. I am kind of using benchmark tests that our school takes.

Research Question 1: Instructional Adaptations for Students With an IEP in Mathematics in a Standards-Based Mathematics Classroom

This section describes the findings of this study to answer Research Question 1, inquiring about a fourth-grade general education teacher's instructional adaptations for 3 students who had IEP goals in mathematics and were receiving their mathematics instruction in a standards-based mathematics, general education classroom. Findings were derived from a case analysis of the teacher's instructions for 3 students, which were gathered from four different data sources: (a) observations, (b) interviews, (c) questionnaire survey, and (d) document reviews. To produce more credible findings, this study employed a number of techniques, including triangulation, member checks, peer debriefing, and persistent observations.

Data collection occurred over 12 weeks during December 2005 through March 2006, while the teacher was teaching two mathematics content areas: (a) geometry and spatial reasoning and (b) probability and statistics. Class observations included two baseline observations (one per mathematics content) and eight observations of adaptations, each lasting approximately 60–90 minutes. Interviews included two 1-hour formal interviews and six 15-minute informal conversational interviews. The survey data were gathered from a one-time survey in which the teacher was asked to evaluate geometry and statistics prerequisite skills of each student participant.

Finally, document reviews included reviewing student IEPs, the teacher's lesson plans, and the Math Investigations program used for the teacher's mathematics instruction.

The findings of the teacher's instructional adaptations across three different students were categorized by TEKS mathematics strands (geometry and spatial reasoning, and probability and statistics). Descriptions of each mathematics content area had six major sections: (a) standards, (b) prerequisite skills, (c) difficulties of students with MD with prerequisite skills, (d) typical instruction during the baseline observation, and (e) instructional adaptations. Typical instruction was described in terms of (a) curriculum, (b) instructional routines, (c) instruction for all students, and (d) instruction adapted for students with MD. The section of instructional adaptations was further divided into five subdivisions: (a) identification of instructional adaptations, (b) frequency and settings associated with instructional adaptations for individual students with MD, (c) categories of instructional adaptations, (d) incorporation of evidence-based mathematics instructional components into core instruction, and (e) instructional adaptations addressing students' difficulties in prerequisite skills.

Geometry and Spatial Reasoning

Teaching geometry is an essential component in mathematics education (Oberdorf & Taylor-Cox, 1999). According to the NCTM standards (NCTM, 1989, p. 48), geometry instruction plays a significant role in developing students' systematic representation of their world. Especially for students with MD, geometry is a mathematics area in which students with MD can be successful problem solvers using problem-solving strategies proportionate to their abilities (Grobecker & De Lisi, 2000). Accordingly, a small change in mathematics instruction likely may make valuable contributions to the successful learning of students with MD in general education classrooms. However, little study has been conducted on geometry instruction for students with MD (Rivera, 1997), let alone instructional adaptations for students with MD in standards-based mathematics classroom. Thus, it is significant to examine how a general education teacher adapts her instruction for her students with MD in a standards-based mathematics, general education classroom.

This section describes findings on instructional adaptations that Ashley Hamilton made for her 3 students with an IEP in mathematics during her geometry instruction. She taught geometry for 2 weeks, during February 2006. During this period, her three lessons using Math Investigations (TERC, 1998) were observed for a total observation time of approximately 270 minutes. These four lessons were focused on developing some basic concepts and the language needed to reflect on and communicate about spatial relationships in 3-D environments. Prior to conducting observations of her geometry lessons, this study gathered teacher interview data about teaching geometry to fourth graders, one-time survey data about student difficulties with prerequisite skills for geometry, and document review data on student difficulties (e.g., IEP and lesson plans).

Standards for Geometry

Geometry is a branch of mathematics that deals with the measurement, properties, and relationships of points, lines, and 2-D and 3-D figures. According to NCTM (2000), fourth-grade students are expected to be able to (a) analyze characteristics and properties of 2-D and 3-D geometric shapes and develop mathematical arguments about geometric relationships; (b) specify locations and describe spatial relationships using coordinate geometry and other representational systems; (c) apply transformations and use symmetry to analyze mathematical situations; and (d) use visualization, spatial reasoning, and geometric modeling to solve problems.

The state standards for mathematics education, TEKS (TEA, 2006), have very similar expectations for mathematics learning of fourth-grade students. In Texas, students in fourth grade are expected to (a) identify and describe attributes of geometric figures using formal geometric language, (b) connect transformations to congruence and symmetry, and (c) recognize the connection between numbers and their properties and points on a line after receiving regular mathematics instruction. Specifically, they are expected to (a) identify and describe right, acute, and obtuse angles; (b) identify and describe parallel and intersecting lines using concrete objects and pictorial models; (c) use essential attributes to define 2-D and 3-D geometric figures; (d) demonstrate translations, reflections, and rotations using concrete models; (e) use translations, reflections, and rotations to verify that two shapes are congruent; (f) use reflections to verify that a shape has symmetry; and (g) locate and name points on a number line using whole numbers, fractions such as halves and fourths, and decimals.

Such standards assume that students bring prerequisite skills required for learning geometry content to be taught at a given grade to their mathematics class. The following sections describe prerequisite skills for learning fourth-grade geometry and MD student participants' difficulties with the prerequisite skills.

Prerequisite Skills

Prerequisite skills refer to skills required to accomplish a new task (VGCRLA, 2001). Research on the development of children's mathematical thinking has shown that children tend to be naturally motivated to learn about the geometry, because it helps them think of their world in a systematic way (Fuys & Liebov 1993; Oberdorf & Taylor-Cox, 1999). In school, students start to learn geometry and its related skills from kindergarten (NCTM, 2000; TEA, 2006). Accordingly, students are expected to have more knowledge about geometry and to prepare for advanced geometry skills as they grow up.

According to the state standards in Texas, TEKS (TEA, 2006), fourth-grade students are expected to bring geometry knowledge and skills including (a) describing the relative positions of objects (e.g., describing one object in relation to another using informal language such as *over, under, above,* and *below* and placing one object in a specific position) (; (b) using attributes to determine how objects are alike and different by informal language; (c) recognizing geometric shapes in their real life; (d) using attribute to identify, compare, and contrast 2-D and 3-D geometric figures by using informal and formal language (e.g., circles, triangles, rectangles, squares, spheres, rectangular prisms, cubes, cylinders, and cones); (e) using attributes to identify, compare, and contrast 2-D and 3-D geometric figures (e.g., describing attributes such as the number of vertices, faces, edges, and sides of 2-D and 3-D geometric figures such as circles, polygons, spheres, cones, cylinders, prisms, and pyramids; using attributes to describe how two 2-D figures or 3-D geometric figures are alike or different; and cutting 2-D geometric figures apart and identifying the new geometric figures formed); (f) recognizing that a line can be used to represent a set of numbers and its properties (e.g., using whole numbers to locate and name points on a number line); (g) using formal geometric vocabulary to identify, classify, and describe 2-D and 3-D figures by their attributes; (h) recognizing congruence and symmetry (e.g., identifying congruent 2-D figures, creating 2-D figures with lines of symmetry using concrete models and technology, and identifying lines of symmetry in 2-D geometric figures); and (i) recognizing that a line can be used to represent numbers, fractions, and their properties and relationships (e.g., locating and naming points on a number line using whole numbers and fractions, including halves and fourths).

In comparison to the Texas state standards and expectations for prerequisite skills required for learning fourth-grade geometry, the classroom teacher's expectation was relatively low. During interviews, she stated that she would expect her students bring at least the following from their previous learning: (a) a basic understanding of geometry vocabulary such as *polygon*; (b) understanding the concepts of geometric shapes; (c) recognizing examples of geometric shapes in their real world; (d) familiarity with basic shapes such as a square, a rectangle, and a triangle; and (e) a basic understanding of the attributes of the common shapes (e.g., side and corner). It appeared that the classroom teacher's expectations were not the same as those of the state standards. Her expectations of prerequisite skills for learning fourth-grade geometry corresponded to kindergarten to the second-grade level of geometry. Especially with geometry vocabulary, she did not expect her students to bring even the second-grade level of geometry vocabulary (e.g., *vertex* or *edge*). She stated,

> Umm, a basic understanding of, you know, the geometric vocabulary like polygon. They need to understand or have been introduced about polygons. And, shapes, just to know, what the definition of a shape is. Umm, I will hope that they will be able to, you know, find examples of shapes in their real world, in their homes, in the classrooms. They should be familiar with basic shapes like a square, a rectangle, a triangle, all the fundamental shapes, 2-D shapes. They may not know what 2-D means, but they need to know that, be familiar with the 2-D shapes before we go on to 3-D shapes and build from there. Umm, and then, I will hope that they will be familiar with, they don't know that vocabulary like *vertex* or *faces,* but at least they will be able to point to corners, just the common names like *corners* or *sides*. You know sometimes, have an understanding at least of 2-D shapes. Most fourth graders know how many sides a square or a rectangle has. They don't necessarily know the names of hexagon, but basic understanding of the most common shapes. They don't have to know the actual fourth-grade vocabulary, but they need to be able to point to these parts of the shape.

The teacher's instruction was supposed to be aligned with the standards adopted by NCTM or TEA, and the teacher's expectations were likely to reflect what students actually brought from their previous learning. Thus, it was apparent that there would be a considerable gap between what her students brought from their previous learning and what they should learn. To students who had IEP goals in mathematics, the gap might not be negligible. However, it was important to make an effort to reduce the gap between what students brought in and what they were expected to learn. The following section describes difficulties of 3 students with an IEP in mathematics related to the prerequisite skills mentioned above.

Difficulties With the Prerequisite Skills of Students With an IEP in Mathematics

Data about difficulties of students with an IEP in mathematics with the prerequisite skills for learning fourth-grade geometry were collected by examining student IEPs in mathematics, teacher interviews, and a one-time teacher survey using a questionnaire developed based on TEKS geometry standards for kindergarten through Grade 3. The survey questionnaires were given to the teacher before starting the class observations and clinical interviews with the students in mid-December. Even though the students had an IEP in mathematics, their IEPs did not contain goals for learning geometry. Difficulties of the 3 MD students with prerequisite skills for learning fourth-grade geometry were analyzed in two aspects: (a) their difficulties with the knowledge and skills expected to be achieved by third grade in standards, compared to those of their typically achieving peer, and (b) their difficulties with the minimum prerequisite skills recognized by the classroom teacher.

For the first analysis, the teacher was asked to rate her typically achieving student's performances on the questionnaire. The rating on each item was compared with that of the individual student with an IEP in mathematics. The comparative survey data were triangulated by data from interviews with the teacher. Table 4.1 shows descriptive statistics about the teacher's ratings on geometry prerequisite skills of individual students with an IEP in mathematics (MD) and those of her typically achieving students.

Overall, the teacher rated MD student participants (students with an IEP in mathematics) as possessing lower prerequisite skills across all 12 items than she rated her typically achieving student. Compared to the typically achieving student's mean score of 2.75 over all the items, individual MD participants' mean scores were between 1.83 and 2.00.

Table 4.1

Teacher Ratings on Student Participants' Possession of Prerequisite Skills for Learning Fourth-Grade Geometry

Prerequisite skills for learning fourth-grade geometry	Typically achieving student	MD students		
		Lee	Kevin	Tina
1. Describe and identify objects in order to sort them according to a given attribute using informal language.	3	2	2	2
2. Identify circles, triangles, and rectangles, including squares, and describe the shape of balls, boxes, cans, and cones.	3	2	2	3
3. Combine geometry shapes to make new geometry shapes using concrete models.	3	2	2	2
4. Identify attributes of any shape or solid.	2	2	2	2
5. Use attributes to describe how two shapes or two solids are alike or different.	3	2	2	2
6. Cut geometric shapes apart and identify the new shapes made.	3	2	2	2
7. Use whole numbers to locate and name points on a line.	2	1	2	2
8. Name, describe, and compare shapes and solids using formal geometric vocabulary.	3	2	2	2
9. Identify congruent shapes.	3	2	2	2
10. Create shapes with lines of symmetry using concrete models and technology.	3	2	2	2
11. Identify lines of symmetry in shapes.	3	2	2	2
12. Locate and name points on a line using whole numbers and fractions such as halves.	2	1	1	1
Average	2.75	1.83	1.92	2.00

Note. Rating scale was 3 = *all the time*, 2 = *sometimes,* and 1 = *not at all. MD = mathematics disabilities.*

More specifically, the students with MD were rated as having lower levels of skills than the typically achieving student on all the items except Item 4 (the skill of identifying attributes of any shape or solid). Especially, all 3 MD student participants were rated as 1 on the skill of locating and naming points on a line using whole numbers and fractions such as halves (Item 12). This showed that reteaching or supplementary teaching of the skill should be arranged before fourth-grade geometry was introduced to the students. In addition, compared to the teacher's ratings on Item 7, the teacher's ratings on the Item 12 showed that 2 students with MD had the skills of using whole numbers to locate and name points on a line at the same level with the typically achieving student, but the 2 MD students showed problems with using fractions to do the same thing. Lee had problems with using both whole numbers and fractions to locate and name a point on a line. Accordingly, the teacher was expected to make instructional adaptations to address most of the prerequisite skills, especially the skills of locating and naming a point on a line using fractions to help the students with MD in her class.

For the second analysis, teacher interview data were examined in terms of the teacher's thoughts about minimum skills required for learning fourth-grade geometry in her class and her perceptions on the MD students' difficulties in the minimum skills. Minimum prerequisite skills, which the teacher identified as essential for learning fourth-grade geometry knowledge and skills, included the following: (a) understanding geometry vocabulary (e.g., *polygon*), (b) capability of discriminating a polygon from nonpolygon circles, (c) understanding the concepts of shapes, (d) skills of finding examples of shapes in the real world, (e) understanding 2-D shapes (e.g., how many sides a square has), (f) capability of counting the number of sides and understanding the facts that the number of sides determines the name of shapes, (g) capability of drawing geometry shapes, and (h) capability of pointing to corners or sides.

The teacher also identified difficulties with which individual student participants with an IEP in mathematics were struggling with regard to these minimum prerequisite skills for learning fourth-grade geometry. Geometry vocabulary was the weakness that the teacher commonly identified with all 3 MD students. Particularly, they had problems in remembering terms indicating parts of a shape (e.g., *vertex, side, edge,* and *face*). Other than that, the teacher recognized Kevin's struggle with describing angles.

In summary, the MD student participants were found to struggle with most prerequisite skills that were supposed to be mastered before learning fourth-grade geometry. Especially, geometry vocabulary and the skills of locating and naming points on a line using fractions such as halves were the most severe struggles of the students.

However, standards adopted for contemporary mathematics education in Texas (e.g., NCTM standards, TEKS) and the mathematics program employed by Ashley Hamilton for her lesson seemed to assume that most fourth-grade students would bring prerequisite skills required for learning fourth-grade geometry content from their previous learning. Consequently, a substantial gap might exist between what the fourth-grade students with MD were prepared for and what they should be prepared for. Thus, it was expected that the teacher would make efforts to fill the gaps for her students with MD in her mathematics classroom.

Typical Standards-Based Mathematics Instruction

This study defined typical standards-based mathematics instruction as instruction implemented during the baseline observation on each mathematics content, including both instruction provided for all students and instruction adapted for students with MD. Typical standards-based mathematics instruction might include the teacher's verbal or nonverbal interactions with her students with or without involving assistances or remedial efforts for students with MD, occurring in mathematics instruction using a standards-based mathematics curriculum or program. In this study, typical standards-based instruction was used to provide descriptions of the teacher's ordinary instruction (e.g., instruction for all students and instruction adapted for students with MD) and to provide a reference to identify and analyze instructional adaptations that Ashley implemented for her individual students with MD during mathematics instruction.

Observation of typical standards-based instruction. Before starting to observe the teacher's instructional adaptations for students with MD, the researcher visited the classroom twice to become familiar with the students in her class and to observe her typical instruction. Ashley was teaching measurement (e.g., measurement units such as yard and inches, measuring self using a yard stick and a ruler) during the first visit. She was teaching a lesson on geometry and spatial reasoning (e.g., making 3-D cube building) based on the standards-based mathematics curriculum, Math Investigations (TERC, 1998), during the second visit. Because instruction observed during the first visit was not related to instruction on geometry and spatial reasoning, the instruction was excluded in analysis of baseline instruction on geometry and spatial reasoning.

After observing the teacher's typical instruction on geometry and spatial reasoning, the researcher had informal conversations with the teacher for approximately 40 minutes during the students' special time to see if a specific interaction or instruction was made targeting for the students with MD in her class. If she recognized a specific interaction as not targeting for the students with MD, the interaction was identified as instruction for all students. A specific interaction which the teacher recognized as targeting for the students with MD was categorized into instruction adapted for students with MD.

Curriculum. The typical standards-based geometry lesson was based on a lesson (*Building with cubes*) from Investigations in Number, Data, and Space (TERC, 1998). The teacher taught this lesson over two sessions, one of which was observed as typical standards-based geometry instruction. The typical standards-based geometry lesson was based on one activity from the textbook lesson, "making cube building". In this activity, students were expected to put interlocking cubes together to form cube buildings shown in drawings and discuss their strategies to build the cubes. For this activity, teachers were supposed to (a) provide materials (interlocking cubes and student sheet), (b) have each student in a pair make their own version of the building, (c) have students in a pair compare the two versions, (d) illustrate (or have students illustrate) viewing positions for comparisons, (e) have students complete the 7 more buildings , (f) implement progress monitoring to assess students' interpretations of the cube drawings and their ability to put together the buildings shown, and (g) have students engage in a class discussion about their construction strategies and ask them to compare the buildings shown in different drawings. Tips for the linguistically diverse classroom (e.g., a nonverbal comparison method of pointing to parts of the buildings that are the same) were provided in the teacher's manual of the textbook.

Instructional routines. The typical standards-based geometry instruction observed consisted of five instructional routines: (a) warm-up activity, (b) overview of the lesson or presentation of the objectives of the lesson, (c) explanation of the new skills, (d) guided practice, and (e) independent practices. Ashley started her mathematics instruction with a warm-up activity. The warm-up activity involved reviewing skills, two-step word-problem solving, which had been taught previously but were not related to the new skill that would be taught on that day. The warm-up activity lasted for approximately 10 minutes.

After finishing the warm-up activity, Ashley provided the overview of the lesson or the objectives of the lesson for approximately 5 minutes. While Ashley was providing the outline or the objectives of the lesson, she provided a time for reviewing vocabulary such as strategy and volume as the prerequisite skills for learning the lesson.

After that, the teacher started to teach the new skills of the day. This portion of instruction occurred in a whole class. The teacher provided instructional materials (e.g., interlocking cubes and a student sheet containing drawings of buildings) to individual students, provided a time for getting familiar with the materials to the students, and explained the day's activity. During the explanation of the activity, the teacher reviewed two prerequisite skills that were needed for completing the task. First, she stated the importance of positioning two cube buildings when the students would be comparing them and showed how to position two cube buildings for comparisons. Second, she taught a way to figure out the number of cubes shown in the drawing of a building.

After explaining the activity, the teacher provided a time for guided practice with two examples but did not provide any demonstration of how to solve the problem before having the students work on the tasks. While the teacher was trying to provide explanation of the activity and guided practices, she used direct questioning frequently to check students' understanding of the task and to prompt class discussions. After finishing two practice examples, the teacher encouraged the students to share their strategies about making cube buildings shown on 2-D drawings.

As a final routine of the day's instruction, the teacher asked the students to complete eight independent practice items with their partners and place their works on the teacher's desk. For this practice, every two students were randomly paired, and a sufficient practice time (45 minutes) was provided for completing the practice items.

Instruction for all students during the typical standards-based geometry instruction. Typical standards-based geometry instruction included both instruction targeting all students and instruction adapted for students with MD. Overall, typical standards-based instruction for all students involved instructional features from two different approaches of instruction: the constructivist instructional approach and the direct-strategy instructional approach. Table 4.2 provides an overview of instructional components used in instruction for all students during typical standards-based instruction in geometry and spatial reasoning.

Table 4.2

Instructional Components Used in Typical Standards-Based Instruction for All Students on Geometry and Spatial Reasoning

Instructional routines	Instructional grouping	Instructional time (min.)	Instructional components
Warm-up activity	Whole	10	Review of skills taught
Overview of the lesson	Whole	5	Advance organization
			Review of prerequisite skills
Explanation of the new skills or activity	Whole	15	Review of prerequisite skills
Guided practice	Whole	15	Strategy instruction
			Use of direct questioning
			Use of manipulatives
			Discourse-driven instruction
			Use of multiple teaching examples
Independent practices	Pair	45	Use of multiple examples
			Sufficient time for practices

Based on an analysis of typical standards-based geometry instruction and curriculum for all students, a constructivistic profile most frequently shown in the typical standards-based geometry lesson for all students was *discourse-driven instruction*. Discourse-driven instruction refers to an instructional practice where a teacher encourages students to learn mathematics concepts or procedures by engaging in the construction of shared mathematics knowledge in their classrooms, which is usually accomplished by verbal interactions between a teacher and students or among students (Baxter et al., 2001). For example, during the lesson, all students in her class were encouraged to engage in the whole-class sharing of their strategies to determine the number of cubes in a building. They were also asked to share their strategies to create buildings with the whole class after they finished all practice items. Inquiry-based instructional practice, which emphasizes exploration or discovery of multiple strategies for problem solving, was not apparent in the baseline instruction, even though standards-based mathematics instruction emphasizes discovery learning as well as learning from class interactions.

Direct-strategy instruction was another feature of the typical standards-based geometry instruction targeting for all students. The typical instruction for all students included some direct instructional features, including advance organization, direct questioning, use of manipulatives, guided practice using multiple examples, repeated independent practice with multiple problems, and review of prerequisite skills. Advance organization and review of prerequisite skills were evidence-based instructional features employed during the routine warm-up activity or overview of the day's lesson.

The teacher used *direct questioning* as a way to get students' attention or to prompt class discussions. The teacher's questions used for these purpose could be categorized into two levels:

cognitive memory and convergent. The teacher used direct questioning to determine the student knowledge about factual information-cognitive memory level or to determine the students' understanding or comprehension of a subject-convergent level. For example, the teacher asked her students to predict how many cubes it would take to make two different buildings in 2-D drawings. Then, she asked them to compare the two buildings (e.g., "How are they related?" "Which one is bigger?").

The typical standards-based geometry instruction for all students also involved *hands-on activity* using interlocking cubes and an opportunity of repeated independent practices using multiple problems (eight problems). The instruction contained *reviewing prerequisite skills,* including geometry vocabulary previously taught. For example, the teacher provided her students with two reviews of geometry vocabulary, including faces, vertices and rectangular prisms during the instruction.

In addition to some features of direct instruction, the typical standards-based geometry instruction for all students involved features of *strategy instruction*. During this instruction, the teacher taught her students about a strategy that could be used for determining the number of cubes to create a building in a 2-D drawing and also reminded them to use specific strategies for counting the cubes in 2-D drawing buildings. For example, the teacher taught her students a way to determine the number of cubes in 2-D drawings (counting the cubes in successive layers) and later reminded them of using the strategy for their work. Also, the teacher reminded them to use counting strategies they had used in other contexts, such as counting by 2s, 3s, or 4s in determining the number of cubes in a building in a 2-D drawing. However, modeling of an activity or problem-solving procedures by the teacher or students was not observed. Likewise, neither corrective feedback nor group instruction was observed in this instruction.

In summary, the typical standards-based geometry instruction targeting all students contained a feature of constructivistic instruction (discourse-driven instruction), features of direct instruction (advance organization, direct questioning, use of manipulatives, guided practice using multiple examples, repeated independent practices with multiple problems, and review of prerequisite skills), and features of strategy instruction (teaching strategies for problem-solving, and prompting to use strategies). However, modeling, corrective feedback, and purposive group instruction, which have been demonstrated as effective for teaching students with MD, were not present in the typical standards-based geometry instruction targeting all students. The following section provides the findings on the typical standards-based geometry instruction targeting students with MD which was observed during the baseline observation.

Instruction for students with MD during the typical standards-based geometry instruction. For her 3 students with MD, the teacher implemented two instructional adaptations during the observation of typical standards-based geometry instruction. First, she purposively paired individual students with MD with a high achieving student for the activity of making cube building. Although the activity was suggested as a pair work in the teachers' manual, pairing individual students with MD with a high achieving student was not suggested in the manual but purposively implemented for the students with MD by the teacher. Second, the teacher provided corrective feedback to the wrong answers by a student with MD during the observation of typical standards-based geometry instruction. For example, when the teacher asked Tina to position her cube buildings as shown in the drawings, Tina was not able to position her buildings correctly. In response to Tina's incorrect answers, the teacher prompted correct responses by asking Tina to think about if her buildings were the same with the buildings shown in the drawings and turn around her buildings little by little to match them to the drawings. This corrective feedback led Tina to find the correct answers at last.

In summary, the typical standards-based geometry instruction implemented by Ashley included instructional adaptations for students with MD as well as instruction targeting all students. The instructional adaptations observed during the observation of typical standards-based geometry instruction involved grouping students with MD with high achieving students and providing corrective feedback.

Instructional Adaptations for Students with an IEP in Mathematics Geometry and Spatial Reasoning

Findings on Ashley's instructional adaptations for her 3 students with MD during lessons on geometry were derived from data from observations, interviews, and document reviews (e.g., lesson plans). During the observational period of this study, the teacher provided three 90-minute geometry instructions from *Investigations in Number, Data, and Space* (TERC, 1998; see Appendix F for an example lesson), which she had been trained for teaching at a school district professional development program. The titles of three lessons were (a) Building With Cubes (Investigation 1, Session 1), (b) Making Mental Pictures (Investigation 1, Session 2), and (c) Silhouettes of Geometric Solids (Investigation 2, Session 1). Appendix G provides an overview of Ashley's instructional adaptations for 3 students with an IEP in mathematics in her standards-based mathematics, general education classroom.

Identification of instructional adaptations. Instructional adaptations refer to appropriate adjustments, accommodations, and modifications to instruction or supports that allow students to meet academic requirements and conditions of the curriculum, most often the general education curriculum (VGCRLA, 2001). In this study, instructional adaptations were identified by the existence of the teacher's interactions adjusted for individual students with MD or small groups involving the students with MD. To determine if a portion of instruction was adjusted to target the individual students with MD or groups involving them, the portion was compared with typical standards-based instruction targeting all students. Additionally, the teacher was interviewed about why she implemented the specific interactions with the individual students with MD after each observation was conducted. Adaptation of instruction for the individuals with MD was also determined by the teacher's written records, including notes for accommodations or adaptations in her lesson plans.

A portion of instruction was considered as an instructional adaptation if it satisfied all of the following three criteria: (a) It involved interactions with the individual students with MD or small groups involving the students with MD, which was not shown in typical standards-based instruction targeting all students; (b) it included teacher-identified adjustments or modifications in terms of instructional content, instructional activity, delivery of instruction, or instructional materials; and (c) the teacher recognized the occurrence of an adjustment or modification through interviews or lesson plans.

Frequency and context of instructional adaptations for individual students with MD. This section provides overall descriptions of the teacher's instructional adaptations for the individual students with MD in terms of (a) frequency per each lesson, (b) group setting, and (c) situations inducing adaptations. Overall, the teacher made instructional adaptations in terms of grouping formats in 2 out of 3 lessons. Table 4.3 summarizes the occurrence of instructional adaptations made for the individual students with MD in either whole-group or small-group settings during the whole observational period of instruction on geometry and spatial reasoning.

Table 4.3.

Ashley's Instructional Adaptations for Three Individual Students With MD Across Three Geometry Lessons

	Frequency		
Group and context	Lee	Kevin	Tina
Whole group			
Correcting student wrong responses (unplanned)	7	0	2
Assisting student to understand concepts or procedures (planned)	2	2	1
Small group			
Correcting student wrong responses (unplanned)	3	0	0
Assisting student to understand concepts or procedures (planned)	0	0	0
Total	12	2	3

Across three lessons, a total of 12 adaptations were observed relating to Lee, who was rated as the most struggling students in the prerequisite skills required to learn fourth-grade geometry and spatial reasoning by the teacher. At least 2 adaptations were made for her in each lesson (2, 2, and 5 adaptations in Lesson 1, Lesson 2, and Lesson 3, respectively). Nine adaptations out of 12 were made in a whole-group setting; 3 adaptations occurred in a small-group setting. Particularly, 7 of 9 adaptations that occurred in whole-group instruction were made as part of the process of correcting student response, after Lee showed misunderstanding of a concept or procedure or provided an incorrect answer. These adaptations were not planned prior to the lesson but were to respond to the student difficulties identified through class interactions. For example, when Lee answered incorrectly the teacher's question about how many cubes they would use for building a building in a 2-D drawing during Lesson 1, the teacher tried to provide direct questions related to the problem-solving process to prompt Lee to arrive at a correct answer. In those interactions, the teacher employed direct questioning at cognitive-memory level. She adjusted difficulty of the task by segmenting the task into smaller stages (identifying the number of cubes at the bottom and at the top separately and then adding them together). She also had Lee answer a question about each smaller stage. These interactions could be categorized as prompting.

> Teacher (T): Okay, let's go ahead and someone tell me how many cubes we are going to use for the #1 building? Lee?
> Lee: Four.
> T: Okay, she counted these. Let's point to the one. How many numbers in the bottom?
> Lee: One.
> T: One. How many cubes in the bottom?
> Lee: One.
> T: Just one. How many on the top? One, two, three…
> Lee: Four.
> T & Lee: Four, five, six.
> T: So, there are six plus one, so how many?
> Lee: Seven.
> T: Seven cubes. What did you say, Lee?
> Lee: Four (Laugh).
> T: So, we are going to need seven cubes.

Similarly, during Lesson 1, Ashley used the tricky part of the process to figure out the number of cubes in a building in order to teach students about a cognitive strategy to count the number of cubes. In reaction to Lee's nonresponse, Ashley tried to correct student wrong response by having a typically achieving student answer her question. Also, the strategy was demonstrated with one example.

> T: Okay, making it sure that each side has correct number of cubes, that might be tricky. One side has only two, left side. One side has three, right side. Make it sure you are using the correct number of cubes from the correct sides. Something else you may think of might confuse you. Just count the volume. What might be tricky? I saw S8 had this problem so I brought this up. What might confuse you? Lee? (No response.) What's the most confusing part? (No response.) Amy?
> Amy: This is 3-D on 2-D.
> T: Right, it's very tricky, isn't it? When you look at it, sometimes it confuses you. Look at number two in the middle. You started the top and counted down one, two, three, four, so four cubes in the middle, isn't it? But it almost looks like that's more than one cube, doesn't it? Because you are seeing the top of the cube and the side of the cube. So, you have to make it sure that you still count that cube as how many?
> All students: One.
> T: Look at the top cube in the middle. It almost looks like two cubes. However, this is one cube, even though you see all the side of the cube. So, you have to count it only once. Some kids want to count the top as one, the side (right) is how many?
> Student 1: One.
> T: Well, they count this side as two and this side as one, two, three, three. You don't count every side.

Another example of adaptations involved in the process of correcting student wrong responses was implemented during Lesson 3. When Lee failed to provide a correct mathematics vocabulary definition for *corner,* the teacher attempted to prompt her to generate a correct answer by providing verbal cues and/or asking leading questions. For example, when Lee was struggling with remembering the mathematics vocabulary word *vertex*, the teacher provided prompts for the correct answer by pronouncing the first sound of the vocabulary.

> T: All of you probably have different angles of this cube right now, don't you? Student 1, you see what?
> Student 1: I see a little bit of back sides, and—
> T: What is the side called?
> Student 1: Faces.
> T: Right. Lee, what do you see? You see the back?
> Lee: No.
> T: No, what do you see?
> Lee: I see the bottom, I see the corner.
> T: Can you tell me what you are seeing in mathematics words? What is the corner called? (No response.) What do you see? (No response.) You see the ver…?
> Lee: Vertex.
> T: She sees the vertices.

On the other hands, two of nine adaptations that Ashley made for Lee in a whole group setting were to help students with MD understand concepts or procedures being taught during introducing or explaining the concepts or procedures. Compared to the adaptations occurred in the process of correcting student's wrong responses, these adaptations were planned prior to the lessons for the students with MD. For example, while Ashley was explaining the concept of silhouette as a shadow, she attempted to make her instruction more explicit by using manipulatives, multiple teaching examples (e.g., her hand and cube), and multiple practice items (e.g., cube, can, and cone).

Interview data with Ashley and data from her lesson plans supported these adjustments as adaptations purposively implemented for Lee, Kevin, and Tina.

> T: Okay, before we get it started, I would first like to talk about little bit more about silhouette. And, I want to talk about this cube, okay? Use this cube. First of all, the silhouette, we call it as a shadow or outline. (Put her hand on the overhead projector) I am going to put my hand on here. What do you see here? What's my hand like?
> All students: A shadow.
> T: A shadow. You can trace the outline of my hand. It is a silhouette of my hand. Why is its shape like that?
> Student 2: Because its shape is like that.
> T: Yeah, because its shape is like that. So, basically, the shape of the object determines the shape of the…?
> Some students: Shadow.
> T: Silhouette, shadow. So, if you want to predict what type of shape of this shadow makes, remember this is the 3-D object, it's going to make a 2-D shadow, right? I am going to turn the light off, and put this (showed a cube) right here. …I want you to put a piece of paper on your desk, use your top of your paper, do not use a whole sheet, predict, I want you to predict what the shape of the shadow of this object is going to look like when I turn the light on.

On the other hand, Ashley purposively paired Lee with a high-achieving student for group work in Lessons 1 and 3. Three adaptations were implemented in a small-group setting, and all the adaptations were made during Lesson 1. All three adaptations in a small-group setting occurred in the process of providing corrective feedback during progress monitoring of each group's work. During Lesson 1, Ashley paired Lee with a high-achieving student for partner work, in which they were asked to create 3-D building based on 2-D drawings. Ashley frequently monitored Lee's understanding of the task and task-related skills (e.g., prerequisite skills) and her performance on the task. She checked Lee's prerequisite skills for making 3-D cubes using 2-D drawings by having her point to a specific cube in the 3-D building corresponding to a specific section of 2-D drawing. Also, Ashley frequently checked Lee's understanding of the procedures or strategies for completing the task by asking her about her strategies for task completion (e.g., "How did you make this building?") or validation of her work (e.g., "What can you do to make sure if you are correct?").

> T (to Lee and Student 2, her partner): You started with the bottom three. Right now, can you try to match this with this so far?
> Lee & Student 2: (Made it).
> T: There we go. Now, you are going to build the back. That's something you can do to make it sure if you are correct. You already did that. What can you do to check yourself?
> Student 2: Counting the volume of the building.
> T: It can be, but it would be better to match your work with the building on your sheet. Lee, can you point that cube for me? (Pointed at the second cube on the left side on the sheet). Let's put it the way looks first of all. Position it as it looks. Doublecheck it. There we go. (Lee makes it.) Good job. Can you point this cube (the second in the middle) for me? (Lee makes it.) Very good. (Turning to Student 2.) Once you finish the number 2, try to label the building. You are going to go ahead to label that.

For Kevin, the teacher adjusted her instruction twice for the lessons on geometry and spatial reasoning. The two adaptations, one in Lesson 2 and the other in Lesson 3, were made in a whole-group setting and purposively implemented while Ashley was explaining geometry concepts or procedures. One of these two adaptations was also induced by Kevin's incorrect answer. While the teacher was trying to teach strategies to remember mental images of buildings after a quick glance, she attempted to provide explicit teaching by using an explicit example (Kevin's building) and concrete representation. In addition, when Kevin incorrectly responded to the teacher's direct question ("When I say look at it carefully, what might you pay attention to?") during these interactions, the teacher reacted with prompting. The process of prompting included using a concrete example (Kevin's building) to reduce the difficulty of the question.

> T: Umm, today's pictures are going to be very similar ones you made yesterday. Okay? You remember all the cube buildings you made yesterday. They are going to be very similar to them. Umm, I will flash one picture of a cube building overhead for 3 seconds. Then, when I do that, you are going to look at it very carefully. When I say look at it carefully, what might you pay attention to? Kevin? When I shot the picture for 5 seconds, what part are you going to pay attention to? (Kevin pointed at a specific part). Well, can you tell me some important parts of shapes of the building? For example, go back to the table and take one you made yesterday. Hold it up so that we can see it. If I flash the Kevin work on the screen for 3 seconds, what do you think you need to remember? Student 8?
> Student 8: One side is shorter than another.
> T: Okay, he is noticing that one side is shorter than another. You don't have a time to count the number of cubes for 3 seconds. You may realize when I put it on this side, I need one more for it. What else are you going to look for? Lee?

For Tina, three instructional adaptations were made during three geometry lessons. All of the three occurred in whole-group instruction. Two adaptations were induced by Tina's incorrect response during whole-class interactions. Two of these adaptations were related to the review of prerequisite skills including the names of shapes and geometry vocabulary (e.g., *square prism, edge*).

> T: What's the name of this shape? Tina?
> Tina: Rectangle.
> T: Not a rectangle, this is 3-D.
> Tina: A cube.
> T: Not a cube. This has rectangular faces here. What faces does it have, Student 4?
> Student 4: A square.
> T: So, square prism. We call this square prism.

One instructional adaptation for Tina was previously planned and implemented for all 3 students with MD during instruction on concepts or procedures. The instance was illustrated in the description of adaptations for Lee.

In summary, the teacher's instructional adaptations differed across students with MD in terms of frequency and contexts inducing the adaptations. The teacher made more adaptations for the students whom she rated as the most struggling students with the prerequisite skills required to learn fourth-grade geometry and spatial reasoning.

Categories of instructional adaptations. For this analysis, this study employed the AF by Bryant and Bryant (2001). According to Bryant and Bryant, instructional adaptations can occur in at least one of four categories: (a) instructional content, (b) instructional activity, (c) delivery of instruction, and (d) materials or technology. Instructional content means skills and concepts that are the focus of teaching and learning. The instructional content is related to the instructional objective and the state's curriculum. Instructional activity refers to the procedure, lessons, or strategy to teach the content. Instructional delivery consists of how the activity is taught, such as instructional grouping, instructional routines, and instructional language. Materials include textbooks or other manipulatives used for mathematical representation, and technology includes computer software for drill and practice on basic math facts, calculators, or Internet facility used for mathematics activities, for example. Frequencies of Ashley's instructional adaptations by category are provided in Table 4.4; actual cases ware shown in Appendix F.

Table 4.4

Frequencies of Ashley's Instructional Adaptations by Category

	Frequency			
Category	Lee	Kevin	Tina	Average
Instructional content	0	0	0	0.0
Instructional activity	3	1	1	1.7
Delivery of instruction	11	5	6	7.3
Instructional materials or technology	1	1	1	1.0
Total	15	7	8	

According to Table 4.4, Ashley adjusted her instruction for the students with MD in terms of delivery of instruction (average of 3 students = 7.3), instructional activity (average = 1.7), and instructional materials or technology (average = 1) for the three lessons on geometry and spatial reasoning. Adaptations of instructional activity included (a) discussing possible errors and tricky parts that could transpire during task completion (e.g., miscounting of the number of cubes in creating cube buildings as shown in 2-D drawings), (b) teaching cognitive or metacognitive strategies to correctly complete the task (e.g., matching the cube buildings with 2-D drawings by counting the number of cubes in the cube building and the 2-D drawings), and (c) reviewing prerequisite skills before or during the instructional activity (e.g., reviewing vocabulary such as *vertex, faces,* and *edges*). For data of the instances of instructional adaptations cited in this section, refer to the section including the descriptions of the occurrence of the teacher's instructional adaptations.

Ashley's efforts to adapt her instruction for the students with MD were also found in modifications or adjustments of delivery of instruction. Ashley's instructional adaptations occurred most frequently in this adaptation category. Adaptations in this category involved (a) providing or modifying teacher examples when she explained concepts or procedures (e.g., concrete and meaningful examples such as Kevin's cube building or multiple number of examples for teaching the concepts or procedures), (b) providing prompting, (c) providing practice opportunity (e.g., extended time for practices, multiple practice items, and repeated reviews), (c) controlling task difficulty, (d) providing group instruction (e.g., pair work), and (e) monitoring students' understanding or performances. For data of the instances of instructional adaptations cited in this section, see the section of incorporation of evidence-based mathematics instructional components into typical geometry standards-based instruction.

In addition, Ashley used more manipulatives (e.g., her hands, cans, and objects in the classroom) to provide more explicit explanations of concepts or procedures. For example, she had a soda can and a dictionary brought by students projected to provide meaningful and concrete examples of silhouettes.

In summary, Ashley's instructional adaptations found during instruction on geometry and spatial reasoning were categorized into three categories of adaptations: (a) instructional activity, (b) delivery of instruction, and (c) instructional materials or technology. Most instances of adaptations implemented by her transpired in terms of delivery of instruction. Adjustments or modifications of instructional content did not occur during instruction on geometry and spatial reasoning. The following section provides findings on instructional components that Ashley used to adapt her instruction during geometry instruction.

Incorporations of Evidence-Based Mathematics Instructional Components Into Typical Geometry Standards-Based Instruction

Ashley's instructional adaptations were categorized by evidence-based mathematics instructional components. The instructional components emerging from data on Ashley's instructional adaptations during geometry instruction included (a) prompting, (b) control difficulty, (c) direct questioning, (d) review of prerequisite skills, (e) group instruction, (f) strategy instruction, (g) progress monitoring, (h) use of manipulatives, (i) use of teaching examples, and (j) vocabulary instruction.

Some components, including prompting, control difficulty, direct questioning, review of prerequisite skills, and vocabulary instruction, were incorporated into her typical standards-based instruction in order to help the students with MD as a reaction to the student wrong response mainly during whole group instruction. The components of group instruction, strategy instruction, progress monitoring, use of manipulatives, and use of teacher examples were employed to adjust her instruction for the students with MD during her instruction on concepts or procedures. Of these components, group instruction and progress monitoring were related to providing small-group instruction, whereas strategy instruction, use of manipulatives, and use of teacher examples occurred mainly in whole-group instruction. Table 4.5 provides the frequencies of Ashley's instructional adaptations across three geometry lessons by evidence-based mathematics instructional components.

Table 4.5

Frequency of Ashley's Instructional Adaptations Using Evidence-Based Mathematics Instructional Components Across Three Geometry Lessons

Evidence-based mathematics instructional component	Lesson 1	Lesson 2	Lesson 3	Average
Prompting	5	3	5	4.33
Control difficulty	1	1	1	1.00
Direct questioning	3	2	3	2.66
Review of prerequisite skills	1	0	2	1.00
Group instruction	3	0	3	2.00
Strategy instruction	2	1	0	1.00
Progress monitoring	3	0	0	1.00
Use of manipulatives	0	1	1	0.66
Practice opportunity	0	0	0	0.00
Teacher examples	0	1	1	0.66

Vocabulary instruction	1	0	1	0.66

Prompting. Prompting refers to providing verbal, physical, or written cues to assist students in generating correct response (Rivera & Smith, 1998). Teachers may provide prompting by asking leading questions, repeating and rephrasing lesson content, pointing to a specific word or number, providing examples and nonexamples, giving feedback, doing tasks partially, doing a task with students, or providing manual guidance (Mercer & Mercer, 2006).

Prompting was a component of evidence-based mathematics instruction that Ashley most frequently utilized for adjusting her instruction for 3 students with MD (average = 4.33 per lesson). For example, Ashley prompted the student to generate the correct answer in most cases in which students incorrectly answered by providing cues or leading questions.

> T: All of you probably have different angles of this cube right now, don't you? Student 1, you see what?
> Student 1: I see a little bit of back sides, and—
> T: What is the side called?
> Student 1: Faces.
> T: Right. Lee, what do you see? You see the back?
> Lee: No.
> T: No, what do you see?
> Lee: I see the bottom, I see the corner.
> T: Can you tell me what you are seeing in mathematics words? What is the corner called? (No response from Lee.) What do you see? (No response.) You see the ver…?
> Lee: Vertex.

T: What's an edge? Tina? (No response.) Which one shows you edge right here? It can be made on top, front, or bottom. What's the edge?
Tina: Bottom.
T: The bottom, right. So, what is an edge? (No response from Tina.) What forms an edge? (No response.) Student 2?
Student 2: When two faces come together.
T: Good, the bottom and the middle face, when they come together, they form an?
All students: Edge.
T: Edge. Good.

Control difficulty. Control difficulty refers to the adjustment of task difficulty by sequencing tasks from easy to difficult and providing only necessary hints to students, segmenting the task into smaller steps or units and then synthesizing the parts into a whole, or providing simplified demonstration (Swanson, Hoskyn, & Lee, 1999). During the geometry lessons, Ashley adjusted task difficulty for the students with MD three times, one per each lesson. She adjusted task difficulty by (a) segmenting the task into smaller parts and prompting the student to complete each smaller part, (b) providing necessary prompts, or (c) providing an example to help the student's understanding about the task. The following is an example of control difficulty by segmentation:

T: Okay, let's go ahead and someone tell me how many cubes we are going to use for the #1 building? Lee?
Lee: Four.
T: Okay, she counted these. Let's point to the one.
T: How many numbers in the bottom?
Lee: One.
T: One, How many cubes in the bottom?
Lee: One.
T: Just one.
T: How many on the top? (No response.) One, two, three…

Lee: Four…
T & Lee: Four, five, six.
T: So, there is six plus one, so how many?
Lee: Seven.
T: Seven cubes. What did you say, Lee?
Lee: Four (Laugh).
T: So, we are going to need seven cubes.

The following is an instance of control difficulty by providing a concrete example:

T: Umm, today's pictures are going to be very similar ones you made yesterday. Okay? You remember all the cube buildings you made yesterday. They are going to be very similar to them. Umm, I will flash one picture of a cube building over head for 3 seconds. Then, when I do that, you are going to look at it very carefully. When I say look at it carefully, what might you pay attention to? Kevin? When I shot the picture for 5 seconds, what part are you going to pay attention to? (Kevin pointed to a specific part). Well, can you tell me some important parts of shapes of the building? For example, go back to the table and take one you made yesterday. Hold it up so that we can see it. If I flash the Kevin work on the screen for 3 seconds, what do you think you need to remember? Student 8?

Direct questioning. Direct questioning is defined as providing process-related or content-related questions to students (Swanson et al., 1999). Teacher's questions may be classified into a category of questions according to what the teacher is trying to get the student to do in response. These categories include (a) cognitive-memory level, (b) convergent level, (c) divergent level, and (d) evaluative level (Callahan & Clarke, 1988). Questions at the cognitive level have a simple answer that the students are expected to know. Questions at the convergent level ask the students to explain, interpret, give examples, or summarize concepts in their own words. Divergent questions have students apply principles in new settings and involve problem solving or decision making.

Finally, evaluative questions require the student to make a value judgment (for detailed information, see the glossary).

In Ashley's instructional adaptations for the students with MD, direct questioning was an evidence-based mathematics instructional component she frequently used (average use = 2.66 per each lesson). Direct questioning was usually involved in the process of prompting the student to get the right answer. In six out of eight instances, Ashley utilized direct questioning, especially for prompting the student to get to the correct answer. Questions in five of the six instances appeared to be at the level of cognitive memory. The remaining instance included questions at the divergent level (e.g., "When I say look at it carefully, what might you pay attention to?"). The following illustrates direct questioning at the cognitive memory level, which Ashley employed in the process of providing corrective feedback to Tina:

T: What's the name of this shape? Tina?
Tina: Rectangle.
T: Not a rectangle, this is 3-D.
Tina: A cube.
T: Not a cube. This has rectangular faces here. What faces does it have, Student 4?
Student 4: A square.
T: So, square prism. We call this square prism.

The other two instances of using direct questioning also included questions at the level of cognitive memory, but they were used for checking for understanding rather than for prompting. During Ashley's monitoring of student's understanding about correspondence between a 2-D drawing and a cube building, she said, “Kevin, please point to this cube [the second on the left side on the drawing] in your building. Where is this cube?” Kevin pointed at the correct cube, and Ashley responded, “Very good. “

Review of prerequisite skills. Prerequisite skills are background knowledge necessary for applying the target skills (Jitendra et al., 1999). Reviewing prerequisite skills for learning geometry was an evidence-based mathematics instructional component employed in Ashley’s instructional adaptations. Ashley’s review of prerequisite skills mainly focused on geometry vocabulary. For example, she adjusted her instruction to include reviews of the names of geometry shapes (e.g., a square prism and a cube), and geometry vocabulary (e.g., *edge* and *vertex*).

Group instruction. Group instruction is instruction using pairs or small groups as an alternative to whole-group or independent seatwork (Gersten, Schiller, & Vaughn, 2000). Group instruction has been reported as a critical component of evidence-based mathematics instruction (Swanson et al., 1999). Out of three lessons on geometry, the teacher employed group instruction in two lessons. Both group instructions included pairing a student with MD with a high- or average-achieving student and were purposively implemented according to Ashley’s lesson plan. In addition, Ashley always paired Lee with a high-achieving student, whereas the other 2 students were paired with an average-achieving student. Ashley also stayed with groups involving the students with MD during most of group instruction time.

Strategy instruction. Strategy refers to a broad range of routines that facilitate both knowledge acquisition and utilization, including various heuristic techniques that allow one to more easily access relevant information during problem solving as well as general control strategies such as planning, monitoring, checking, and revising (Prawat, 1989). In this sense, strategy instruction involves teaching or cueing to use strategies for solving problems and for verifying the problem solution. The teacher used strategy instruction for helping the students with MD correctly perform tasks. The following instance illustrated the teacher's instruction on a metacognitive strategy of verifying answers, which occurred during progress monitoring:

> T (to pair of Lee & Student 2): You started with the bottom three. Right now, can you try to match this with this so far? (Lee and Student 2 did so.) There we go. Now, you are going to build the back. That's something you can do to make it sure if you are correct. You already did that. What can you do to check yourself?
> Student 2: Counting the volume of the building.
> T: It can be, but it would be better to match your work with the building on your sheet. Lee, can you point that cube for me? (Pointed at the second cube on the left side on the sheet). Let's put it the way it looks first of all. Position it as it looks. Doublecheck it. There we go. (Lee did so). Good job. Can you point this cube for me? (Lee did so). Very good. (Turning to Student 2.) Once you finish the number 2, try to label the building. You are going to go ahead to label that.

Progress monitoring. Progress monitoring is described as a teacher's checking whether students understand task requirements and the procedures needed to complete the task correctly. Ashley's progress monitoring was found in only one lesson, especially during small-group instruction, and was implemented to check the students' understanding of the task or activity. In addition, as with group instruction, the groups involving the individual students with MD were targeted for Ashley's progress monitoring and feedback.

> T (to the pair of Lee and Student 2): What did you do to make the buildings?
> Student 2: Before we tried to do the number 1, we did the number 2 first. Because they look the same.
> T: Okay, that's what I want to say. What's the same between them? (No response from either student). What's a volume for both buildings?
> Student 2 & Lee: 7.
> T: So, they must be pretty similar. So, what's the difference?
> Student 2: We took one from the bottom and add another one to it.
> T: Okay, I am trying to get something else too. You probably didn't have to do too much to it. Could we take cubes from the number 2 to make the number 1?
> Student 2: No.
> T: No, you can just turn it around to make the building number 1. So, basically, number 1 and number 2 are the same building. You have to build the same building and then decide to have them positioned in different ways on your paper.

Use of manipulatives. Manipulatives are concrete objects to represent a skill or concept or to provide hands-on instruction (Resnick & Ford, 1981). Representation tools including manipulatives are used to provide complete, consistent, and logical explanations for concepts, skills, or activities (Jitendra et al., 1999). Even though Ashley consistently stated that her favorite adaptations included use of manipulatives during interviews, she actually used manipulatives only once for adjusting her geometry instruction for students with MD. During Lesson 3, she used many manipulatives available for comparing shapes of silhouettes to help the students with MD understand the concept of silhouette, which was shown in her lesson plan.

Practice opportunity. In this study, practice opportunity refers to providing time for guided practice or independent practice. In Ashley's instruction on geometry and spatial reasoning, adjustments or modifications of guided practices were not found. Similarly, Ashley used all independent practice items proposed by the mathematics curriculum, Math Investigations (TERC, 1998), but did not attempt to provide supplemental items for the purpose of independent practice of the students with MD.

Teacher examples. Teacher examples refer to instances that illustrate a rule or method, as a mathematical problem proposed for solution (Mish, 1994). In one lesson out of three geometry lessons, Ashley used the increased number of teacher examples for the students with MD. During Lesson 3, she provided eight teaching examples (e.g., her hands, cubes, cylinder, can, etc.) to help the students with MD understand the concept of a silhouette of an object. She had the objects projected.

Vocabulary instruction. Vocabulary instruction includes teaching or reviewing geometry vocabulary that is new or previously taught. Ashley provided supplemental instruction on geometry vocabulary when a student with MD showed difficulty with it during whole-class instruction. The following instance illustrates geometry vocabulary instruction that occurred as a reaction to Tina's struggle:

> T: What's an edge? Tina? (No response.) Which one shows you edge right here? It can be made on top, front, or bottom. What's the edge?
> Tina: Bottom.

T: The bottom, right. So, what is an edge? (No response.) What forms an edge? (No response.) Student 2?
Student 2: When two faces come together.
T: Good, the bottom and the middle face, when they come together, they form an…?
All students: Edge.
T: Edge. Good.

Summary of evidence-based mathematics instructional components used for adapting standards-based geometry instruction. In summary, Ashley appeared to incorporate evidence-based mathematics instructional components into her standards-based geometry core instruction to assist the students with MD to understand the concepts or skills being taught. The components included (a) prompting, (b) control difficulty, (c) direct questioning, (d) review of prerequisite skills, (e) group instruction, (f) strategy instruction, (g) progress monitoring, (h) use of manipulatives, (i) use of teacher examples, and (j) vocabulary instruction. Of these components, prompting and direct questioning were the most frequently found in her adaptations. Different from expectations based on interview data, Ashley rarely adjusted her instruction in terms of use of manipulatives and opportunities of guided or independent practice for the students with MD. Also, she was not observed using modeling or demonstrations of a skill in adapting her instruction for the students with MD.

Compared to the variety of evidence-based instructional components used in instruction for all students during the observation of typical standards-based geometry instruction, Ashley attempted to use more, various, evidence-based instructional components when she adapted her mathematics instruction for the students with MD. Some instructional components, including prompting and control difficulty were not observed in her standards-based instruction for all students during the observation of typical standards-based geometry instruction and were used only for instructional adaptations for the students with MD during the three observations of instructional adaptations. Ashley used the other instructional components of direct questioning, review of prerequisite skills, group instruction, strategy instruction, progress monitoring, use of manipulatives, use of teaching examples, and practice opportunity for instruction for all students in standards-based baseline instruction and instructional adaptations for the students with MD.

Interestingly, the findings of this study suggested a relationship between instructional components used for instructional adaptations and their context. For example, prompting, control difficulty, direct questioning, review of prerequisite skills, and vocabulary instruction were found to be mainly related to instructional adaptations that occurred as the teacher's reaction to student wrong responses during whole-group instruction. Group instruction (pair work), progress monitoring, use of manipulatives, and use of teacher examples were found to be associated with instructional adaptations made during the teacher's instruction on concepts or procedures. Particularly, pair work and progress monitoring were found to be associated with small-group instruction.

Instructional adaptations addressing student difficulties in prerequisite skills. Data on difficulties of students with MD in prerequisite skills for learning geometry and spatial reasoning (e.g., teacher survey data, student IEP, and teacher interview data) showed that the most severe struggles of the students were geometry vocabulary, especially mathematics words indicating parts of a shape (e.g., *vertex, side, edge,* and *face*) and the skills of locating and naming points on a line using whole numbers or basic fractions. Because the findings were derived mainly from data that the teacher produced, it was expected that the teacher would recognize the students' difficulties in those prerequisite skills and would consider these difficulties when she planned and implemented her instruction. Table 4.6 provides an overview of the difficulties of individual students with MD in prerequisite skills for learning fourth-grade geometry and spatial reasoning and instructional adaptations that might be linked to the difficulties.

Table 4.6

Instructional Adaptations Addressing Student Difficulties Related to Prerequisite Skills for Learning Fourth-Grade Geometry and Spatial Reasoning

Prerequisite skills	Instructional adaptations
Skills to locate and name points on a line using whole numbers (Lee)	None
Skills to locate and name points on a line using fractions (Lee, Kevin, & Tina)	None
Knowledge about geometry vocabulary (Lee, Kevin, & Tina)	Teaching geometry vocabulary (e.g., names of shapes such as polygon and rectangular pyramid) to Lee and Tina in Lesson 1
	Teaching geometry vocabulary (e.g., *face* and *vertex*) to Lee and Tina in Lesson 3

Regarding the difficulties of students with MD in prerequisite skills for learning fourth-grade geometry and spatial reasoning, Ashley made two adaptations across three lessons and implemented them in whole-group instructional settings. For example, during Lessons 1 and 3, Ashley spent her instructional time reviewing the prerequisite geometry vocabulary, including the names of shapes (e.g., *cube, square prism, rectangle*) and mathematics words indicating part or components of a geometry shape (e.g., *face, vertex, edge*).

However, the teacher's instructional adaptations addressing the student difficulties with geometry prerequisite skills seemed not to be evenly balanced across prerequisite skills and among the students with MD. For example, even though the teacher recognized that all 3 students were struggling with geometry vocabulary at teacher interviews, only 2 of the 3 students were targeted for interactions of reviewing geometry vocabulary. Ashley's instructional adaptations relating geometry vocabulary did not include interactions with Kevin. In addition, the other difficulties in skills of locating and naming a point on a line using whole numbers or fractions were not addressed in her instruction.

In summary, it was repeatedly observed that the teacher attempted to address one of the student difficulties relating to geometry prerequisite skills, geometry vocabulary, through interactions with 1 of the students with MD during whole-group instruction. However, the instructional adaptations addressing the other student difficulties with geometry prerequisite skills did not appear to be implemented evenly across student difficulties and among the 3 students.

Probability and Statistics

The ability to analyze data is becoming an important prerequisite skill of living and working in contemporary society with a superfluity of data and information (J. D. Baker & Beisel, 2001; TERC, 1998). In this context, the NCTM (2000) recommended that students in Grades 3–5, including students with MD, attain statistics-related skills, including data collection, data analysis using appropriate statistics methods, data comparisons, and statistical inferences.

Learning probability and statistics is based on various prerequisite skills (TERC, 1998). For example, figuring out a typical value of a data set involves understanding the concept of typical as an average and producing a value for typical based on computation skills. However, research in the field of mathematics disabilities has revealed that students with MD struggle even with basic mathematics skills, including computations, and their difficulties continue to exist throughout their school years (Cawley & Miller, 1989). Consequently, students with MD likely struggle with learning statistics and its related skills in standards-based mathematics, general education classrooms. These students not only do not understand the concepts or procedures taught at a given grade, but also are not equipped with the prerequisite skills for learning the grade-level content. Accordingly, general education teachers should ensure that students with MD in their classrooms posses the prerequisite skills and should address these skills, if necessary, before or during statistics and probability instruction.

This section describes findings on instructional adaptations that Ashley made for her 3 students with an IEP in mathematics during her probability and statistics instruction. Ashley provided statistics and probability instruction for approximately a month, from the end of February 2006 to the end of March. Because this period included a week of spring break, the last observation was conducted after spring break. During this period, five statistics and probability lessons (approximately 450 minutes) were observed, which used *Investigations in Number, Data, and Space* (TERC, 1998). The five lessons emphasized developing the skills of making quick sketches of the data, describing the shape of the data, summarizing the data in terms of typical values, inventing ways to compare and represent two sets of data, and finding the median in a set of data. This section includes descriptions of (a) standards for learning probability and statistics, (b) prerequisite skills for learning the content, (c) student difficulties related to the content, (d) typical standards-based statistics instruction, and (e) findings on the teacher's instructional adaptations for the students with MD during instruction on probability and statistics. Typical standards-based statistics instruction was described in terms of (a) curriculum, (b) instructional routines, (c) instruction for all students, and (d) instruction adapted for students with MD. The findings on instructional adaptations were further divided into five subdivisions: (a) identification of instructional adaptations, (b) frequency and settings associated with instructional adaptations for individual students with MD, (c) categories of instructional adaptations, (d) incorporation of evidence-based mathematics instructional components into core instruction, and (e) instructional adaptations addressing students' difficulties in prerequisite skills.

Standards for Probability and Statistics

Probability is defined as the likeliness or chance of an event occurring. Statistics is a branch of applied mathematics concerned with the collection and interpretation of quantitative data and the use of probability theory to estimate population parameters. It is the mathematics of collecting, organizing, and interpreting numerical data as well as estimating population parameters based on the sample data.

In the standards-based mathematics curriculum, *Investigations in Number, Data, and Space* (TERC, 1998), fourth-grade students are taught about some critical concepts about data during instruction on probability and statistics, including describing and comparing the patterns and special features of data—the shape of data. According to the NCTM (2000) standards, fourth-grade mathematics instruction should enable the students to (a) formulate questions that can be addressed with data and collect, organize, and display relevant data to answer them; (b) select and use appropriate statistical methods to analyze data; (c) develop and evaluate inferences and predictions that are based on data; and (d) understand and apply basic concepts of probability.

In Texas, TEKS (TEA, 2006) for mathematics education recommends that all fourth-grade students should attain the knowledge and skills to solve problems by collecting, organizing, displaying, and interpreting sets of data by the end of the school year. More specifically, the students are expected to be capable of (a) using concrete objects or pictures to make generalizations about determining all possible combinations of a given set of data or of objects in a problem situation and (b) interpreting bar graphs.

As in other areas, including geometry and spatial reasoning, these standards set the same level of expectation for learning of students with MD about probability and statistics, even though they are not in the same start line with their peers in terms of prerequisite skills for learning the content at a given grade as well as cognitive functions. In order to maximize the access of students with MD to the standards-based general mathematics curriculum as required by law (IDEA, 2004), general education teachers should make efforts to make up for the deficiency of the prerequisite skills required for learning probability and statistics at a given grade.

Prerequisite Skills

As in the analysis of prerequisite skills on geometry and spatial reasoning, prerequisite skills for learning fourth-grade probability and statistics are described based on comparison of prerequisite knowledge and skills specified by the state standards for mathematics education (TEKS; TEA, 2006) to the teacher's perceptions or beliefs. According to the mathematics portion of the TEKS (TEA, 2006), fourth-grade students are expected to be conversant with some background mathematics knowledge and skills related to probability and statistics at or before beginning the fourth grade. The followings are involved in the background knowledge and skills: (a) collecting and sorting data; (b) using organized data to construct real object graphs, picture graphs, and bar-type graphs; (c) drawing conclusions and answering questions using information organized in real-object graphs, picture graphs, and bar-type graphs; (d) identifying events as certain or impossible, such as drawing a red crayon from a bag of green crayons; (e) constructing picture graphs and bar-type graphs; (f) using data to describe events as more likely or less likely, such as drawing a certain color crayon from a bag of seven red crayons and three green crayons; (g) collecting, organizing, recording, and displaying data in pictographs and bar graphs, where each picture or cell might represent more than one piece of data; (h) interpreting information from pictographs and bar graphs ; and (i) using data to describe events as more likely, less likely, and equally likely.

Compared to the prerequisite knowledge and skills for learning probability and statistics in fourth grade, which were specified in the state standards, Ashley's list of the prerequisite skills was inclining toward probability. Her expectations did not include a thorough knowledge of statistics-related skills listed by the state standards. Also, there was no overlap between the prerequisite skills expected by the state standards and Ashley's expectations about what her students might bring from previous mathematic learning.

A list of prerequisite skills perceived by Ashley included (a) remembering orders and steps (e.g., find total first and focus on just one part), (b) understanding the meaning of probability related to chance and probability vocabulary (e.g., *probable, probably*), (c) understanding of a total and part of a total, and (d) understanding that probability can be in fractions and how to express it in fractions. Teacher seemed to perceive the basic understanding of probability, especially understanding of the meaning of probability, the procedures of figuring out a probability, and the expression of a probability in fractions as prerequisite skills for learning probability and statistics in fourth grade. Other skills related to statistics (e.g., collecting data, organizing data, etc.) were not included in her list of prerequisite skills for learning fourth-grade probability and statistics.

> Again, it's a skill that they do cover in third grade, not as in depth. In fourth grade, they might add more, you know, to the total, maybe have more that you are choosing from third grade, they might be choosing red, green, and yellow, and then, fourth grade might be

adding red, green, and yellow, and then, may be purple, blue, and pink. So, they have to choose from more. You know when they come to fourth grade, I expect them to at least understand that they are choosing one variable. You know, they are not looking at the whole picture. They need to understand we are just focusing on one part. So, being able to understand that concept that we are just looking at one bit of information, instead of all of objects. And a lot of the time, I will expect them, third grade, to have practice time using manipulatives? I will hope that it would not be the first time that they have seen them. …

It's a hard one. I just walked down the third-grade hallway and heard a third-grade teacher talking to her students. "What is the probability that someone here is wearing a pair of tennis shoes?" I know the third grade. They introduce the term *probability*. They need to understand what that means. We talk about that means a chance that something happens. I just hope that they will be able to relate those two words, *probability* and *chance*. Umm, and understanding of a total and then single out one, part of a total. A lot of them don't know that when they come to the fourth grade. They may not be able to write them as a fraction, but they might be able to say two out of six socks or oranges. Some may be able to express it poorly. But by the end of fourth grade, they may be able to write it as fractions or as a decimal. Umm, when we talk about probability, I will ask them most fourth graders will know, they recognize the words like probable, probably, and they will be able to relate the word probable to the word, probability.

In summary, based on the mathematics portion of kindergarten through Grade 3 TEKS and the teacher's perceptions of prerequisite skills for learning probability and statistics in fourth grade, fourth-grade students should be equipped with kindergarten through Grade 3 probability and statistics skills as well as understand the meaning of probability, the procedures for figuring out a probability, and expression of a probability in fractions in order to learn probability and statistics in standards-based mathematics, fourth-grade general education classrooms. However, as the teacher mentioned, many students come to the fourth grade without possessing these prerequisite skills.

Difficulties of Students With an IEP in Mathematics With the Prerequisite Skills

The findings of difficulties of students with MD in prerequisite skills for learning probability and statistics in fourth grade were derived from data from student IEP in mathematics, teacher interviews, and a one-time teacher survey using a questionnaire on the possession of prerequisite skills required for learning fourth-grade probability and statistics. The items on the questionnaire were developed based on the mathematics portion of TEKS for kindergarten through Grade 3.

As with geometry and spatial reasoning, none of the students' IEPs included goals for learning probability and statistics. Two analyses were conducted to identify difficulties of the 3 students with MD in prerequisite skills related to probability and statistics. The first analysis was to compare the students' difficulties in the knowledge and skills that were expected to be mastered by Grade 3 with those of their typically achieving peers. The second analysis focused on the students' difficulties with the minimum prerequisite skills identified by their classroom teacher.

For the first analysis, this study compared the teacher's ratings on mathematics knowledge and skills of a typically achieving student in her class (Amy) and those of individual students with MD. Data from teacher interviews were used to establish the reliability and the validity of the survey data. Table 4.7 shows descriptive statistics about the teacher's ratings on probability and statistics prerequisite skills of individual students with an IEP in mathematics (MD) and those of her typically achieving students.

Overall, Ashley rated the 3 students with MD as possessing lower levels of prerequisite skills for learning probability and statistics in fourth grade (average = 1.93) than the typically achieving student (2.4). However, the difference between an average of the 3 students with MD and that of the typically achieving student was not as large as in geometry and spatial reasoning (0.47 in probability and statistics vs. 0.83 in geometry and spatial reasoning). Ashley identified all 3 students with MD as not showing mastery in any prerequisite skills for learning fourth-grade probability and statistics. They were identified as occasionally showing the abilities to use most of the prerequisite skills for learning probability and statistics (the teacher's ratings on most items for each student were 2). In addition, Lee was identified as the most struggling student with prerequisite skills related to geometry and spatial reasoning, but it was Kevin who struggled most with prerequisite skills in probability and statistics (average score of the teacher's ratings for Kevin = 1.8). The typically achieving student was identified as showing mastery in some of the prerequisite skills (the teacher's ratings on some items were 3), while occasionally showing the ability to use the other skills (the teacher's ratings on these items were 2).

Table 4.7

Teacher's Ratings of Student Participants' Possession of Prerequisite Skills for Learning Fourth-Grade Probability and Statistics

	Frequency of showing skill			
Prerequisite skills for learning fourth-grade probability and statistics	Typically achieving student	Lee	Kevin	Tina
1. Collect and sort data.	3	2	2	2
2. Use organized data to construct real object graphs, picture graphs, and bar-type graphs.	2	2	1	2
3. Draw conclusions and answer questions using information organized in real-object graphs, picture graphs, and bar-type graphs.	2	2	2	2
4. Identify events as certain or impossible such as drawing a red crayon from a bag of green crayons.	3	2	2	2
5. Construct picture graphs and bar-type graphs.	2	2	2	2
6. Use data to describe events as more likely or less likely, such as drawing a certain color crayon from a bag of seven red crayons and three green crayons.	3	2	2	2
7. Collect, organize, record, and display data in pictographs and bar graphs, where each picture or cell might represent more than one piece of data.	2	2	2	2
8. Interpret information from pictographs and bar graphs.	2	2	2	2
9. Use data to describe events as more likely, less likely, and equally likely.	3	2	2	2
Average	2.4	2.0	1.8	2.0

Note. Rating was as follows: 1 = *not at all*; 2 = *sometimes*; 3= *all the time*.

More specifically, the students with MD were rated as having lower levels of skills on Items 1, 2, 4, 6, and 9 than the typically achieving student. However, the students with MD were rated as having the same level of skills as the typically achieving student on the other four items (Items 3, 5, 7, and 8). Accordingly, if Ashley planned to make an instructional adaptation for the students with MD, she was likely to address the skills represented by Items 1, 2, 4, 6, and 9, the skills of (a) collecting and sorting data; (b) using organized data to construct real object graphs; (c) identifying events as certain or impossible; (d) using data to describe events as more likely or less likely; and (e) using data to describe events as more likely, less likely, and equally likely. Especially for Kevin, it was necessary to provide instruction to enhance the skills of using organized data to construct real object graphs.

For the second analysis, teacher interview data were examined in terms of the teacher's thoughts about minimum skills required for learning fourth-grade probability and statistics in her class. Ashley identified the following knowledge and skills as minimum prerequisite skills for learning probability and statistics: (a) remembering orders and steps, (b) understanding the meaning of probability related to chance and probability vocabulary, (c) understanding of a total and part of a total, and (d) understanding that probability can be in fractions and how to express it in fractions. In terms of these minimum prerequisite skills for learning fourth-grade probability and statistics, Ashley identified two areas with which individual students with MD were struggling. First, students had difficulty in remembering orders and steps of probability (e.g., add to get the total and then sort out one piece of information out of the total). She identified all 3 students as having problems with the skills. Second, two students, Lee and Tina, had difficulty in understanding probability in fractions and decimals.

In summary, the MD student participants struggled with all prerequisite skills that were supposed to be mastered before starting fourth grade probability and statistics by the standards for mathematics education in Texas. They did not show mastery in any skills that they were supposed to have learned before entering fourth grade. They were also rated as showing difficulties in more than half of the minimum prerequisite skills for learning fourth-grade probability and statistics that were identified by their classroom teacher. Lee and Tina had difficulties in the skills of (a) remembering orders and steps of probability and (b) understanding probability in fractions and decimals. Kevin was found to struggle with the skills of (a) remembering orders and steps of probability and (b) using organized data to construct real object graphs. Ashley needed to address the difficulties of the students with MD during her mathematics instruction.

Typical Standards-Based Instruction

Findings on the teacher's baseline instruction on probability and statistics were derived from data from an observation and a review of probability and statistics lesson. The curriculum used was *Investigations in Number, Data, and Space* (TERC, 1998). This study defined typical standards-based mathematics instruction as instruction implemented during the baseline observation on each mathematics content, including both instruction provided for all students and instruction adapted for students with MD. Typical standards-based mathematics instruction might include the teacher's verbal or nonverbal interactions with her students with or without involving assistances or remedial efforts for students with MD, occurring in mathematics instruction using a standards-based mathematics curriculum or program. In this study, typical standards-based instruction was used to provide descriptions of the teacher's ordinary instruction (e.g., instruction for all students and instruction adapted for students with MD) and to provide a reference to identify and analyze instructional adaptations that Ashley implemented for her individual students with MD during mathematics instruction.

Observation of typical standards-based instruction. An observation of typical standards-based instruction on probability and statistics was conducted a day before the commencement of observing the teacher's instructional adaptations for five consecutive lessons on the topic. After observing the teacher's typical instruction on probability and statistics, the researcher had informal conversations with the teacher for approximately 35 minutes during the students' special time to see if a specific interaction or instruction was made targeting for the students with MD in her class. If she recognized a specific interaction as not targeting for the students with MD, the interaction was identified as typical standards-based statistics instruction for all students. A specific interaction which the teacher recognized as targeting for the students with MD was categorized into instructional adaptations for students with MD during typical standards-based statistics instruction.

Curriculum. The typical standards-based statistics instruction observed was based on a lesson (*How many raisins in a box)* from *Investigations in Number, Data, and Space* (TERC, 1998). The teacher taught this lesson over two sessions, the earlier session of which was observed as typical standards-based geometry instruction. The observation of the latter session was used to examine the teacher's instructional adaptations for the students with MD. In the session observed as typical standards-based statistics instruction, students were expected to involve two activities in which they counted the raisins in a sample of small boxes of raisins, record and organize the results. The line plot was introduced as a useful way to make first-draft visual representation of a set of data.

Neither warm-up activities nor reviews of prerequisite skills were included in the teachers' manual. In the first activity, the manual suggested that teachers start their lesson with talking about the meaning of statistics and provide examples of everyday situations that needed statistics, and discuss methods to collect data (e.g., counting, measuring, or doing experiments). In this activity, teachers were suggested to encourage students think of and discuss examples of everyday situations that needed statistics, and think of measures involving weight, volume, time, or temperature, as well, which may be prerequisite skills for learning statistics. In summarizing the first activity, the teachers' manual suggested that teachers provide the outline of the lesson to students.

In the second activity, teachers were expected to provide materials (e.g., a box of raisins to each student), ask students to estimate the number of raisins in the box, encourage students to share their ideas or strategies to estimate the number of raisins in a box, lead the discussion about their methods for organizing data, and have some students to demonstrate their methods for organizing data. Pairing up students or grouping students in groups of 3 and providing enough time for class discussion were suggested for this activity.

Instructional routines. The typical standards-based statistics instruction observed in this study was implemented for approximately 60 minutes. It was based on a lesson included in *Investigations in Number, Data, and Space* (TERC, 1988). The lesson, titled How Many Raisins in a Box, was taught across two consecutive mathematics instructional times. The earlier session was observed to identify typical standards-based statistics instruction; the latter session was observed to explore the teacher's instructional adaptations for the students with MD. Typical standards-based instruction on probability and statistics included three instructional routines: (a) overview of the lesson or presentation of the objectives of the lesson, (b) explanation of the new skills, and (c) independent practice.

At the outset of the lesson, Ashley spent approximately 10 minutes in providing an overview of the lesson. While she was providing an overview of the lesson, she reviewed non-mathematics-specific vocabulary that was used in the lesson objectives. For example, she put an outline of the lesson on the overhead projector and explained about the word *prediction*, which was included in the outline. The first routine occurred in a whole-group setting.

Explanation of the new skills was composed of instruction on new mathematics vocabulary and two activities which were designed to introduce the skills of statistics, estimating data and organizing data. Before starting the activities, the teacher provided instruction on mathematics vocabulary and statistics, which included the definition of statistics and examples using statistics. During the activities, strategies to implement data estimation were discussed and demonstrated by the teacher or student groups. The teacher spent 25 minutes in teaching the new skills.

The final routine of typical standards-based statistics instruction was to provide a time for independent work and sharing student work in class. The students, in small groups of three, were asked to organize data on the number of raisins in a box, listed on the chalkboard. The teacher had each group choose one strategy to organize data, write down three important things they could say about their data, and share the information with the whole class. Instructional time assigned to this routine was 25 minutes.

Typical standards-based statistics instruction for all students. As in typical standards-based geometry instruction for all students, instructional components featuring instruction for all students during typical standards-based instruction on probability and statistics were rooted in two different approaches of instruction: constructivist instruction and direct-strategy instruction. Table 4.8

presents an overview of instructional components used in instruction for all students during typical standards-based instruction on probability and statistics.

Table 4.8

An Overview of Instructional Components Used in Typical Standards-Based Statistics Instruction for All Students

Routine	Grouping	Time (min.)	Instructional components
Overview of the lesson	Whole	10	Review of prerequisite skills (e.g., vocabulary such as *predict*, *statistics*)
			Direct questioning at cognitive memory level (e.g., "What is statistics?")
			Multiple teacher examples during vocabulary instruction
			Discourse-driven instruction (e.g., share ideas about statistics)
			Advanced organization (e.g., purpose setting, an overview of the lesson)
Explanation of the new skills or activity	Whole	25	Discourse-driven instruction (e.g., share their strategies to estimate the number of raisins in a box)
			Modeling of strategies to collect data and organize data by the teacher
			Teaching strategies
			Direct questioning at divergent level (e.g., "How did each of you arrive at your estimate?")
			Use of manipulatives (e.g., raisins)
Independent practice	Small group of 3 students	25	Group instruction
			Inquiry-based instruction (develop strategies to organize data)
			Discourse-driven instruction (e.g., share their strategies to organize data with class)

Apparently, typical standards-based statistics instruction for all students featured two instructional components stemming from a constructivist approach of instruction: discourse-driven instruction and inquiry-based instruction. Discourse-driven instruction was the most notable feature shown in typical standards-based statistics instruction for all students. Discourse-driven instruction refers to an instructional practice where a teacher encourages students to learn mathematics concepts or procedures by engaging in the construction of shared mathematics knowledge in their classrooms, which is usually accomplished by verbal interactions between a teacher and students or among students (Baxter et al., 2001). For example, during the lesson, all students repeatedly were encouraged to participate in class discussions to share their ideas and strategies related to statistics. They were encouraged to discuss their knowledge about the definition of statistics and examples of statistics and to share their strategies to estimate the number of raisins in a box. Unlike the typical standards-based geometry instruction for all students*, inquiry-based instructional practice* was found in typical standards-based statistics instruction for all students. Inquiry-based practice emphasizes exploration or discovery of multiple strategies for problem solving. For example, during independent practice, students in small groups were asked to explore and demonstrate strategies to organize data.

Direct-strategy instruction was another feature of typical standards-based statistics instruction for all students. Direct-strategy instructional components were found, such as advance organization, manipulatives, group instruction, modeling, multiple teacher examples, teaching strategies for problem solving, direct questioning, and review of prerequisite skills (e.g., vocabulary).

Ashley provided advance organization including purpose setting (e.g., “We are learning statistics to find out information about ourselves or the world around us”) and presentation of an outline or overview of the lesson. In addition, she provided a review of vocabulary, which could be regarded as a review of prerequisite skills. Typical standards-based statistics instruction for all students also involved teacher modeling and use of multiple teacher examples, and instruction on strategies that were not found in typical standards-based geometry instruction for all students. Ashley demonstrated a strategy to collect data, a strategy to count data, and a strategy to organize data and used five examples to show how to use statistics (pets, brown eyes, speaking Spanish, taking shuttle bus, and lunch choice).

In addition, Ashley used direct questioning as a way to guide student discussions. The teacher’s questions used in this instruction were at the level of cognitive memory (e.g., “What’s statistics?”) or divergent level (e.g., “How did each of you arrive at your estimate?”). Group instruction was also an instructional component used in typical standards-based statistics instruction for all students. The students were grouped into four groups of three students; they collaborated on developing strategies for organizing data and on writing down three important things about the data.

In summary, typical standards-based statistics instruction for all students featured instructional components stemming from constructivist and direct-strategy instructional approaches. Two constructivist components, discourse-driven instruction and inquiry-based instruction, were found in typical standards-based statistics instruction for all students. Eight direct or strategy instructional components used in the instruction were (a) use of advance organization, (b) use of manipulatives, (c) group instruction, (d) modeling, (e) multiple teacher examples, (f) direct questioning, (g) teaching strategies, and (h) review of prerequisite skills (e.g., vocabulary). In addition, discourse-driven instruction was the component found most frequently in typical standards-based statistics instruction for all students. As in typical geometry instruction for all students, some instructional components effective for teaching students with MD, including prompting and control difficulty, were not present in the instruction.

Instructional adaptations for students with MD during typical standards-based statistics instruction. Typical standards-based instruction on probability and statistics included instructional adaptations for students with MD as well as typical standards-based instruction for all students. Instructional adaptations for students with MD during typical standards-based statistics instruction included reviews of skills taught and reviews of vocabulary. For example, the teacher provided a review of methods for organizing data including drawing a bar graph, tables, and charts, which were not included in the teachers' manual of the textbook. In addition, before introducing the concept of statistics, the teacher reviewed vocabulary such as “estimate” and “organize” which were related to the concept of statistics. In the informal interview conducted after the observation of typical standards-based statistics instruction, the teacher recognized these two instructional practices as adapted instruction for students with MD in her class.

Instructional Adaptations for Students With an IEP in Mathematics During Lessons on Probability and Statistics

Findings on Ashley’s instructional adaptations for her 3 students with MD during 5 consecutive instructions on probability and statistics were derived from data collected through observations, interviews, and document reviews (e.g., lesson plans). The teacher was observed for five consecutive lessons on this content, which were based on the standards-based mathematics curriculum, *Investigations in Number, Data, and Space* (TERC, 1998). Instructional time spent on each lesson was 60–90 minutes. Five lessons were observed. Investigation 1, Session 1 was How Many Raisins in a Box? Investigation 1, Sessions 2 and 3 covered How Many People in a Family? Investigation 2, Session 1 was entitled, How Tall Are Fourth Graders? Investigation 2, Session 2 and 3 covered the unit, Fourth and First Graders: How Much Taller? Table 4.9 provides an overview of Ashley’s instructional adaptations for 3 individual students with an IEP in mathematics in her standards-based instruction on probability and statistics.

Table 4.9

Ashley's Instructional Adaptations for Three Individual Students With MD Across Five Statistics Lessons

Group setting and context	Frequency		
	Lee	Kevin	Tina
Whole group			
Correcting student wrong responses (unplanned)	4	4	1
Assisting student to understand concepts or procedures (planned)	3	4	1
Small group			
Correcting student wrong responses (unplanned)	2	5	0
Assisting student to understand concepts or procedures (planned)	0	0	0
Total	9	13	2

Identification of instructional adaptations. Instructional adaptations were defined as appropriate adjustments, accommodations, and modifications to instruction or supports that allow students to meet academic requirements and conditions of the curriculum, most often the general education curriculum (VGCRLA, 2001). To determine if a portion of instruction was adjusted targeting individual students with MD or groups involving them, the portion was compared with typical standards-based instruction for all students. Additionally, the teacher was interviewed about why she implemented the specific interactions with the individual students with MD after each observation was conducted. Adaptations were also determined by the teacher's written recognitions including notes for accommodations or adaptations in her lesson plans.

A portion of instruction was considered an instructional adaptation if it satisfied all the following three criteria: (a) It involved interactions with the individual students with MD or small groups involving the students with MD, which were not shown at typical standards-based statistics instruction for all students; (b) it included teacher-identified adjustments or modifications in terms of instructional content, instructional activity, delivery of instruction, or instructional materials; and (c) the teacher recognized the occurrence of an adjustment or modification through interviews or lesson plans.

Frequency and contexts of instructional adaptations for individual students with MD. This section provides overall descriptions of the teacher's instructional adaptations for the individual students with MD in terms of (a) frequency per lesson, (b) group setting, and (c) situations inducing adaptations. Overall, the teacher made instructional adaptations in terms of grouping formats in four out of five lessons. Table 4. 9 summarizes the occurrence of instructional adaptations made for the individual students with MD in whole-group or small-group settings during the whole period of observations of lessons on probability and statistics.

The teacher rated Kevin as a student most struggling with the prerequisite skills required for learning fourth-grade probability and statistics. Ashley made the largest number of adaptations for Kevin among the 3 students with MD during the statistics lessons. A total of 13 adaptations were observed relating to Kevin across five lessons. Ashley's adaptations for Kevin were unevenly distributed across lessons. At least 2 adaptations were found in Lessons 4–7, but no adaptation was found in Lesson 8.

Eight adaptations out of a total of 13 were made in a whole-group setting, whereas 5 adaptations occurred in a small-group setting. Particularly, half of the adaptations that occurred in whole-group instruction were part of the process of prompting to correct student wrong responses, after Kevin showed difficulty in understanding a concept or procedure or provided an incorrect answer. For example, when Kevin failed to give an answer for a multiplication problem of 15 times 4, the teacher prompted him to get to the right answer by segmenting the procedures of multiplication into easier steps, computing 15 times 2 and then multiplying by 2. In these interactions, the teacher employed direct questioning at cognitive-memory level to prompt the correct answer. She changed task difficulty by breaking the problem into easier steps. Overall, these interactions were made as efforts to provide prompting.

> T: Maybe, every class has about 15. And then, you might think how many classes are there in the fourth grade?
> Kevin: Four.
> T: Four. You are going to count 15, how many times?
> All students: Four.
> T: Can you tell me what it is, 15 times 4? Kevin, what is 15 times 4?
> Kevin: (Murmured).
> T: 15 times 2 is what?
> Student 1: 30.
> T: 15 times 4 is 15 times 2 times 2. So, 30 times 2. What is 30 times 2? Kevin?
> Kevin: 60.

Another adaptation in a similar situation was implemented when Kevin failed to provide a correct answer to the question asking a typical number of raisins in a box. The teacher attempted to provide prompting using a direct questioning method. Questions used for direct questioning were at the level of cognitive memory. The teacher also reexplained the way to figure out a typical number shown in a graph or a container (pick a number between the smallest number and the largest number).

> T: Kevin, would you say 41 is typical number of raisins in your box? Typically, you get 41 raisins in a box. What would you say, true or false? (No response from Kevin.) Think about 44. What would you say about more typical number, 41 or 44? What's more typical?
> Kevin: 40.
> T: Not 40, 41 and 44. What do you think is the better choice?
> Kevin: 44.
> T: 44?
> Kevin: 33.
> T: I will go with the numbers between 33 and 44; 33 is the lowest and 44 is the highest. So, you don't want to go with either the lowest or the highest number for the most typical number of raisins.
> Kevin: Most people got 33.
> T: Right.

The other four adaptations out of the eight whole-group adaptations made for Kevin occurred during the teacher's instruction on concepts or procedures. These adaptations were planned prior to the lessons and were shown in the teacher's lesson plans. For example, during Lesson 4, the teacher made her question clear and understandable through using a specific example and explicit directions. She replaced the question, "What is prediction?" with, "What if I said how many students in the fourth grade? Would you just say probably 500?" These interactions also involved providing prompting through direct questioning at the level of cognitive memory.

> T: What is prediction? We are going to talk about prediction today. What are predictions, Linda?
> Linda: Guess.
> T: Okay, guess. How do you guess? Kevin? We just randomly pick any number in the world? Or do you think about before you guess? You know, what if I said how many students in the fourth grade? Guess how many kids in the fourth grade. I don't know if you guys know that, but you are going to predict what you thought the number would be. Would you have some ways of thinking about that? Or, would you just say probably 500?
> Kevin: I will look at the class.
> T: You will look at the class. Which class?
> Kevin: Our class.
> T: Our class. And then you will think what?
> Kevin: There is more than one.
> T: Of course, there's more than one. You are closer. How many students do you think each class has?
> Kevin: 10.

An instance of adaptations in a similar situation (whole-group instruction on concepts or procedures) was found during Lesson 5. While explaining procedures for making a bar graph, Ashley demonstrated how to make a bar graph using an example (e.g., a bar graph of family size in the class) and guided practice, which involved direct questioning at the level of cognitive memory. These adaptations were implemented not only for Kevin, but also for the other students with MD. These adaptations were shown in the teacher's lesson plan.

> T: (Starting to draw an example of a bar graph of family size on the board). Okay, we are talking about the title. At the bottom, you have numbers. So, we are going to have one side, maybe have what kind of an angle?
> Some students: A right angle.

T: A right angle, this, we are going to start here always with 0. Now, we are going to tell how many people had 2, 3, 4, 5, etc. I think we need to start with 1. So, we can number all the way up to what?

Some students: 15.

T: 15. (Put the numbers 1–15 at the bottom). Okay, now, from here to here (pointing at the right line of the graph), how many students do we have all in our class?

All students: 13.

T: So, I will need 13 people. I am putting numbers from 0 to 13 on the left line. Now, I am going to use bars, because this is a bar graph, so we need to use bars. I start with 1. We have any having one family?

All students: No.

T: Two?

All students: No.

T: Three?

All students: Yes.

T: Yes, we have one. So, my bar should go up to the...?

All students: One.

T: One.

T: How many have four?

All students: One.

T: Again, my bar should go up to one. How many have five?

All students: Two.

T: You color up to the…?

Some students: Two.

T: Two. You are going to all the way go through up to 15. How many people have 15?

All students: One.

T: The only thing else you need to do, do you know what the numbers mean?

Some students: No.

T: We do, but somebody from outside our class they don't know what the numbers mean.

Student 2: We need a title.

T: Yes. What do we put up here (bottom) on this bar graph?

Some students: Number of people in our family.

T: Number of people in our family (puts it on the bottom). What do we put up here (left side)?

Some students: People in class.

T: Number of people in class. What if we decide to flip around this graph? Can we put the number of people in our family here and put the number of students here?
All students: Yes.
Student 2: Can we color this?
T: Yes, you will color yours. You may color every bar having one student in yellow, every bar having two students in brown something like that. We try to have some systems when we color it. Do you have any questions?
All students: No.

All the five adaptations made in small-group instructional settings occurred in the process of prompting after identifying their difficulties through small-group interactions. Four of the adaptations occurred on providing prompting after conducting progress monitoring of each group work. For example, during Lesson 5, Ashley paired Kevin with a high-achieving student for partner work to collaborate to make a bar graph representing the size of family in the class. While monitoring the group work, the teacher checked Kevin's understanding of making a bar graph from a T-chart by asking him how many students had three family members in his class on the T-chart. When Kevin showed difficulty in making a bar graph from the T-chart, Ashley adjusted her instruction by providing prompting procedures and reteaching about the way to make a bar graph (e.g., look the numbers at the bottom to find three people in the family and look at the one from the left line).

T: (to Kevin group) How many people have 3, Kevin?
Kevin: Four.
T: These (at the bottom) are the numbers representing the number of family. These numbers (on the left side) are representing the number of students in our class. How many students have three? (No response from Kevin.)

Student 9: One.
T: One. So, you will look the numbers at the bottom to find three people in the family and look at the one from the left line. That represents one student has three family members. Four has one. So, you look for four at the bottom numbers and find one from the left line.

The remaining adaptations occurred during small-group instruction in prompting Kevin when he showed difficulties in understanding of concepts or procedures being taught. For example, during Lesson 6, Ashley attempted to teach the concept of typical and the procedure of finding a typical number on a bar graph. When Kevin showed difficulty, the teacher adjusted her instruction by using prompting and direct questioning.

T: You may pick one. How many brothers do you think might be a typical in our class?
Kevin: Two.
T: Why, Kevin? Why 2?
Kevin: I see a lot of 2 there.
T: You see 2 a lot there? I see 14 has 2. Eleven has 2. And that's it. I see a lot of 0. I see number 2, number 7, 8, 9, 10, 12, and 13. These people have no brother. A couple of you have two and a couple of you have 1. What is between 0 and 3?
Some students: One.
T: Maybe 1. Maybe 2. I am going to go with 1. One brother may be typical in our class. Could 0 be a typical number?
Some students: Yes.
Kevin: No.
T: Yes, it could be a typical number. More people in this class have 0 brothers..

For Lee, Ashley adjusted her instruction nine times in either whole-group (seven times) or small-group instruction (two times). At least one adaptation was made by Ashley for Lee in each lesson, except Lesson 4. Four out of seven adaptations made during whole-group instruction were induced by Lee's incorrect response. For example, the teacher attempted to prompt Lee to get to the right answer to her question about the numbers on the X-axis of a bar graph when Lee failed to answer the question. These interactions involved prompting procedures which included asking direct questions at the level of cognitive memory.

> T: What do you all have to have in your graph?
> All students: Title.
> T: The title, just like a story, you have to have a title of your graph. At the bottom, you are going to have numbers. What does the numbers represent, Lee? (No response.) At the bottom.
> Lee: The numbers of the thing we are going to do.
> T: I don't know what that means. Don't say things. The numbers of …what?
> The numbers tell us….? The numbers of people in…?
> Lee: People in your family.
> T: People in your family. So, at the bottom, you are going to write your numbers and label it number of…?
> Some students: People in my family.

Similarly, when the teacher reviewed skills of finding operations to solve word problems using key words, she attempted to prompt Lee to get the correct answer and provide an example to explain the meaning of a key word (e.g., *joined*).

> T: (Showing the word *heavier*), Lee? (No response.) Heavier, ier, er…
> Lee: Subtract.
>
> T: (Showing the word *joined*), Lee? (No response.) You and Student 4 (Lee's twin sister in the classroom) might be joined together since you were born. (No response.) Joined means add together.

Three adaptations identified during whole-group instruction were implemented as the teacher provided explanations of concepts or procedures. One of these adaptations also involved her reaction to Lee's incorrect response, which was identified during Ashley's instruction on concepts or procedures. For example, while Ashley was trying to teach how to make a bar graph of student heights in her class, Lee showed difficulty in marking a data point on the bar graph using a ruler. In response to Lee's difficulty, Ashley prompted Lee using direct questions at the level of cognitive memory and modeled how to do it.

> T: 125. So, maybe, Student 1 lay down here (drawing a bar for Student 1's height), so you may want to get a ruler or yard stick, to make it sure that your bars are even. Let's do Student 7? Let's do Student 7'stogether. Student 7 says, 141 centimeters. Lee, come on here. I will let you do this one. (Lee comes to the front.) Linda said 141 centimeters. And we will make a bar graph. You are going to make a bar. You are going to make it come up to where?
> Lee: 141.
> T: 141. You may want to come up and put a little dot where you want to stop that. (No response or action from Lee.) This (pointing at 140 on the Y axis) is 140. You are going to go little bit above that. This is 140. Do you have a ruler? (Put a ruler below the 140). You should put a dot right on the top of this ruler to show 141. Okay, when you take the ruler away, now you got your dot. And you may want to use your ruler to make your bar (Showed how to do this to Lee). And then, you color that. What else could you do to make this even easier to read?
> All students: Label it.
> T: Yes, just like we do in a math problem, you have to label your bar graph, so like Student 2 said, you can come up here and put 141. If you've got room, you can go ahead and put cm here.
> T: Okay, any questions about what you are going to do?
> All students: No.
> T: When you are done, you are going to have every single person in our class on your class, even though Student 7 is absent, let's go ahead to make his bar. You should make it sure you can fit every person's name on your bar graph.

During small-group instruction, the teacher adjusted her instruction for Lee twice. Both adaptations were implemented as reactions to Lee's wrong responses or incorrect performances identified during Ashley's progress monitoring of group work. For example, when Ashley found Lee struggle with drawing a bar representing a student's height in monitoring group work during a lesson, she attempted to prompt Lee to perform correctly. In this instance, while attempting to graph 141 cm for Student 1, Lee put a dot around 145 on the Y axis and did not show the skills of using a ruler to draw a dot representing the coordinate of 141(Y) and Student 1(X).

> T: Wait a minute. Where is your dot? Put your ruler under the dot horizontally. (Showed where to place the ruler to the group. Lee corrected her ruler's position.) Wait a minute. Where did you put your dots? What's the height of Student 1?
> Lee: 141.
> T: You put your dot around 145 (Helped Lee find 141).

For Tina, two instructional adaptations were made for five statistics lessons. Both occurred in whole-group instruction. One adaptation was induced by Tina's incorrect response during the interactions. For example, as Ashley reviewed the skills of finding operations of solving word problems using key words, Tina showed difficulty in figuring out an appropriate operation using a key word (e.g., *and* indicates adding together, addition). The teacher attempted to prompt her to get to the correct answer and adjusted the level of difficulty of the task by presenting a simpler problem than the original problem.

> T: (Showed *and*) I have seven candies and three bananas. Tina, how many foods do I have to eat? (No response.) Four tickets in this pocket, and three tickets in that pocket. What would you do to find out how many tickets I have now?
> Tina: Five.
> T: What will be four and three tickets? (No response.) We need to add them together.
> Tina: Seven.

The other instructional adaptations made for Tina occurred to assist students with MD including Tina to understand concepts or procedures being taught. The instance was illustrated in the descriptions of instructional adaptations for Kevin.

In summary, the numbers of instructional adaptations made by the teacher were different across 3 students (12 for Kevin, 9 for Lee, and 2 for Tina). The teacher made the largest number of adaptations for the student (Kevin) whom she rated as the most struggling student with the prerequisite skills required for learning fourth-grade probability and statistics. The grouping format and the situations in which adaptations occurred were different across students. While Ashley adjusted her instruction for Kevin relatively evenly across grouping formats (whole group vs. small group) and situations inducing adaptations (reaction to student response vs. assistance of students' understanding of concepts or procedures), she adjusted her instruction for Lee and Tina more in the whole-group setting. In addition, Ashley tended to make adaptations for Lee when she identified Lee's difficulties through Lee's incorrect responses rather than to plan and implement adaptations for helping Lee understand concepts or procedures. For Tina, adaptations were distributed in both situations evenly, one per situation.

Categories of instructional adaptations. For this analysis, this study employed the AF by Bryant and Bryant (2001). According to Bryant and Bryant, instructional adaptations can occur in at least one of four categories: (a) instructional content, (b) instructional activity, (c) delivery of instruction, and (d) materials or technology. Instructional content means skills and concepts that are the focus of teaching and learning. Frequencies of Ashley's instructional adaptations by category are provided in Table 4.10.

According to Table 4.10, Ashley adjusted her instruction for the students with MD in terms of delivery of instruction (average of 3 students = 11.3) and instructional activity (average of 3 students = 1.7) for the five lessons on probability and statistics. Adaptations of instructional activity included (a) reviewing other skills taught for the students with MD while the other students were working on an activity of the lesson (e.g., the teacher reviewed key words associated with operations to solve word problems using flash cards, while a group of student was measuring their heights to collect data for making a bar graph), (b) reviewing prerequisite skills related to the instructional content (e.g., the teacher provided times for reviewing how to use a ruler to measure a length and reviewing measurement units including centimeters and inches), (c) teaching or prompting to use a metacognitive strategy to verify their performances (e.g., when Kevin's group finished transferring T-chart data on family size in their class into a bar graph, the teacher provided instruction on a metacognitive strategy to ensure that their graph was correct as a wrap-up activity), and (d) changing a portion of an activity.

Table 4.10

Frequencies of Ashley's Instructional Adaptations by Category

Category	Lee	Kevin	Tina	Average
Instructional content	0	0	0	0.0
Instructional activity	2	3	2	1.7
Delivery of instruction	12	16	6	11.3
Instructional materials or technology	0	0	0	0.0
Total	14	19	8	

First, Ashley reviewed the key-word strategy to solve word problems for her struggling students, including the students with MD, while the other students were engaged in the activity planned for the day's lesson. Because the skill of using the word-problem-solving strategy, key-word strategy, was not related to the skills being taught on that day and was not like prerequisite skills for the lesson, this type of modification was categorized and presented separately from instructional adaptations by supplemental teaching of prerequisite skills.

> T: Again, your choices are adding, subtracting, multiplying, and dividing. (Showed a card including a keyword for solving a word problem, the word *heavier*). Lee? (No response.) Heavier, ier, er…
> Lee: Subtract.
> T: (Showed the word *product*) Student 8? (No response.) Product is an answer of what?
> Student 8: Subtraction.
> T: No, it's the answer of multiplication. (Showed the word *left*) Student 1?
> Student 1: Subtract.
> T: (Showed the word *joined*) Lee? (No response.) You and [Student 4] (Lee's twin sister in the classroom) might be joined together since you were born. (No response.) Joined means

> add together. (Showed the word *and.*) I have seven candies and three bananas. Tina, how many foods do I have to eat? (No response.) Four tickets in this pocket, and three tickets in that pocket. What would you do to find out how many tickets I have now?
> Tina: Five.
> T: What will be four and three tickets? (No response.) We need to add them together.
> Tina: Seven.

The second type of adaptations was made by providing supplemental instruction on prerequisite skills (e.g., teaching how to use a ruler and read a ruler to measure length) for learning the skills being taught (e.g., measuring length). The following instance illustrates instructional adaptations made by providing supplemental instruction on prerequisite skills. The teacher had each group of students measure their heights and write it down on their card by turn. The teacher helped Kevin and his partner with measuring their heights, placing the end of the tape measure at the bottom of the wall.

> T: Kevin, you need to start from the right on the bottom. Look at this. Your tape measure should touch the floor. Where are you Kevin right now? That's correct?
> Partner: No.
> T: No. Kevin. Look at your tape measure. Kevin, you may be holding it. Hold it right here. (To partner) You are going to touch with the floor and then go straight up to, you are actually 142 cm (showed how to measure it again.

Another type of change that Ashley made in her mathematics instructional activity was to provide supplemental assistances in using a metacognitive strategy to verify their performances. During Lesson 5, Ashley monitored work by the group involving individual students with MD. As she found Kevin's group struggle with the group assignment (transferring T-chart information to a bar graph), Ashley assisted them to complete the assignment correctly and provided a time to check the correctness of their procedures and outputs.

As well, Ashley changed her instruction in terms of student roles in the activity of measuring heights during Lesson 7. In the activity suggested in the curriculum, the students were supposed to be in groups of three students, with one person being measured, a person measuring, and a person recording. However, Ashley changed this activity into partner work of two students and had each student take all three responsibilities. Ashley identified this activity as being adapted for her struggling students, including students with MD, during informal conversation after the lesson. Ashley expected that these changes would provide multiple opportunities of practicing the measurement skills (e.g., measuring things and understanding measurement) to the struggling students. The following was cited from the teacher's introduction of the activity, which included changes in the activity.

> T: Right, you want your legs to be straight, your feet together, put your back against the back of the closet, and don't look down until your partner finish measuring your height (Showed right posture). Your partner, what your partner will do is to take a pencil and put it on the top and then just make a little line (showed it). Now, the line, that's what you are going to measure. You and your partner will measure it together. I would say that you are going to have to measure it twice. Make it sure that you get the same measurement with your partner, okay?

Delivery of instruction was a category in which Ashley made adaptations for her students with MD most frequently. Adapting delivery of instruction involved changes in various aspects of her instruction. This included the following nine aspects:

1. She explicitly provided teacher examples that directly corresponded to the specified learning objective. During Lesson 5, the teacher provided explicit explanations of concepts or procedures related to creating a bar graph from the T-chart data in whole group, using explicit modeling and a teacher example of family size in class. The teacher recognized this as an instructional adaptation for her students with MD later at informal interviews.

2. She provided practice opportunities, such as providing a time for guided practice when she taught about how to draw a bar graph.

3. She prompted the students to get to the right answer when they provided wrong answers.

4. She controlled the level of task difficulty. She attempted to control the level of task difficulty by prompting or providing easier examples.

5. She used a direct questioning method.

6. She monitored student learning on the objective.

7. She provided explicit modeling.

8. She provided reteaching or reexplanation.

9. She provided group instruction. Except for Lesson 4, the teacher purposively paired individual students with MD with highly achieving students for four lessons on probability and statistics.

The specific instances of the practices mentioned above were illustrated in the section of incorporation of evidence-based mathematics instructional components into typical statistics lessons. In summary, instructional adaptations in two categories—instructional activity and delivery of instruction—were identified in Ashley's five instructions on probability and statistics. Most adaptations were classified into delivery of instruction. Neither adjustment of instructional content nor that of instructional materials occurred during instruction on probability and statistics. Unlike instructional adaptations in geometry lessons, Ashley used explicit explanations using modeling for assisting the students with MD in her statistics instruction and did not use manipulatives. The following section provides findings on instructional components that the teacher used to adapt her instruction during instruction on probability and statistics.

Incorporations of Evidence-Based Mathematics Instructional Components Into Typical Statistics Standards-Based Instruction

The instructional components emerging from data on the teacher's instructional adaptations during instruction on probability and statistics included (a) prompting, (b) control difficulty, (c) explicit modeling, (d) direct questioning, (e) review of prerequisite skills, (f) group instruction, (g) strategy instruction, (h) progress monitoring, (i) use of manipulatives, (j) use of teaching examples, (k) practice opportunity, (l) explicit explanations of concepts or procedures, (m) reteaching or reexplanation, and (n) review of skills taught previously. Table 4.11 provides an overview of evidence-based mathematics instructional components that were used for Ashley's instructional adaptations for teaching probability and statistics to students with MD.

Table 4.11

Frequency of Ashley's Instructional Adaptations Using Evidence-Based Mathematics Instructional Components Across Five Probability and Statistics Lessons

Evidence-based mathematics instructional component	Frequency					
	Lesson 4	Lesson 5	Lesson 6	Lesson 7	Lesson 8	Avg.
Prompting	4	3	5	4	2	3.6
Explicit explanations	1	4	2	2	1	2.0
Explicit modeling	0	3	1	1	1	1.2
Control difficulty	2	3	0	2	0	1.4
Direct questioning	4	7	5	1	1	3.6
Review of prerequisite skills	1	0	0	1	1	0.6
Group instruction	0	3	3	3	3	2.4
Strategy instruction	0	0	0	0	0	0.0
Progress monitoring	0	1	3	0	0	0.8
Use of manipulatives	0	0	0	0	0	0.0
Practice opportunity	0	0	1	1	0	0.4
Teacher examples	0	0	0	2	0	0.4
Reteaching	0	2	1	0	2	1.0
Vocabulary instruction	0	0	0	0	1	0.2

Prompting. Teachers assist students in generating correct response by prompting them with verbal, physical, or written cues (Rivera & Smith, 1998). Prompting may be implemented by asking leading questions, repeating and rephrasing lesson content, pointing to a specific word or number, providing examples and nonexamples, giving feedback, doing tasks partially, doing a task with students, and providing manual guidance (Mercer & Mercer, 2005). As in the lessons on geometry and spatial reasoning, prompting was a component of evidence-based mathematics instruction that the teacher most frequently utilized for adjusting her instruction for 3 students with MD to teach probability and statistics (average frequency per lesson = 3.6).

In accordance with its definition, the use of prompting was found to be associated with an instructional situation in which a student with MD showed difficulties in understanding concepts or procedures or in correctly answering the teacher's question, especially in the whole-group setting.

Most of the time (12 instances out of 18), Ashley provided prompting when a student with MD showed errors or difficulties. The following examples were cited as examples of instructional adaptations including prompting procedures which involved asking leading questions, providing feedback, pointing a specific number or word, and/or providing cues. For instance, during the activity of solving a word problem using a key word, the teacher showed the word *heavier* to Lee and asked her about an operation related to this key word. When Lee showed difficulty in producing the right answer, the teacher prompted Lee: "Heavier, ier, er…" and Lee responded, "Subtract." The following two examples were cited from the teacher's interactions with Kevin.

> T: Somebody give me the number in fraction? How many of you have pets as a fraction? Kevin?
> Kevin: 10 out of—
> T: How many people are in this room?
> Student: (Counted them) 16.

T: 10 out of what?
Kevin: 16.
T: 10 out of 16. Can you put this in another way, Kevin? (No response.) Student 8?
Student 8: Ten over 16.

T: You may pick one. How many brothers do you think might be a typical in our class?
Kevin: 2.
T: Why, Kevin? Why 2?
Kevin: I see a lot of 2 there.
T: You see 2 a lot there? I see 14 has 2, 11 has 2 and that's it. I see a lot of 0. I see number 2, number 7, 8, 9, 10, 12, and 13. These people have no brother. A couple of you have two and a couple of you have 1. What is between 0 and 3?
Some students: One.
T: Maybe one. Maybe 2. I am going to go with 1. One brother may be typical in our class. Could 0 be a typical number?
Some students: Yes.
Kevin: No.
T: Yes, it could be a typical number. More people in this class have 0 brothers.

During the activity of solving a word problem using key words, the teacher showed a key word *and* to Tina and asked her to answer an operation associated with *and*:

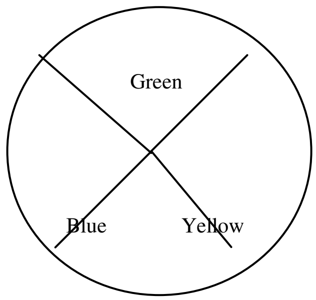
T: I have seven candies *and* three bananas. Tina, how many foods do I have to eat? (No response.) Four tickets in this pocket, and three tickets in that pocket. What would you do to find out how many tickets I have now?
Tina: Five.
T: What will be four and three tickets? (No response.) We need to add them together.
Tina: Seven.

Explicit explanations. Explicit explanations were related to providing complete, consistent, and logical explanations for concepts, skills, or activities through verbal direction, examples, or representation tools such as manipulatives (Downey, 2001; Jitendra et al., 1999). During the lessons on probability and statistics, the teacher employed explicit explanations in adjusting her instruction for students with MD 10 times. The 10 instances of instructional adaptations using explicit explanations were further divided into three subcategories; (a) providing logical, consistent, and complete verbal direction (5 instances); (b) providing examples that directly corresponded to the lesson objective (5 instances); and (c) rephrasing directions in easier words (1 instance). One of the 10 instances was found to be associated with two categories (providing examples and rephrasing directions).

Half of instances of instructional adaptations including explicit explanations were related to adjusting verbal directions in a logical, consistent, and complete way. The following two instances presented how explicit explanations were used in Ashley's instruction to assist the students with MD in learning probability and statistics.

> T: (To Lee's group; they did not have four different colors of yarns or four brown yarns representing one) You should have four different colors. Where is your yellow? Then, you have to have four little ones (brown). You have three. Go out to the hallway to measure one more this. (Lee and Student 1 measured one more brown yarn.)
> Student 1: Now, we make a circle with these yarns. (Lee tried to put all together to make a random shape).
> T: We are going to put all together to make this shape (Brought the example of a pie chart from the overhead projector). Do you remember this? We are going to put all together to make this chart and divide it into four different sections. We are going to make a circle as best as you can. Remember this yarn (brown) represents how much?
> Lee & Student 1: one.
> T: So, this represents 1. This represents 2, this represents 3, and this represents 4. Make a circle as best as you can.

T: These all represent 1. So, they will be put together to show your one group. Then, your yellow section will start from here and go to there (modeled how to draw a pie chart using the yarn).



T: Do not glue your yarns. Just draw a best circle you can make. So, your first step is to make a best circle you can make and trace the circle using your pencil. (Lee & Student 1 made a circle and divided it into each section).
T: When you are done, you are going to have this kind of pie chart (Showed a model chart not including information for each section).

T: 125. So, maybe, Student 1 lay down here (Drew a bar for Student 1's height), so you may want to get a ruler or yard stick, to make it sure that your bars are even. Let's do Student 7? Let's do Student 7's together. Student 7 says, 141 centimeters. Lee, come on here. I will let you do this one. (Lee came to the front.) Student 2 said 141 centimeters. And we will make a bar graph. You are going to make a bar. You are going to make it come up to where?
Lee: 141.
T: 141. You may want to come up and put a little dot where you want to stop that. (No response or action.) This (pointing at 140 on the Y axis) is 140. You are going to go little bit above that. This is 140. Do you have a ruler? (She put a ruler below the 140). You should put a dot right on the top of this ruler to show 141. Okay, when you take the ruler away, now you got your dot. And you may want to use your ruler to make your bar (Showed how to do this to Lee). And then, you color that.
T: What else could you do to make this even easier to read?
All students: Label it.
T: Yes, just like we do in a math problem, you have to label your bar graph, so like Student 2 said, you can come up here and put 141. If you've got room, you can go ahead and put cm here.
T: Okay, any questions about what you are going to do?

All students: No.

The other half of instances of instructional adaptations using explicit explanations involved providing examples directly corresponding to the skills being taught. The following instances illustrated instructional adaptations involving explicit explanations with examples:

> T: (Showed the word *joined*) Lee? (No response.)
> You and Student 4 (Lee's twin sister in the classroom) might be joined together since you were born. (No response.) Joined means add together.

Conducting progress monitoring of group work, the teacher found Lee and Tina's group struggling with putting titles on their bar graphs. In response to their struggles, she drew an example of a bar graph of family size on the board.

> T: Okay, we are talking about the title. At the bottom, you have numbers. So, we are going to have one side, maybe have what kind of an angle?
> Some students: A right angle.
> T: A right angle, this, we are going to start here always with 0. Now, we are going to tell how many people had 2, 3, 4, 5, etc. I think we need to start with 1. So, we can number all the way up to what?
> Some students: 15.
> T: 15. (Put the numbers 1–15 at the bottom). Okay, now, from here to here (pointed at the right line of the graph), how many students we have all in our class?
> All students: 13.
> T: So, I will need 13 people. I am putting numbers from 0 to 13 on the left line. Now, I am going to use bars, because this is a bar graph, so we need to use bars. I start with 1. We have any having one family?
> All students: No.
> T: Two?
> All students: No.
> T: Three?
> All students: Yes.

T: Yes, we have one. So, my bar should go up to the…?
All students: One.
T: One.
T: How many have four?
All students: One.
T: Again, my bar should go up to one.
T: How many have five?
All students: Two.
T: You color up to the?
Some students: two.
T: Two.
T: You are going to all the way go through up to 15. How many people have 15?
All students: One.
T: The only thing else you need to do, do you know what the numbers mean?
Some students: No.
T: We do, but somebody from outside our class they don't know what the numbers mean.
Student 2: We need a title.
T: Yes. What do we put up here (bottom) on this bar graph?
Some students: Number of people in our family.
T: Number of people in our family (Put it on the bottom).
T: What do we put up here (left side)?
Some students: People in class.
T: Number of people in class.

Rephrasing directions in easier words was a method Ashley used to provide explicit explanations to her students with MD.

T: What is prediction? We are going to talk about prediction today. What are predictions, Linda?
Linda: Guess.
T: Okay, guess. How do you guess? Kevin? (No response.) We just randomly pick any number in the world? Or do you think about before you guess? You know, what if I said how many students in the fourth grade? Guess how many kids in the fourth grade. I don't know if you guys know that, but you are going to predict what you thought the number

would be. Would you have some ways of thinking about that? Or, would you just say probably 500?

Kevin: I will look at the class.

T: You will look at the class. Which class?

Kevin: Our class.

T: Our class. And then you will think what?

Kevin: There is more than one.

T: Of course, there's more than one. You are closer. How many students do you think each class has?

Kevin: 10.

Explicit modeling. Using explicit modeling involved the teacher's demonstration of the skill, process or steps to solve a problem, or how to do a task using thinking aloud (Butler et al., 2003). The teacher might solve a problem with students through a question-and-answer format after a short demonstration of the skill, algorithm, or strategy. During most lessons, except Lesson 4, the teacher used explicit modeling to adjust her instruction for students with MD. Four out of six instances using explicit modeling occurred as reactions to students' errors or difficulties in completing the task during whole-group instruction, while the other two occurred during instruction on concepts or procedures in small groups. The first example below was cited to illustrate the instance of using explicit modeling to address student wrong responses or difficulties in whole-class instruction, and the second example showed the instance of using explicit modeling to teach concepts or procedures in small groups.

T: 125. So, maybe, Student 1 lay down here (Draw a bar for Student 1's height), so you may want to get a ruler or yard stick, to make it sure that your bars are even. Let's do Student 7's. Let's do Student 7's together.

Student 7 says, 141 centimeters. Lee, come on here. I will let you do this one.
Lee: (Came to the front)
T: Student 2 said 141 centimeters. And we will make a bar graph. You are going to make a bar. You are going to make it come up to where?
Lee: 141.
T: 141. You may want to come up and put a little dot where you want to stop that.
Lee: (Did not do any).
T: This (pointing at 140 on the Y axis) is 140. You are going to go little bit above that. This is 140. Do you have a ruler? (She put a ruler below the 140). You should put a dot right on the top of this ruler to show 141. Okay, when you take the ruler away, now you got your dot. And you may want to use your ruler to make your bar (Showed how to do this to Lee). And then, you color that.
T: What else could you do to make this even easier to read?
All students: Label it.
T: Yes, just like we do in a math problem, you have to label your bar graph, so like Student 2 said, you can come up here and put 141. If you've got room, you can go ahead and put cm here.
T: Okay, any questions about what you are going to do?
All students: No.

The teacher had each group of students measure their heights and write it down on their card by turn. When Kevin and his partner were measuring their heights, the teacher helped them by placing the end of the tape measure at the bottom of the wall.

T: Kevin, you need to start from the right on the bottom. Look at this (Showed how to do it). Your tape measure should touch the floor. Where are you Kevin right now? That's correct?
Some students: No.
T: No. Kevin. Look at your tape measure. Kevin, you may be holding it. Hold it right here. Kevin, you are going to touch with the floor and then go straight up to, you are actually 142 cm.

Control difficulty. Control difficulty refers to the adjustment of task difficulty by sequencing tasks from easy to difficulty and providing only necessary hints to students, segmenting the task into smaller steps or units and then synthesizing the parts into a whole, or providing simplified demonstration (Swanson et al., 1999). During the lessons on probability and statistics, Ashley adjusted task difficulty for the students with MD at least once per lesson, except during Lesson 8. She adjusted task difficulty by (a) segmenting the task into smaller parts and prompting the student to complete each smaller part, (b) providing an example to help the student's understanding about the task, and (c) providing an easier example or problem to the student who showed struggles with the original question or problem. Of these subtypes, control difficulty by providing a concrete example was most frequently found in her instruction on probability and statistics (five out of seven instances). The following example showed an instance in this category:

> T: (Showed the word *joined*) Lee? (No response.) You and Student 4 (Lee's twin sister in the classroom) might be joined together since you were born.

Another way of controlling task difficulty by Ashley was related to segmenting the tasks into smaller parts and prompting a student to complete each smaller part. The following example was cited to illustrate this type of task difficulty control:

> T: Maybe, every class has about 15. And then, you might think how many classes are there in the fourth grade?
> Kevin: Four.
> T: Four. You are going to count 15, how many times?
> All students: Four.
> T: Can you tell me what it is, 15 times 4? Kevin, what is 15 times 4?
> Kevin: (Murmured).
> T: 15 times 2 is what?
> Student 1: 30.
> T: 15 times 4 is 15 times 2 times 2. So 30 times 2. What is 30 times 2? Kevin?

Kevin: 60.

Direct questioning. Direct questioning is defined as process-related or content-related questions asked to students (Lee et al., 1999). A teacher's questions may be classified into four categories of questions according to what the teacher is trying to get the student to do in response: (a) cognitive memory level, (b) convergent level, (c) divergent level, and (d) evaluative level (Callahan & Clarke, 1988; for detailed information, see the glossary). Among the components of evidence-based mathematics instruction, direct questioning was frequently incorporated into the teacher's instruction to adjust the typical standards-based instruction on probability and statistics for the students with MD (average frequency per lesson = 3.6).

In most cases (17 out of 18 instances), Ashley used questions at the level of cognitive memory to (a) check student understanding of facts, concepts, or procedures or (b) to prompt students to provide the correct answers. Only once the teacher used a question at the divergent level to prompt the correct answer (e.g., "Why? Why 2?"). In half of instances involving direct questioning, Ashley utilized direct questioning, especially for prompting the individual students with MD to derive the correct answer. The following were cited to illustrate this case:

T: Kevin, would you say 41 is typical number of raisins in your box? Typically, you get 41 raisins in a box. What would you say, true or false?
Kevin: (No response.)
T: Think about 44. What would you say about more typical number, 41 or 44? What's more typical?
Kevin: 40.

T: Not 40. 41 and 44. What do you think is the better choice?
Kevin: 44.
T: 44?
Kevin: 33.
T: I will go with the numbers between 33 and 44. 33 is the lowest and 44 is the highest. So, you don't want to go with either the lowest or the highest number for the most typical number of raisins.

T: Maybe, every class has about 15. And then, you might think how many classes are there in the fourth grade?
Kevin: Four.
T: Four. You are going to count 15, how many times?
All students: Four.
T: Can you tell me what it is, 15 times 4? Kevin, what is 15 times 4?
Kevin: (Murmured).
T: 15 times 2 is what?
Student 1: 30.
T: 15 times 4 is 15 times 2 times 2. So 30 times 2. What is 30 times 2? Kevin?
Kevin: 60.

In the other half of instances involving direct questioning, Ashley employed direct questioning to check the students' understanding of the content and help them understand the content more easily.

T: Okay, guess. How do you guess? Kevin? (No response.) We just randomly pick any number in the world? Or do you think about before you guess? You know, what if I said how many students in the fourth grade? Guess how many kids in the fourth grade. I don't know if you guys know that, but you are going to predict what you thought the number would be. Would you have some ways of thinking about that? Or, would you just say probably 500?
Kevin: I will look at the class.
T: You will look at the class. Which class?
Kevin: Our class.
T: Our class. And then you will think what?

Kevin: There is more than one.
T: Of course, there's more than one. You are closer. How many students do you think each class has?
Kevin: 10.

Review of prerequisite skills. Prerequisite skills are background knowledge necessary for applying the target skills (Jitendra et al., 1999). The teacher reviewed prerequisite skills for learning probability and statistics for the students with MD, especially when they gave incorrect answers to the questions regarding the skills or showed difficulty with using the skills. Ashley's review of prerequisite skills for her students with MD was found three times during the whole period of observation of probability and statistics lessons (Lessons 4, 7, and 8). The prerequisite skills reviewed by Ashley were related to (a) expressing a probability in fractions (e.g., transferring 10 out of 16 to 10/16ths), (b) measuring lengths (or heights) as a process of collecting data, and (c) marking data points using a ruler. The following examples illustrated the situations involving review of prerequisite skills as an instructional adaptation for students with MD.

T: Somebody give me the number in fraction? How many of you have pets as a fraction? Kevin?
Kevin: 10 out of _____
T: How many people are in this room?
S: (Counted them) 16.
T: 10 out of what?
Kevin: 16.
T: 10 out of 16. Can you put this in another way, Kevin? (No response.) Student 8?
Student 8: Ten over 16.
T: Another way, Student 2?
Student 2: Eleven sixteenths.

Another instance was when the teacher helped Kevin and his partner measure their heights.

Group instruction. Group instruction is instruction using pairs or small groups as an alternative to whole-group or independent seatwork (Gersten et al., 2000) and has been reported as a critical component of evidence-based mathematics instruction (Swanson et al., 1999). In four out of five lessons on probability and statistics, the teacher employed group instruction including pairs and small groups of three students. The group instruction was purposively planned prior to the lessons. As in the instruction on geometry and spatial reasoning, the teacher stayed with groups involving the students with MD during most group instructional time. Both formats of group instruction included pairing up a student with MD with a high-achieving student.

Strategy instruction. Strategy refers to a broad range of routines that facilitate both knowledge acquisition and utilization, including various heuristic techniques that allow one to more easily access relevant information during problem solving as well as general control strategies such as planning, monitoring, checking, and revising (Prawat, 1989). In this sense, strategy instruction involves teaching or cueing to use strategies for solving problems and verifying the problem solution. During the teacher's instruction on probability and statistics, she did not use strategy instruction to adapt her instruction for the students with MD.

Progress monitoring. Progress monitoring includes the teacher's checking whether students understand the task requirements and the procedures needed to complete the task correctly. The teacher's progress monitoring of the students with MD were found in only two lessons (Lessons 5 and 6) and were associated mainly with checking for understanding of the task or activity during small-group instruction. For example, most progress monitoring (three out of four cases) was found during small-group instruction and was implemented to check students' understanding of the task or the group activity. The following example was cited to provide an illustration of typical progress monitoring implemented during instruction on probability and statistics:

> T: It will be a circle. Now, you are going to have 8 pieces?
> Some students: No.
> T: Lee, How many pieces do you think we are going to have today in our pie chart? How many parts?
> Lee: 7.
> T: Why are you saying 7? (No response.) Where is 7 from? (No response.) Okay, let's think about it. How many sections are we going to divide this into? Student 2?
> Student 2: Four.
> T: It is four. Why four?
> Student 2: The number of colors.
> T: Do you remember how many colors did we use for our bar graphs?
> Student 2: Four.
> T: So, you are going to tell me how much of this each color we have out there. Your pie chart today will have four sections. Your pie chart should be matched with your what?
> Some students: Bar graphs.
> T: The bar graph. So, if on the bar graph you only have one person who has four, hold on, that's not going to work. How many people had one?
> Student 1: 0.
> T: How many had two?
> Some students: 0
> T: How many had three?
> Some students: 1.
> T: So, that's going to be, let say "red", represent one.
> T: What color do we use for this one?
> Some students: Red.

T: Red, this has only one person. What color do we use for this one?
Some students: Red.
T: Red. So, really, how many people have red?
Student 2: four.
T: One, two, three, four. That's going to be four. We are going to divide your pie chart according to color. Let's go and have seats in front of your bar graph with your partners.

Use of manipulatives. Manipulatives are concrete objects to represent a skill or concept or to provide hands-on instruction (Resnick & Ford, 1981). Representation tools including manipulatives are used to provide complete, consistent, and logical explanations for concepts, skills, or activities (Jitendra et al., 1999). The teacher did not adjust her instruction for the students with MD by using manipulatives during instruction on probability and statistics.

Practice opportunity. In this study, practice opportunity refers to providing times for guided practice or independent practice. As in the lessons on geometry and spatial reasoning, supplementary items or opportunities for independent practice were not found in probability and statistics lessons. However, the teacher adjusted her instruction in terms of guided practice during instruction on probability and statistics. In these cases, guided practice was provided in the process of furnishing prompting to the student in response to his errors during small-group instruction. The following example was provided to illustrate a guided practice adapted for a student with MD during small-group instruction:

T: (To Kevin's group) What is a 10 yarn? Kevin?
Kevin: (Selected wrong one).
T: Is it 10?

Kevin: No (Selected right one).

T: (Showed a yellow paper and told him) so, what you are going to do is to group your colors together. Let's start with 10 (yarn representing 10).

Kevin: Ten, ten, ten.

T: Put it on here.

Kevin: This one?

T: Ten.

Kevin: (Put the color for 10 on the chart).

T: Here we go. We are trying to make what shape, Kevin?

Kevin: Uh…a circle.

T: Circle, because we are going to make a pie chart. So, that's going to be a top of our circle. What color do you want to use next? (No response.) What are other ones going with this? (No response.) You are making a circle. Now, you might have to make this circle smaller even. There's our four.

Kevin: I will do the yellow next.

T: Do the black next. (Kevin did so.) And then, what do we have left?

Kevin: Yellow.

T: Yellow. You might have to make it little bit bigger. (Kevin did so.) Okay, now, what this is telling us is how big our pieces are going to be in our pie chart. You kind of see what is going with this?

Kevin: Yes.

T: Now, we are going to take a line and with our pencil we are trying to draw a pie chart. Okay, basically, this is all one, isn't it?

Kevin: Yes.

T: Because this is all the same. So, you are going to go from here to the center.

Kevin: Is it equal to this one?

T: That's one piece. Okay, now, black is going through from here to here, isn't it?

Kevin: Yes.

T: Then, the yellow goes from here to here. Then, you are going to move the yarn and draw a circle. Alright, you take the yarns and draw a circle. You might want to color the pie chart. You have to make sure that the colors on your pie chart are matched with the colors on your bar graph.

Teacher examples. Examples refer to instances that illustrate a rule or method, as a mathematical problem proposed for solution (Mish, 1994). Teacher examples include those gradually sequenced from familiar to new in guided practice (Rosenshine, 1983). The teacher used an increased number of teacher examples for the students with MD in one lesson out of five on probability and statistics. For example, while she was reviewing the concept of *compare* during Lesson 7, she provided several examples of comparisons to help the students more clearly understand the concept.

> T: Now, we are going to go little bit further than that. (Wrote *compare* on the board) Let's look at this word on the board. Raise your hands to tell me what it is saying and what it means.
> All students: (Raised their hands).
> T: Lee, what is that word?
> Lee: Compare.
> T: Compare. What are you doing when you compare things?
> Lee: I take one thing and look at another.
> T: Okay, you basically need two things when you compare. What do you do with the two things? (No response.) You take one thing and then you look at something else, and you can find how they are the same. What if I ask you to compare things we can graph? For example, you said we graphed the number of family members in our class. Can you think of another graph so that we can make a comparison with our class graph? With maybe?
> Student 2: Ms. Raymond's class.
> T: Ms. Raymond's class. We can do graph with another fourth grade class on how many family members live in their house. Okay, that will be a comparison. Can you think of something else we can compare in schools other than family members. Maybe, we may want to compare how many people in our class have blue eyes, compared to how many people in Ms. Bloom's class have blue eyes. What else can we compare? Kevin?
> Kevin: Umm, pets.
> T: Okay, what about pets? (No response.) Well, do you want to just do dogs? Maybe, we can compare how many, what types of pets we have compared to types of pets the next class have. Good. Can we compare anything else? Student 3?
> Student 3: Shoe size.
> T: Shoe size. That will be the good one. Does Ms. Hebert's class have bigger shoe sizes than Ms. Kim's class? One more, can you think of something else we can compare? Student 4?
> Student 4: Polar bears and rabbits in terms of feeding babies?

> T: Very good. She chose two different animals. We can graph, maybe, umm, polar bears, types of foods for babies, compared to types of food for rabbit babies. So, we can compare lots of things.

Vocabulary instruction. Vocabulary instruction includes teaching or reviewing mathematics vocabulary that may be new or previously taught. Unlike in instruction on geometry and spatial reasoning, supplemental vocabulary instruction for the students with MD was not found in instruction on probability and statistics. As illustrated in the section of teacher examples, the teacher provided vocabulary instruction (e.g., *compare*) but not targeting the students with MD, and the teacher did not provide instruction on extra vocabulary with which that the students might be struggling.

Reteaching. In this study, reteaching refers to repeating instruction on the content that has been already taught. Reteaching is guided by the information obtained during continuous progress monitoring (Downey, 2001). Unlike in the lessons on geometry and spatial reasoning, the teacher provided reteaching to the students with MD in three out of five lessons on probability and statistics, especially when the student showed difficulty in understanding or mastering the skills already taught. In the following example, the teacher repeated her instruction on how to read information from a bar graph, which had been taught at the beginning of the lesson, for the student with MD who showed difficulty with the skill.

> T: (To Kevin's group) How many people have 3, Kevin?
> Kevin: 4.

T: These (at the bottom) are the numbers representing the number of family. These numbers (on the left side) are representing the number of students in our class. How many students have three?
Kevin: (No response.)
Student 9: One.
T: One. So, you will look the numbers at the bottom to find three people in the family and look at the one from the left line. That represents one students have three family members. Four has one. So, you look for four at the bottom numbers and find one from the left line.

Summary of evidence-based mathematics instructional components used for adapting standards-based statistics instruction. In summary, it was found that the teacher adjusted her instruction on probability and statistics for the students with MD by incorporating some components of evidence-based mathematics instruction into standards-based instruction on these areas. The components were (a) prompting, (b) explicit explanations, (c) explicit modeling, (d) control difficulty, (e) direct questioning, (f) review of prerequisite skills, (g) group instruction, (h) progress monitoring, (i) practice opportunity, (j) teacher examples, and (k) reteaching.

Compared to her typical standards-based statistics instruction for all students, Ashley used more diverse evidence-based instructional components when she adapted her instruction for the students with MD. The components that were not used in typical standards-based statistics instruction for all students but were used in instructional adaptations for the students with MD were (a) prompting, (b) explicit explanations, (c) control difficulty, (d) progress monitoring, (e) practice opportunity, and (f) reteaching. Evidence-based instructional components of (a) group instruction, (b) modeling, (c) multiple teacher examples, (d) direct questioning, and (e) review of prerequisite skills were used for both teaching all students in typical standards-based statistics instruction and adapting instruction to teach the students with MD in standards-based mathematics instruction.

Compared to instructional adaptations occurred during instruction on geometry and spatial reasoning, instructional adaptations during instruction on probability and statistics showed more differences from typical standards-based instruction for all students in terms of the number of evidence-based instructional components used just for adapting instruction. In addition, instructional components of explicit explanations, modeling, practice opportunity, and reteaching were used to adjust instruction for the students with MD only during the lessons on probability and statistics. Whereas the teacher used strategy instruction, manipulatives, and vocabulary instruction to adapt instruction for the students with MD during the geometry lessons, she did not employ these components for adapting instruction on probability and statistics. Six components were found in both subject areas: (a) prompting, (b) control difficulty, (c) direct questioning, (d) review of prerequisite skills, (e) group instruction, and (f) teacher examples.

Of these components, the teacher used prompting and direct questioning most frequently to adjust her probability and statistics instruction for the students with MD. Explicit explanations, explicit modeling, and control difficulty were also utilized at least once per lesson to help the students with MD learn probability and statistics in this standards-based mathematics general education classroom.

As well, some of components were found in association with specific instructional situations. For example, prompting and review of prerequisite skills were found mainly in instances of instructional adaptations that occurred as the teacher's reactions to student errors or difficulties. However, the associations were not as clear as in instruction on geometry and spatial reasoning.

Instructional Adaptations Addressing Student Difficulties in Prerequisite Skills

For this analysis, findings on student difficulties in prerequisite skills for learning probability and statistics (e.g., findings from teacher interview and survey questionnaire) were synthesized to produce a summary of the student difficulties, and then data on the teacher's instructional adaptations were analyzed in terms of addressing the difficulties of the students with MD. According to the summary of findings on difficulties of students with MD relating to probability and statistics, the most severe struggles of the students were (a) remembering orders and steps of probability; (b) understanding of probability in fractions and decimal; (c) using organized data to construct real object graphs; (d) collecting and sorting data; (e) identifying events as certain or impossible; (f) using data to describe events as more likely or less likely; and (g) using data to describe events as more likely, less likely, and equally likely. Table 4.12 provides an overview of the teacher's instructional adaptations that might address these difficulties of the students with MD during instruction on probability and statistics.

According to Table 4.12, only one of the student difficulties listed was addressed through Ashley's instructional adaptations, and only once across the five lessons on probability and statistics. For example, Ashley provided reviews of the skills of transferring a probability to fractions while she was introducing the concept and the procedures of statistics in Lesson 4. The other difficulties were not targeted in Ashley's instructional adaptations across the lessons on probability and statistics.

However, it is notable that some of these difficulties were covered by standards-based lessons suggested by the curriculum. For example, four of five probability and statistics lessons included an activity in which students were engaged in collecting and sorting data as part of the activity (e.g., data collection of the number of raisins in a box during Lesson 4, data collection of the number of family members during Lesson 5).

In summary, it was observed that the teacher attempted to address only one of the student difficulties relating to statistics prerequisite skills, transferring a probability into fractions, through interactions with a student with MD while she was introducing the concept and the procedure of statistics. However, it should be noted that standards-based lessons on probability and statistics addressed some of the skills with which the students with MD struggled in activities of each lesson.

Table 4.12

Instructional Adaptations Addressing Student Difficulties Related to Prerequisite Skills for Learning Fourth-Grade Probability and Statistics

Prerequisite skills	Instructional adaptation
Remembering orders & steps of probability (Lee, Kevin, & Tina)	None
Understanding probability in fractions & decimal (Lee & Tina)	Teaching transformation of probabilities (e.g., 3 out of 5) to fractions during Lesson 4
Using organized data to construct real object graphs (Kevin)	None
Collecting and sorting data (Lee, Kevin, & Tina)	None
Identifying events as certain or impossible, such as drawing a red crayon from a bag of green crayons (Lee, Kevin, & Tina)	None
Using data to describe events as more likely or less likely, such as drawing a certain color crayon from a bag of seven red crayons and three green crayons (Lee, Kevin, & Tina)	None
Using data to describe events as more likely, less likely, and equally likely	None

Research Question 2: Learning of Mathematics Knowledge and Skills by Fourth-Grade Students with Different Ability in a Standards-Based Mathematics, General Education Classroom

This section describes the findings of Research Question 2, learning of mathematics knowledge and skills by fourth-grade students with different ability (3 students with an IEP in mathematics, 2 teacher-identified struggling students, and 1 typically achieving student) in a standards-based mathematics general education classroom. In this study, student learning was explored in terms of (a) changes in their prerequisite skills, (b) changes in accuracy of problem solutions, (c) changes in concepts or procedures that students used for problem solutions after receiving instruction on the targeted skills in this instructional environment, and (d) transfer of the learned skills to new problems after receiving standards-based mathematics instruction on the skills.

Traditionally, transfer has been defined as cognitive skills of applying knowledge previously acquired in one situation to a different situation (Singley & Anderson, 1989). In this study, transfer of mathematics knowledge and skills by students with different ability in the standards-based mathematics general education classroom was explored by investigating how the students with different ability use mathematics knowledge and skills taught in class to solve the curriculum-based problems after they have received classroom instruction. Findings were derived from data from clinical interviews (e.g., interview transcripts, student permanent products, field notes). Data from the clinical interviews were analyzed using both quantitative and qualitative method within a case study design.

Clinical interviews with the student participants were conducted over 2 months, from January 2006 through March 2006, while they were receiving instruction on geometry and statistics. Four curriculum-based tasks for clinical interviews were developed: two tasks on geometry and two on statistics. Each task consisted of four alternative transfer problems on a mathematics topic taught in class. Especially on Clinical Interview Task 1 (geometry) and Clinical Interview Task 3 (statistics), three different types of problems were included to examine the students' knowledge transfer according to the similarity to the original problem taught in class: one base problem, one near-transfer problems, and two far-transfer problems. The base problem was exactly same with the problem used to teach the skill in class. Near-transfer problems had the same structure (e.g., problem solutions) but different surface features (e.g., context and the numbers shown on the problem) from the original problems taught in class. Far-transfer problems were different from the original problems in both problem surface features and problem structures (see chapter 3 for more information about clinical interview tasks).

A total of 42 interviews (8 each for 5 students and 6 for 1 student) were conducted, each interview lasting approximately 20 minutes. Interviews were conducted with individual students a week before (baseline interviews) and a day after they were taught on the topic (postinstruction interviews). Performances at baseline clinical interviews served as baseline data to be used to provide a comparison for evaluating student performances at postinstruction interviews.

The following sections presented the findings on learning of mathematics knowledge and skills by students with different ability in a standards-based mathematics, general education classroom by mathematics content areas: (a) geometry and spatial reasoning and (b) probability and statistics. Each mathematics content area included descriptions about four major categories: (a) prerequisite skills, (b) accuracy in problem solutions, (c) concepts or procedures used for problem solutions, and (d) transfer of problem solutions taught in class to problems with different similarity to the original problems across students with different ability. Each major category was divided into two subdivisions, baseline performance and postinstruction performance.

Geometry and Spatial Reasoning

Geometry is a fundamental mathematics skill, which serves as an instrument for studying other topics in mathematics and science (NCTM, 2000). According to the national standards (NCTM, 2000) and the state TEKS standards (TEA, 2006), students in fourth grade are expected to achieve various geometric skills, such as using visualization, spatial reasoning, and geometric modeling to solve problems. During the observational period of this study, the teacher taught two specific geometry and spatial reasoning skills: (a) identifying and building a 3-D object from 2-D representations of the object (Clinical interview Task 1) and (b) identifying and drawing a 2-D representation of a 3-D object (Clinical interview Task 2). Clinical Interview Tasks 1 and 2 were created to measure students' performance on either of these two skills. On Clinical Interview Task 1, the students were asked to make a building with cubes based on the 2-D drawing on a card. On Clinical Interview Task 2, the students were asked to find and name 3-D solids that had a specific silhouette shown in the card. Each task consisted of four problems (see Appendix G for the problems on the clinical interview tasks). Individual students' baseline and postinstruction problem-solving performance on each clinical interview task were analyzed, summarized, and synthesized within a group of student ability in terms of (a) his or her prerequisite skill relating to the clinical interview task, (b) accuracy of problem solutions, (c) concepts or procedures used for problem solutions, and (d) transfer of the skills taught to new problems.

Prerequisite Skills

Relating to the mathematics knowledge and skills examined using two clinical interview tasks on geometry and spatial reasoning, two prerequisite skills (one per each task) were examined: (a) the knowledge and skills of identifying the volume of a 3-D cube building (Prerequisite Skill 1, Clinical Interview Task 1) and (b) the knowledge and skills of identifying numbers of 3-D solids and remembering their names (Prerequisite Skill 2, Clinical Interview Task 2).

Each student's performance on each prerequisite skill problem during baseline or postinstruction clinical interviews was scored for a correct answer (1 point) or an incorrect answer

(0 point). The accuracy of individual student's baseline or postinstruction performances on prerequisite skill problems included in each clinical interview was calculated by dividing the number of a student's correct answers by the total number of problems that the student tried to solve (4 problems on Prerequisite Skill 1 and 10 problems on Prerequisite Skill 2), and multiplying it by 100%. An average accuracy was calculated for each group of students with differing ability (MD, struggling, and typically achieving students). In addition, the process of answering to questions about the prerequisite skills (e.g., counting strategies or heuristics for Task 1) at baseline or postinstruction interviews was summarized by the group of students with differing ability and compared across the groups.

Baseline prerequisite skills. Table 4.13 presents the accuracy of each group of students, students with MD, struggling students, and a typically achieving student, in solving baseline prerequisite skill problems. As shown in Table 4.13, the group of 3 students with MD performed at lower levels than the typically achieving student on both skill tasks (41.7% vs. 50.0% on Prerequisite Skill 1; 26.6% vs. 30.0% on Prerequisite Skill 2). They performed slightly better on Prerequisite Skill 2 than did the group of struggling students, but they showed lower performances on the Prerequisite Skill 1 than the group of struggling students. The performance of the group of 2 struggling students on Prerequisite Skill 1 were at the same level with the typically achieving student, but on Prerequisite Skill 2 their performance was lower than the typically achieving student's.

Even though comparisons of average group performances on Prerequisite Skills 1 or 2 did not indicate large differences in the prerequisite skills among the three groups, it should be noted that there were large variations within the group of students with MD. On Prerequisite Skill 1, the 3 students' performances varied from 0% to 75% (0%, 50%, and 75% for Lee, Kevin, and Tina, respectively). On Prerequisite Skill 2, their performances varied from 10% to 40% (30%, 10%, and 40% for Lee, Kevin, and Tina, respectively). These data indicated that Lee rarely had knowledge about counting the number of cubes of 3-D figures shown in 2-D drawings (Prerequisite Skill 1), and Kevin had problems in recognizing and naming 3-D geometric shapes (Prerequisite Skill 2), which influenced their performance on the Clinical Interview Tasks 1 and 2.

Table 4.13

Pre- and Postinstruction Percentage Accuracy in Solving Problems on Geometry Prerequisite Skills Across Groups of Students With Differing Ability

Prerequisite skill	Students with mathematics disabilities	Struggling students	Typically achieving student
1. Identifying the volume of a 3-D solid			
Baseline	41.7	50.0	50.0
Postinstruction	58.3	75.0	75.0
2. Naming a 3-D solid			
Baseline	26.6	25.0	30.0
Postinstruction	46.7	55.0	70.0

Compared to the other groups, the group of students with MD was different in counting skills while they were working on the problems on Prerequisite Skill 1. The struggling students and the typically achieving student did not show problems in using one-to-one correspondence in counting. One-to-one correspondence is one of counting principles suggested by Gelman and Gallistel (1978), who described it as a counting skill involving the ability to assign arbitrary tags to the items in an array. For instance, they used only one number name for each cube in the 2-D drawings of the 3-D buildings. The struggling students and the typically achieving students also did not show problems in using counting strategies such as counting-by-two or counting-by-three strategies (e.g., they counted the number of cubes in the 2-D drawings by counting a set of two cubes or three cubes, two, four, six, etc.). In addition, they showed the skills to use altered counting strategies according to the structures of buildings. For example, with a building with two cubes on the top, four cubes in the middle, and two cubes on the bottom, they utilized the counting-by-two strategy to derive the volume of the building (the number of cubes in the building). However, except Tina, the MD students showed difficulties even with one-to-one correspondence in counting sometimes and did not show the skills of using counting-by-two or counting-by-three strategies. For example, Lee ignored a whole section (e.g., middle or bottom) of a building or skipped some cubes when she was counting cubes, and she counted all cubes one by one. Kevin went back to the cubes that he had already counted and counted them again to produce the number of cubes in the building. However, Tina's counting skills were comparable to the other groups of students.

Postinstruction prerequisite skills. After the students were taught about the geometry skills being examined, they were interviewed again about the skills in Clinical Interview Tasks 1 and 2. The postinstruction clinical interviews were conducted one day after instruction on each content skill. Each student's performances on the problems about prerequisite skills during postinstruction clinical interviews were scored in the same way with the performances on the problems about geometry prerequisite skills during baseline clinical interviews (e.g., score 1 for correct answer and score 0 for incorrect answer). The number of correct answers of each student was divided by the total number of problems in each task and multiplied by 100% to produce the accuracy of solving the problems on the prerequisite skills. A mean accuracy of student performances in each group was calculated and compared with those of the other group as in the analysis of baseline clinical interview data. In addition, the process of answering to questions about the prerequisite skills (e.g., counting strategies or heuristics for Task 1) was summarized for individual students and compared across groups of students with differing ability. Table 4.13 provides a summary of pre- and postinstruction comparisons of geometry and spatial reasoning prerequisite skills among the group of students with MD, the group of struggling students, and the typically achieving student.

Comparisons of prerequisite skills between baseline and postinstruction interviews. Prerequisite skills of all groups of students with differing ability increased between baseline interviews and postinstruction interviews, and the amount of improvement was proportionate to the ability of students (MD students improved less than struggling students, who improved less than the typically achieving student) on both prerequisite skill tasks. Also, it was noted that students having more prerequisite skills at baseline showed more improvements at postinstruction interviews within the group of students with MD.

First, compared to the accuracies at baseline interviews, accuracy on both Prerequisite Skills 1 and 2 was increased across all three groups of students with differing ability. To the problems on Prerequisite Skill 1, the postinstruction accuracy was 58.3%, 75.0%, and 75.0% for the group of MD students, the group of struggling students, and the group of a typically achieving student, respectively. The difference between the mean baseline accuracy (47.2%) and the mean postinstruction accuracy (69.4%) on Prerequisite Skill 1 was 22.2%. In terms of gains on the Prerequisite Skill 1, the group of students with MD made the least gain (16.6%) between baseline and postinstruction. The other two groups showed improvements in Prerequisite Skill 1 by 25%.

On the other hand, after receiving class instruction on the skills, the group of students with MD provided correct answers to 46.7% of problems on Prerequisite Skill 2, whereas the group of struggling students and the typically achieving student provided correct answers to 55% and 70% of problems on Prerequisite Skill 2, respectively. The mean postinstruction accuracy of all three groups of students in the problems on Prerequisite Skill 2 was 57.2%, which was 30% higher than the mean baseline percentage of correct answers. In terms of gains on Prerequisite Skill 2, the group of students with MD had the least gain (20.0%) between baseline and postinstruction. The performances of the other two groups on Prerequisite Skill 2 improved by 30% (struggling students) and 40% (typically achieving student) from baseline interviews to postinstruction interviews.

Second, as shown in Table 4.13, the performances of the three groups of students with different abilities were not quite different on the two geometry and spatial reasoning prerequisite skill tasks at baseline. The students with MD were not quite different from their peers with different ability in terms of prerequisite skills required for solving the two tasks at baseline interviews. However, at postinstruction interviews, the performances of students with MD were much lower than their peers on those prerequisite skills. The performance gaps on the prerequisite skills seemed to get worse, even while they were being taught on the topics related to the prerequisite skills. The phenomenon of "the better at baseline, the better at postinstruction" was observed even within the MD group of students as well as across groups of students with different ability. For example, on Prerequisite Skill 2, Tina got the highest score among the 3 MD students (75.0% correct) at baseline interviews, whereas Lee got nothing correct and Kevin got 50.0% correct. At postinstruction interviews, Tina performed better than Lee and Kevin on Prerequisite Skill 1.

Comparisons of performances of groups of students with differing ability. The findings from comparisons of group mean performances at postinstruction indicated that (a) performances on each prerequisite skill problem were varied across the group of students, (b) within-group variations of the group of students with MD were larger than those of the other groups, and (c) students with MD had problems in counting skills. First, differences in performances of the groups of students were noted across problems on Prerequisite Skill 1. To Problems 1 and 2 on Prerequisite Skill 1, which were about the volumes of buildings with a single layer, all three groups of students provided correct answers (buildings). However, to Problems 3 and 4, which were about the volumes of a building with multiple layers, the group of students with MD did not provide correct answers at all, whereas the other two groups of students (struggling and typically achieving) provided a correct answer to Problem 3, which was less difficult than Problem 4.

Second, variations on Prerequisite Skill 2 within a group were the largest among the 3 students with MD. For example, Lee got eight solids' names correct (square prism, triangular prism, pentagonal prism, large cube, square pyramid, sphere, and cone), Tina got four solids' names correct (cylinder, large cube, square pyramid, and cone), and Kevin got two solids' names' correct (cone and triangle pyramid). As at baseline interviews about Prerequisite Skill 2, Kevin performed at the lowest level among all the student participants. Lee's improvement (from three correct solid names to eight) should be noted. She provided right names for all solids except cylinder and octagonal prism. She was the highest performer on Prerequisite Skill 2 among all students participating in this study. Tina's knowledge about the solids' names at postinstruction interviews stayed at the same level to her knowledge at baseline interviews.

Compared to the group of students with MD, the 2 struggling students were very similar in performances on Prerequisite Skill 2. For example, Laura provided the names of six solids correctly. Jose provided correct names for five solids. The typically achieving student, Amy, provided correct names for seven solids. Even when Amy did not remember correct names, she showed knowledge about the system to name a solid. For example, she named a triangular prism as a "triangular something" and pentagonal prism as "pentagonal something."

Third, compared to the other groups, the group of students with MD was different in counting skills relating to the problems on Prerequisite Skill 1. The struggling students and the typically achieving student were able to use advanced counting strategies, such as the counting-by-two or counting-by-three strategy, and apply the strategy according to the structures of buildings. For example, with a multilayered building of four cubes in each row, Laura and Amy used counting-by-four, even though they failed to provide a correct answer to the problem (Problem 4) due to the difficulty with 3-D perceptions in 2-D drawings. However, the MD students showed difficulties in one-to-one correspondence in counting, let alone the difficulties in 3-D perceptions, and used counting-all strategy to produce the volumes of buildings. Lee did not count the cubes in a whole section (e.g., middle or bottom) or layers of a building or skipped some cubes when she was counting cubes, especially in buildings with multiple layers (Problem 3 and 4). Kevin counted some cubes repeatedly (Problem 3). Tina's counting skills were comparable to the other groups of students, and she did not provide a correct answer to Problem 4 because of her difficulty with 3-D perceptions in 2-D drawings.

The Accuracy of Problem-Solutions

The skills of (a) identifying and building a 3-D object from 2-D representations of that objects (Clinical Interview Task 1) and (b) identifying and drawing a 2-D representation of a 3-D object (Clinical Interview Task 2) were observed and analyzed in this study. Each student's problem-solving performances on problems in Clinical Interview Tasks 1 and 2 were scored for a completely correct problem solution (1 point), a partially correct problem solution (0.5 point), or an incorrect problem solution (0 point). A partial point (0.5) was given to student answers on the problems in Clinical Interview Task 2 that had more than one correct answer. When a student's answer included all possible correct answers, a complete credit (1 point) was given. When a student's answer included at least one correct answer, a partial credit (0.5) was given. The scores of four problems were summed and divided by 4 to produce the accuracy in each clinical interview task. Table 4.14 shows the baseline accuracy in each clinical interview task regarding geometry and spatial reasoning across the three groups of students with different ability.

Table 4.14

Pre- and Postinstruction Percentage Accuracy in Solving Problems on Two Geometry and Spatial Reasoning Clinical Interview Tasks Across Groups of Students With Differing Ability

Clinical interview task	Students with mathematical disabilities	Struggling students	Typically achieving student	Mean of three groups
Task 1				
Baseline	16.7	25.0	50.0	30.57
Postinstruction	41.7	50.0	75.0	55.57
Task 2 without naming				
Baseline	41.6	56.3	62.5	53.4
Postinstruction	70.8	62.5	87.5	73.6
Task 2 with naming				
Baseline	16.6	25.0	25.0	22.2
Postinstruction	58.3	43.8	87.5	63.2

Baseline accuracy in problem solutions. As shown in Table 4.14, the accuracy in problem solutions on Clinical Interview Tasks 1 and 2 was proportionate to the ability of students categorized in this study (MD, struggling, and typically achieving). On Clinical Interview Task 1, the group of MD students provided correct answers to the 16.7% of problems, whereas the group of struggling students and the typically achieving student provided correct answers to 25.0% and 50.0% of problems, respectively. All the students failed to produce correct solutions for Problems 3 and 4 on Clinical Interview Task 1.

Student performance on Clinical Interview Task 2 was analyzed in two ways, because it was observed that students often gave incorrect names of the solids that they selected as answers, even though they got the right solids as answers. The first analysis was conducted in consideration of if the student gave correct solids as answers. When the correctness of the names of solids that they called was not in consideration in the analysis, it was found that most students (except Kevin) provided at least one correct solution to at least two problems (mostly Problems 1 and 4). The group of students with MD showed 41.6% accuracy in problem solutions, the group of struggling students showed the 56.3% accuracy, and the typically achieving student showed 62.5% accuracy in problem solutions.

On the other hand, considering the correctness of the names of the solids as well as the correctness of the solids that were selected as answers to the problems on Clinical Interview Task 2, the students' average percentage of correct answers went down to 22.2%. The group of students with MD showed 16.6% accuracy, whereas the other groups of students showed 25.0% accuracy.

Differences among students with MD were noted. Individual MD students' performances were different in both Clinical Interview Tasks 1 and 2 (see Appendix G for the clinical interview tasks). On Clinical Interview Task 1, Lee and Tina did not make any buildings correctly (0.0% correct), but Kevin produced correct buildings for two of four problems (50.0% correct). On Clinical Interview Task 2, when not considering the correctness of the names of the solids, Lee produced partial solutions for three problems (0.5 points × 3) and complete solutions for one problem (1 point), resulting in 62.5% of correct answers. However, Tina gave partial solutions for two problems (25.0% correct), and Kevin provided partial solutions for three problems (0.5 points × 3 = 1.5 points; 37.5% correct). When considering the correctness of the names of solids, Lee and Tina provided partial solutions for two problems out of four problems (0.5 points × 2 = 1; 25.0% correct), whereas Kevin did not present any solutions for all problems (0.0% correct).

Postinstruction accuracy in problem solutions. Table 4.14 shows the baseline and the postinstruction accuracy in solving each geometry and spatial reasoning clinical interview task of the three groups of students with differing ability.

Comparisons of accuracy between baseline and postinstruction interviews. Comparisons of problem-solving accuracy between baseline and postinstruction interviews revealed that (a) all three groups of students showed improvements in accuracy of problem solving using both targeted skills between baseline and postinstruction interviews; (b) the group of students with MD showed the largest improvements in accuracy of problem solving on both Clinical Interview Tasks 1 and 2; and (c) even though all three groups of students showed improvements, the performance of the students with MD and struggling students were not comparable to that of the typically achieving student after receiving standards-based mathematics instruction.

First, after receiving classroom instruction on the geometry and spatial reasoning skills, and when the knowledge about the names of solids was considered, the three groups of students on average produced correct problem solutions to 55.6% of the problems on Clinical Interview Task 1 (41.7%, 50.0%, and 75.0% for the MD group, the struggling student group, and the typically achieving group, respectively) and 63.2% of the problems on Clinical Interview Task 2 (58.3%, 43.8%, and 87.5% for the MD group, the struggling student group, and the typically achieving group, respectively). When their knowledge about the solids' names was not considered, the mean performance of three groups of students on Clinical Interview Task 2 was 73.6% (70.8%, 62.5%, and 87.5% for the MD, the struggling student, and the typically achieving student groups, respectively). Compared to baseline performances, the students on average showed 25.0% of improvement in accuracy in solving Clinical Interview Task 1 from baseline to postinstruction interviews, 20.2% of improvement considering the names of solids produced by the students, and

41.0% of improvement when not considering the names of solids in solving Clinical Interview Task 2.

Second, on Clinical Interview Task 1, the MD student group showed larger improvement (30.0%) than the other two groups (25.0% for both groups). On Clinical Interview Task 2, including the knowledge about the solids' names, the MD student group showed the largest improvement (MD = 29.2%, struggling = 6.2%, and typically achieving = 25.0%). On Clinical Interview Task 2, not including the knowledge about the solids' names, the typically achieving student made the largest gain among the three groups of students (MD = 42.2%, struggling = 18.8%, typically achieving = 62.5%).

Finally, even though all groups of students showed improvements from baseline interviews to postinstruction interviews, the students with MD and the struggling students did not perform on problem solving using the skills taught as good as well as the typically achieving student did, after receiving mathematics instruction in the standards-based mathematics, general education classroom. For example, the typically achieving student achieved postinstruction accuracy of 75.0% on Clinical Interview Task 1 and 87.5% on Clinical Interview Task 2, whereas the accuracy of the students with MD and the struggling students stayed below 60.0% even after instruction on the targeted skills.

Comparisons of performances of groups of students with differing ability. Comparisons of performances of groups of students with differing ability revealed that problem-solving accuracy was varied within a group, especially in the group of students with MD (on both Clinical Interview Tasks 1 and 2) and the group of struggling students (only on Clinical Interview Task 1). First, group differences on Clinical Interview Task 1 at postinstruction were also found in the group of 3 MD students, as were found at baseline interviews. Individual MD students' performances were different in both Clinical Interview Tasks 1 and 2. On Clinical Interview Task 1, Lee, Tina, and Kevin's accuracy in solving problems was 0.0%, 0.0%, and 50.0%, respectively, at the baseline interviews. However, Lee and Kevin each produced a correct answer to one problem (25.0% correct solution) at the postinstruction interviews. Lee got the right solution for Problem 2, which she failed at the baseline interviews. Kevin provided a correct solution to Problem 1, but failed to provide a correct answer to Problem 2, which he had answered correctly at baseline. Tina correctly answered 75.0% of problems in Clinical Interview Task 1 at postinstruction interviews.

Differences within the group of struggling students in the performances on Clinical Interview Task 1 were also noted. At postinstruction interviews regarding Clinical Interview Task 1, Laura provided correct solutions for only one problem (Problem 2; 25.0% correct solutions), whereas Jose provided correct solutions for three problems (Problem 1, 2, and 3; 75.0% correct solutions).

On Clinical Interview Task 2, within-group differences were found only in the group of students with MD. Considering the correctness of the names of the solids, Lee produced complete solutions for three problems (1 points × 3) and partial solutions for one problem (1 point), resulting in 87.5% of correct answers. However, Tina gave partial solutions for two problems (0.5 points × 2 = 1) and complete solutions for one problem (1 point, 50.0% accuracy). Kevin provided partial solutions for three problems (0.5 points × 3 = 1.5 points), for 37.5% accuracy. Without considering the correctness of the names of solids, Kevin and Tina provided partial solutions for three problems (0.5 points × 3 = 1.5) and complete solutions for one problem (1 point), for a total of 62.5% correct answers. Lee provided complete solutions for three problems (1 point × 3) and partial solutions for one problem (0.5 points × 1 = 0.5) for a total of 87.5% correct.

Concepts or Procedures Used for Problem Solutions

The baseline and the postinstruction concepts or procedures used for problem solutions of individual students with different ability were analyzed and summarized by the ability groups. This analysis produced information about differential problem solutions among the three groups of students.

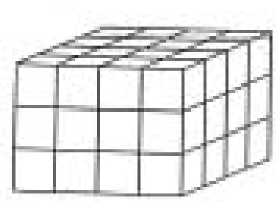
Students with MD. While they were working on Clinical Interview Task 1, the foci of the 3 students with MD were placed only on the cubes or the number of cubes, instead of considering layers of cubes, positions of the layers, or parts of the building, which were later taught in their classroom. To create a 3-D building based on a 2-D drawing on Clinical Interview Task 1, students should look at the drawing, count the number of cubes in the drawing, assemble the cubes to make a part or a layer of the building, figure out the positions of each part of multiple cubes in the whole building, and assemble the parts of the building to match them to the 2-D drawing. Ashley later taught the procedures for creating a 3-D building based on a 2-D drawing, which consisted of two core steps: (a) identifying the number of cubes in each part or layer and (b) positioning the parts of the building correctly so that they can be matched to the 2-D drawing.

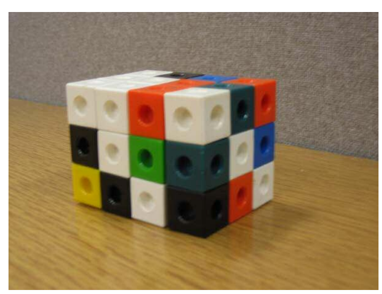
It seemed that students with MD did not consider either of the two necessary steps involved in the procedures of making correct buildings based on 2-D drawings. When they were asked about their strategies to construct the buildings shown in 2-D drawings, all 3 MD students answered that they made the cube buildings by “looking at” the cards and matching their cube buildings to the cards. However, Lee and Tina did not show the understanding of what to look at in 2-D drawings (the number of cubes) and how to match their buildings to 2-D drawings (by positioning the parts of the building correctly). Kevin seemed to understand that he should consider the number of cubes in each part of the building. The following were the answers by the 3 students to the question about how to make their buildings on Problem 1 in Task 1.

> Lee: Well, I just looked at the picture and then I figured I could do either, I could try to make it without paying attention, or I could look at the cubes and see I would have to make the building.
>
> Tina: I just looked at the picture and started putting it together.
>
> Kevin: I looked at the picture and right here (the front side]…and then I saw the one (cube) on bottom, I looked right here at the right (pointed to the front side again] and I saw three (cubes), so I put three (cubes), so that’s how I made it.

Regarding the skills of interpreting 2-D figures of 3-D configurations and making 3-D configurations, the 3 students with MD showed difficulties with at least one of the following processes: (a) identifying the volume of 2-D figure and matching the volume of the 3-D building to that of 2-D figure, (b) positioning and assembling segments of a building to be matched to the 2-D figure, and (c) constructing buildings with multiple layers.

Lee showed problems in all of the three processes. On the first two problems, she made mistakes in figuring out the volumes of the 3-D configurations and produced the outputs with more cubes than the original figures on the cards. Additionally, on the second problem, she put the segments (front, middle, and bottom) together in a wrong position. On the third and fourth problems with multiple layers, she did not count some layers, even in the visible sides. Figure 4.1 illustrates Lee's difficulties with buildings with multiple layers.



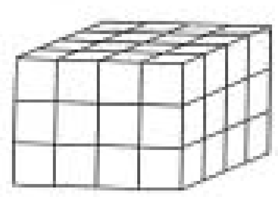


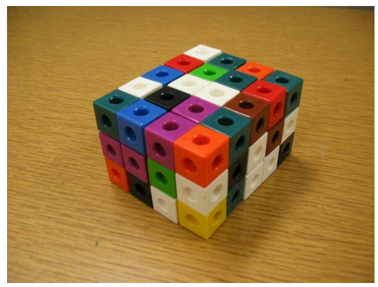
Original Task

Lee's Building

Figure 4.1. Lee's building to Problem 4 in the baseline Clinical Interview 1.

Kevin did not show problems in counting the number of cubes in a building (volume) with buildings of a single layer (Problems 1 and 2 on Clinical Interview Task 1). His difficulty was related to constructing buildings of multiple layers. He seemed to have problems in figuring out the volumes of 3-D buildings of multiple layers and constructing the 3-D buildings. Figure 4.2 illustrates Kevin's difficulty in multilayered buildings.

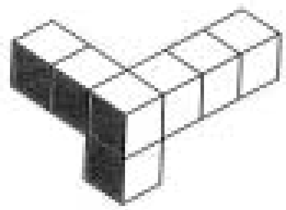


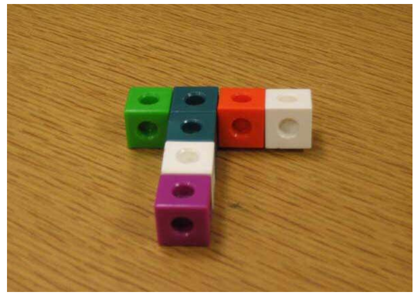


Original Task Kevin's Building

Figure 4.2. Kevin's building for Problem 4 in the baseline Clinical Interview 1.

Tina's difficulties seemed to be related to constructing 3-D buildings with cubes. Except on Problem 4, she did not show difficulties in counting the number of cubes in buildings. On Problem 3, she demonstrated the ability to count the number of cubes in the building, even though it had simple multiple layers. However, when asked to make the buildings shown on the cards, she was not able to do so. She combined all segments of the building in a flat way instead of putting them together in 3-D. On Problem 4, she counted only one of the visible sides (front) and created the building with only one layer. Figures 4.3–4.5 illustrate her problems in combining segments to make a 3-D building or in interpreting and constructing 3-D buildings with multiple layers.

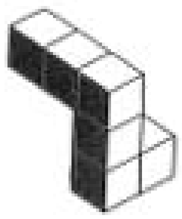


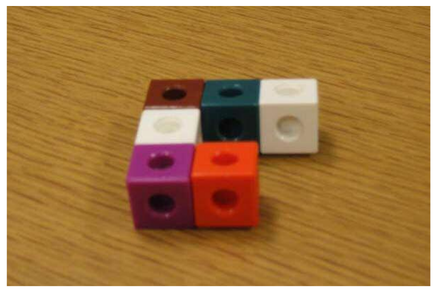


Original Task

Tina's building

Figure 4.3. Tina's building on Problem 1 in the baseline Clinical Interview 1.

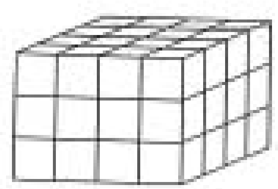




Original Task

Tina's Building

Figure 4.4. Tina's building on Problem 2 in the baseline Clinical Interview 1.



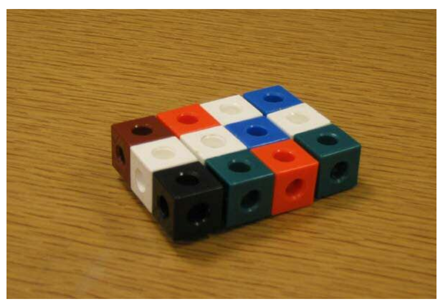


Figure 4.5. Tina's building on Problem 4 in baseline Clinical Interview 1.

Original Task

Tina's Building

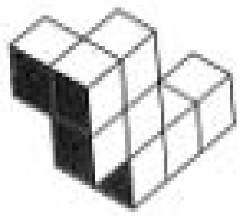
On Clinical Interview Task 2, the students were asked to find solids that would have the silhouette shown on the card. Regarding this task, later in their class, they were taught to think about all sides of a solid or put one of the sides on the overhead projector to find if it would make the specific silhouette. At the baseline, the 3 students with MD used the procedure of placing the bottom of a solid on the card to find 3-D solids that would make a specific silhouette. When a silhouette on a card was shown to them, they attempted to place one of 3-D solids on the card to see if it matched to the silhouette. Notably, they tried to match only the bottom side of 3-D solids to the silhouette on the card. They did not try to match the other sides of the solids to the card. In addition, once they found one solid whose bottom side matched the silhouette on the card, they did not explore to find another solid that would have the silhouette on the card, even though most silhouettes had more than two answers among the 3-D solids. As well, they did not know the names of the solids correctly, except *cube, cylinder,* and *cone*, even though they found some 3-D solids matching the specific silhouettes.

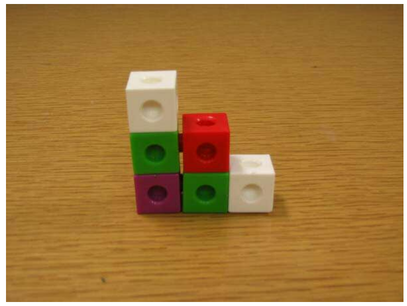
Struggling students. While working on Clinical Interview Task 1, the 2 struggling students seemed to understand that they should consider the number of cubes in each part but did not seem to understand that they should consider the positioning of the parts of the building as well. They counted the number of cubes in each part of the building but did not try to position the parts of the building in the same way as the 2-D drawing. As with the students with MD, their procedures were not accurate and strong enough to constantly produce the correct outputs, even though the procedures used by the struggling students were better than those of students with MD. The following illustrates how the struggling students created their buildings:

Laura: Well, I just looked at the picture and there's one (cube) right here and then right here it has three (cubes). So, then I matched it if it's right. …So it has one, two, three, four. So I put four (cubes) right there.

Jose: Just you know, add that one. And just, first I made this and then I looked on the back and saw this one and add the cube. I just looked at the picture and started putting it together.

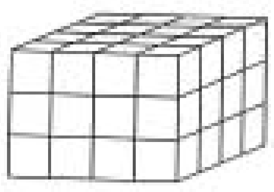
The 2 struggling students showed difficulties with making 3-D buildings. They demonstrated the skills of determining the number of cubes in the buildings with a single layer (Problems 1 and 2 on Clinical Interview Task 1) and the building with simple multiple layers (Problem 3). However, their outputs were incorrect: They made flat shapes instead of 3-D solids. With the building with complicated multiple layers (Problem 4), the students' difficulties got worse. They struggled with counting the number of cubes in the building as well as with constructing the building with cubes. Laura counted cubes only in the front (visible) side, created only the front side, and stated that her building would match the figure on the card (see Figure 4.6). As with Laura, Jose struggled with constructing buildings with multiple layers (Problems 3 and 4). However, his output was different from Laura's in terms of creating 3-D cubes instead of a 2-D rectangle. Figures 4.6–4.8 show Laura and Jose's difficulties in interpreting and constructing 3-D configurations.

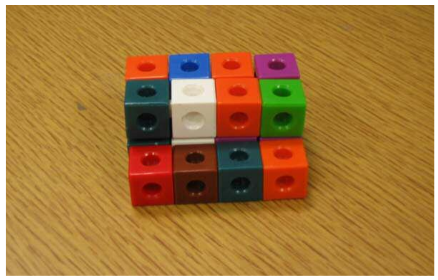




Original Task Laura's Building

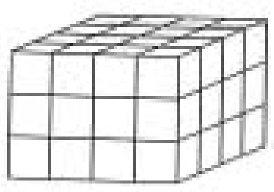
Figure 4.6. Laura's building on Problem 3 in the baseline Clinical Interview 1.

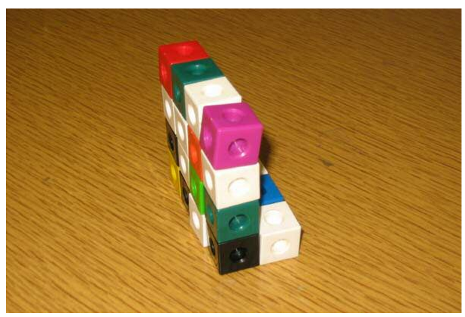




Original Task Laura's Building

Figure 4.7. Laura's building on Problem 4 in the baseline Clinical Interview 1.





Original Task Jose's Building

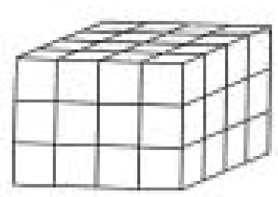
Figure 4.8. Jose's building on Problem 4 in the baseline Clinical Interview 1.

On Clinical Interview Task 2, the 2 struggling students employed the procedure of placing the bottom of a solid on the card to find 3-D solids that would make a specific silhouette, as the students with MD did. They tried to match one of solids on the desk to the silhouette shown on the card. Like the students with MD, they attempted to match only the bottom side of each solid to the silhouette. Also, once they found one answer for one silhouette, they did not explore another answer for the problem. The solids whose names they knew correctly were cylinder, cube, and cone.

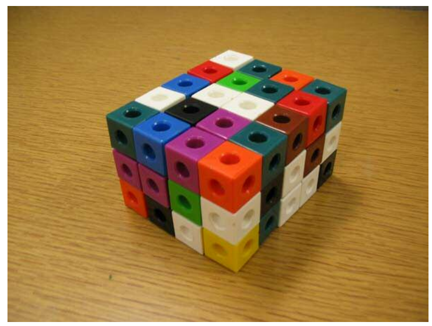
Typically achieving student. On Clinical Interview Task 1, the typically achieving student showed consideration of the direction or position of parts of the building as well as the number of cubes in each part. To remember the positions of parts of the building, she tried to find a shape that was familiar to her and similar to the positions of parts. Instead of attempting to directly match her building to the 2-D picture on the card, she tried to determine the shape of the figure (e.g., L-shape). The following illustrates her strategy to solve the problems in Task 1:

> Amy: Oh, well, I looked at the picture and I was kind of confused about how to put it. But then I looked at the shape and it looked like an L with a cube at the end. So I just thought I'll make an L with only three or four sticks, cubes, and then I'll put one at the end. …
>
> Well, I just looked at the picture again and then I just saw the faces, so I was just thinking even though it's a 3-D object, just look at the 2-D face and just look how it's like this. And then look, and then I looked at the side faces, and then I turned it over and looked at the side faces and that's how I figured it out.

Amy's difficulties in making 3-D configurations based on 2-D representations were related to the multiplicity of the layers of buildings. On the problems of buildings with a single layer (Problems 1 and 2), she was good at computing their volumes and constructing the buildings as shown in the cards. However, on the problems of buildings with multiple layers (Problems 3 and 4), she got confused when counting the number of cubes and constructed incorrect buildings, different from the original figure in terms of a piece of cube or one layer of cubes. Her outputs were very similar to the original figures, but she seemed to fail to keep correct information when she encountered information with multiple features. For example, on the Problem 4, she created a 5 × 4 × 3 building instead of a 4 × 4 ×3 building as shown on the card (Figure 4.9).



Original Task



Amy's Building

Figure 4.9. Amy's building on Problem 4 in the baseline Clinical Interview 1.

On Clinical Interview Task 2, the typically achieving student employed the procedure of placing the bottom of a solid on the card, similar to the other groups of students. However, unlike the other groups of students, she attempted to explore multiple answers for each problem. She knew three solids' names: cone, cube, and cylinder.

Postinstruction Concepts or Procedures

Concepts or procedures used in problem solutions of individual students with different ability at postinstruction interviews were analyzed and summarized by ability group to produce information about differential problem solutions between baseline and postinstruction and among the three groups of students. A summary of concepts or procedures the three different groups of students used for solving Clinical Interview Tasks 1 or 2 at both interview periods is presented in Table 4.15.

Table 4.15

Features of Concepts or Procedures Used by Three Groups of Students with Differing Ability to Solve Two Geometry and Spatial Reasoning Tasks

Task and time	Students with mathematics disabilities	Struggling students	Typically achieving student
Task 1			
Baseline	No specific points of considerations in making a building No strategies used	The number of cubes considered (Jose) No strategies used	The number of cubes and the positions of the parts of a building considered Familiar shape strategy used to remember the positions of the parts
Post	The number of cubes considered (Tina & Kevin) Familiar shape strategy incorrectly used (Lee)	The number of cubes and the positions of parts of a building considered (Jose) Familiar shape strategy used (Laura)	The number of cubes and the positions of parts of a building considered Familiar shape strategy used
Task 2			
Baseline	Only the bottom of a solid considered to find its silhouette (Lee, Kevin, & Tina) Multiple solutions not explored (Lee, Kevin, & Tina) Directly matching the solid on the card (Lee, Kevin, & Tina)	Only the bottom of a solid considered to find its silhouette (Jose & Laura) Multiple solutions not explored (Jose & Laura) Directly matching the solid on the card (Jose & Laura)	Only the bottom of a solid considered to find its silhouette Multiple solutions explored Directly matching the sold on the card
Post	The bottom and the sides of a solid considered to find its silhouettes (Lee, Kevin, & Tina) Multiple solutions explored (Lee, Kevin, & Tina) Directly matching the solid on the card (Lee & Kevin)	The bottom and the sides of a solid considered to find its silhouette (Jose & Laura) Multiple solutions not explored (Jose & Laura) Mentally matching the solid to the silhouette (Jose & Laura)	The bottom and the sides of a solid considered to find its silhouette Multiple solutions explored Mentally matching the solid to the silhouette

As shown in Table 4.15, changes in concepts or procedures used for solving problems on Clinical Interview Tasks 1 and 2 were noted in all three groups of students. Especially on Clinical Interview Task 1, students with MD showed consideration of the number of cubes when they were making a building, which was not found in their performance at baseline, even though those considerations did not lead them to the correct answers all the time. The group of struggling students considered the positions of sections of a building as well as the number of cubes in each section at postinstruction interviews, whereas they considered only the number of cubes in each section at baseline interviews. As well, 1 of the struggling students used a strategy for recognizing and remembering the positions of sections, which was taught in class. The typically achieving student did not show large changes in concepts or procedures because she already knew and applied the desirable concepts or procedures even at her baseline performances.

On Clinical Interview Task 2, the group of students with MD considered the sides as well as the bottom of a building to find a solid that would make a specific silhouette, and they explored multiple answers after they found one solution or answer at postinstruction; at baseline interviews, they considered only the bottom of a solid to find the silhouette and did not explore multiple answers. The changes in performance in both the struggling student group and typically achieving student group between baseline and postinstruction interviews included (a) considering the sides as well as the number of cubes for building creation, and (b) using mental matching of their building to the 2-D figure on the card instead of directly matching their building to the card. The group of struggling students did not explore multiple answers at postinstruction as at baseline interviews. However, the typically achieving student tried to explore multiple answers at both baseline and postinstruction interviews.

Students with MD. Two students with MD started to focus on the number of cubes in the building on Clinical Interview Task 1. The 2 students with MD looked at the card and attempted to make their building matched to the figure on the card. Unlike at baseline, they seemed to know what to look at and match (the number of cubes in each part of the building) at postinstruction interviews, even though it was not perfect knowledge. They looked at each part of the building (front, middle, and bottom or right, left, and middle) on the card, counted the number of cubes in each part, made each part using cubes, and put parts together to make a building matching the picture on the card. However, Lee used the “familiar shape” strategy that was discussed as a good strategy to remember the positions of the parts of a building in class. However, remembering and using the strategy did not lead her to the correct solutions. She just used the L shape, which was taught about in her class as a familiar shape for all problems, without regard to what they looked like or how the parts of a building were positioned. It seemed that she did not know why she was using the L shape, even though she was using it on almost every problem.

Lee's procedures were the least elaborated among the 3 students with MD. For example, even though she seemed to have rough ideas of the procedures to make 3-D buildings shown in 2-D drawings (e.g., make sections using the cubes and put them together in the way that matches the figure on the card), she did not show stable knowledge about specific points that needed to be considered at each step. For example, she seemed not to know where to focus in comparing her building with the figure on the picture (e.g., the number of cubes in each section of the building and the direction of each section). Unlike the other 2 students with MD, Lee did not state specific numbers for the cubes to be assembled. She stated unspecified amounts of cubes (e.g., "some cubes," "the other cubes," etc.). Consequently, her outputs were incorrect in terms of the directions of sections (Problems 1 and 3) and the number of the cubes in each section (Problem 4). The following were answers Lee gave to the question about her procedures or strategies to make the buildings on Task 1:

> Lee: (on Task 1, Problem 1) First I'm going to take some cubes and put them together. Now I'm going to take more cubes and stick them on to my other cubes. Take on more cubes stick it at the corner.
> Tester: How did you know how to make the building?
> Lee: Because I've done it in class before.
> Tester: Why did you make your building like this?
> Lee: Because it looked the same way on the card.
>
> Lee: (on Task 1, Problem 3) I'm going to start with an L piece. I'm going to make an L. A very small L. Now I'm going to make two pieces and stick them at the bottom of my L. Okay, now I'm going to take the cubes and stick them at the front, bottom front of my cubes. I'm not really sure. I'm almost done, I just need one more cube. I am done. I just sort of followed the card.

Compared to Lee's procedures for making buildings shown in 2-D drawings, Kevin and Tina used more elaborate procedures on Clinical Interview Task 1. Kevin sounded to know that he should consider the number of cubes in each section when he compared the building with the figure on the card, but he did not consider the directions of sections. He placed each section of cubes directly on the card to see if they matched each other in terms of the number of cubes.

> Kevin: (on Problem 1) I start from the bottom counting two. I'm not putting it together right now because I want to get the sides done. Now I'm doing the left side, first I'm doing the other side. Now I'm doing the next side…
> Tester: Can you say it a little louder?
> Kevin: I connect it together and now I'm going to count it. [Counting]
> Tester: Why did you count it?
> Kevin: To see this, I counted the card first and then I counted the shape and I counted to see if it had seven.
> Kevin: If you take it a part and just do this, put it on the card and then connect it its going to be correct, put it on the card [mumbling].
>
> Kevin: (On Problem 3) Because I already know how many cubes that I'm going to need so I just started looking at the card counting up cards at the bottom, middle, and top.

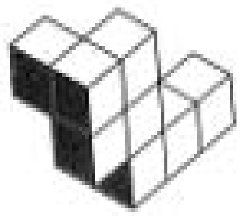
Tina also seemed to know that she should consider the number of cubes to make her buildings match the figures on the cards. Unlike Kevin, she did not place the cubes directly on the card to see if she got the correct number of cubes.

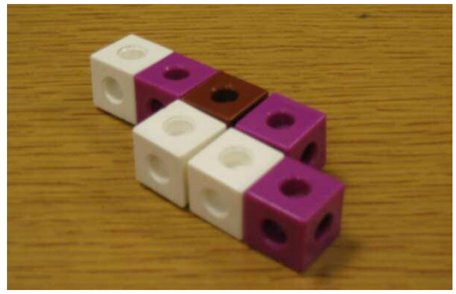
> Tina: (On Problem 1) I put two on the bottom and I add two more to the sides. And then I add three more to the back.
> Tester: Can you talk out loud? Okay, are you done? And how did you know how to make the building?
> Tina: Because I looked at the blocks.

Tester: Look at the picture then what did you do?
Tina: Then I just… I just put the same amount.

Relating to the skills of interpreting 2-D figures of 3-D configurations and making 3-D configurations (Clinical Interview Task 1), the 3 students with MD showed difficulties with at least one of the following processes as at baseline interviews: (a) identifying the volume of a 3-D figure in a 2-D drawing and matching the volume of their 3-D building to that of 2-D figure, (b) positioning and assembling segments of a building to match the 2-D figure, and (c) constructing buildings with multiple layers.

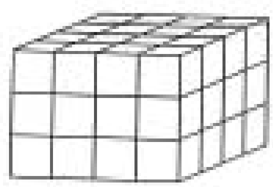
Lee showed problems in all three processes. On Problems 1 and 3, she failed to correctly position the sections of a building to match the picture on the card, although she was able to successfully make the sections (front, middle, and top) of the building. On Problem 4, she showed difficulties in identifying the volume of the building with multiple layers and made a $3 \times 4 \times 3$ building instead of a $4 \times 4 \times 3$ building. Figure 4.10 illustrates Lee's difficulties in correctly combining and positioning the sections of a building, and Figure 4.11 illustrates her difficulties in identifying the volume of the building with multiple layers and constructing the building.

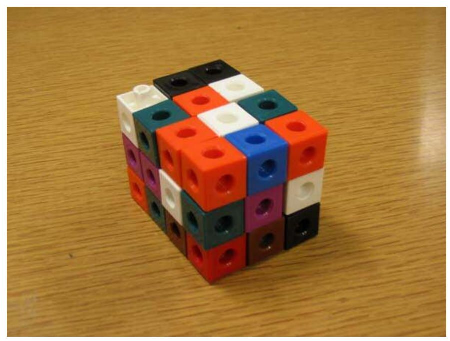




Original Task Lee's Building

Figure 4.10. Lee's building on Problem 3 in the postinstruction Clinical Interview 1.



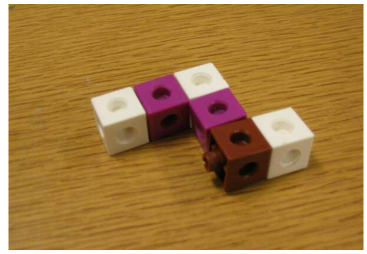


Original Task Lee's Building

Figure 4.11. Lee's building on Problem 4 in the postinstruction Clinical Interview 1.

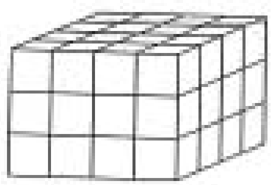
As at baseline interviews, Kevin did not show problems postinstruction in counting the number of cubes in a building (volume) with buildings of a single layer (Problems 1 and 2 on Clinical Interview Task 1). He showed problems in combining the sections of a building in a correct position, even with the building with a single layer (Problem 2). He also showed difficulties in figuring out the volumes of 3-D buildings of multiple layers (Problems 3 and 4) and constructing the 3-D buildings (see Figure 4.12). Interestingly, his understanding about 3-D buildings with multiple layers (Problem 4) was very similar to understanding of 3-D buildings with a single layer.

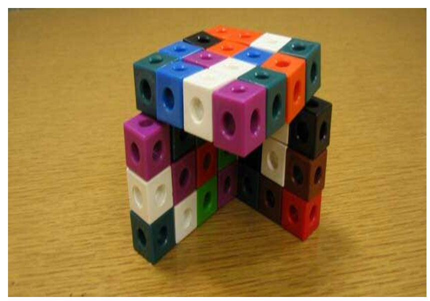
When he was asked to make the building in Problem 4, he made the front side, the top side, and the right side and put them together without considering inside volumes of the sides (see Figure 4.13).



Original Task Kevin's Building

Figure 4.12. Kevin's building on Problem 3 in the postinstruction Clinical Interview 1.

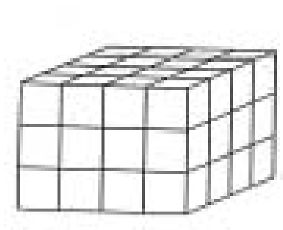


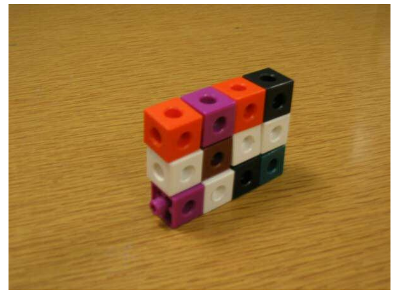


Original Task Kevin's Building

Figure 4.13. Kevin's building on Problem 4 in the postinstruction Clinical Interview 1.

It is notable that Tina's postinstruction performance was different from her baseline performance. At baseline interviews, Tina had problems in constructing all the buildings, and she put the cubes together in a flat way instead of in 3-D. However, at postinstruction interviews, Tina was able to successfully make all the buildings except the building in Problem 4. For Problem 4, she used the same procedures as the one that she used at baseline interviews, just counting the front side of the building, and presented the front side as her output (Figure 4.14).





Original Task   Tina's Building

Figure 4.14. Tina's building on Problem 4 in the postinstruction Clinical Interview 1.

On Clinical Interview Task 2, the 3 students with MD used the strategy of placing the solid on the card to find 3-D solids that would make a specific silhouette, as they did at baseline interviews. During class instruction on this mathematics content, they were taught about how to find the solids that would make specific silhouettes. The methods included (a) directly placing the bottom of the solid on the silhouette and (b) exploring other silhouettes of a solid by projecting it from different points of views. Compared to the students' performance at baseline interviews, their postinstruction performance on this Clinical Interview Task 2 was different in four ways: (a) The side that they tried to match to the card was not limited to the bottom side, (b) they attempted to find multiple answers for each problem after they found one answer, (c) they showed more knowledge about the names of the solids as presented in the section of prerequisite skills, and (d) Lee appeared to have the understanding of the silhouettes projected from different points of views other than the bottom or the sides of the solid .

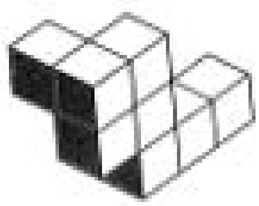
Exploration of multiple answers was apparent in Lee and Kevin's performance. Lee presented three answers to Problem 1 (wide cylinder, narrow cylinder, and cone) and two answers to Problem 3 (cone and square prism). Lee's performance on problem solutions on Task 2 was exactly the same as that of Amy, the typically achieving student. Kevin presented two answers to Problem 1 (wide cylinder and cone) and two answers to Problem 2 (triangular prism and square prism). Lee demonstrated his understanding of silhouettes from different points of views:

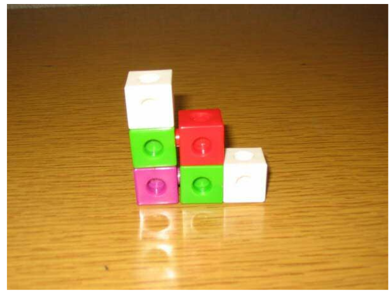
> Lee: First, I'm going to take the cone, I know it's kind of weird but it actually will work because if you set it sideways on the overhead, because it would show this part. Imagine it's not 3-D and cut it, it would look like this thing. Now I'm going to take the square pyramid, just like a little puzzle it fits exactly.

Struggling students. On Clinical Interview Task 1, the 2 struggling students used the procedure of including the steps of considering the number of cubes or the positions of the parts of a building when they were making sections and putting them together. Laura also used the familiar-shape strategy that the typically achieving student used at the baseline interviews and as discussed in their class. Jose showed understanding of what to look at (the number of cubes in each section and the direction of each section), unlike at the baseline interviews. However, although Laura seemed to know what to look at (the number of cubes), she sometimes showed difficulty in counting the number of cubes correctly or positioning the parts of a building correctly, leading her to fail in making buildings on Problems 1, 3, and 4, as at baseline interviews. Compared to his baseline performance, Jose showed increased skills of identifying the volumes of 3-D buildings with a single layer or simple multiple layers and making the buildings (he made correct buildings except on Problem 4), but he still had problems in identifying the volume of the building with multiple layers (Problem 4) and making it. On Problem 4, he made a 3 × 5 × 4 building instead of a 4 × 4 × 3 building. The following are excerpted from postinstruction interviews on Problem 1 with Laura and Jose to show their procedures to make the buildings shown in 2-D drawings:

> Laura: (on Problem 1) I'm starting on the top. This shape looks like an L. I'm doing the top because it has the shape of an L and it's easier…then I matched it. …I count and then I matched it. Well, first I do the easy part, then I count it, then I match it.
>
> Jose: I just counted three right here and four right here and one right here. I counted three bottom, four middle, and one more. I matched it like this.

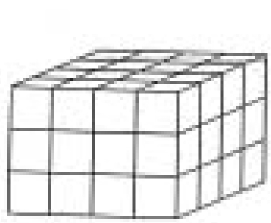
The 2 struggling students showed at least one difficulty in the skills relating to Clinical Interview Task 1: (a) identifying the volume of a building with multiple layers (Problem 4), (b) making a building with multiple layers (Problem 4) using cubes, and (c) combining the sections of a building in the right direction. Laura showed difficulties in all of these skills. Even though she was able to successfully compute the volumes of buildings with a single layer (Problems 1 and 2) and with simple multilayers (Problem 4), she failed to provide correct buildings for those problems because she had difficulties combining the sections of the buildings in the right directions (see Figure 4.15). As in baseline interviews, she made flat shapes instead of buildings having three different directions. With the building with complicated multilayers (Problem 4), she struggled with counting the number of cubes in the building as well as with positioning the sections of the building (Figure 4.16).

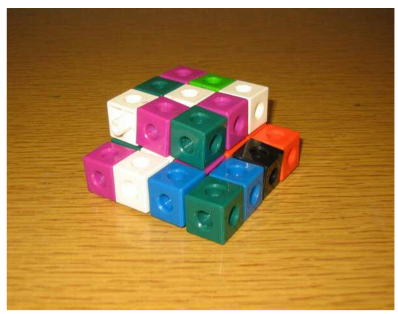




Original Task   Laura's Building

Figure 4.15. Laura's building on Problem 3 in the postinstruction Clinical Interview 1.

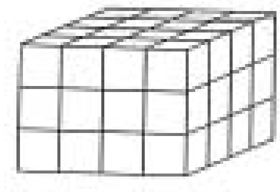


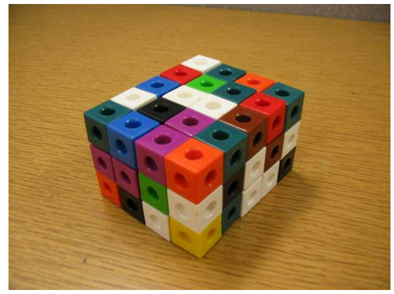


Original Task   Laura's Building

Figure 4.16. Laura's building on Problem 4 in the postinstruction Clinical Interview 1.

Jose struggled with the skills related to making buildings with complicated multilayers (Problem 4). He did not show problems in making the buildings on Problems 1, 2, and 3. However, he showed difficulties in computing the volume of the building in Problem 4 and in making the building; he made a 4 × 5 × 3 building instead of a 4 × 4 × 3 building (Figure 4.17).





Original Task Jose's Building

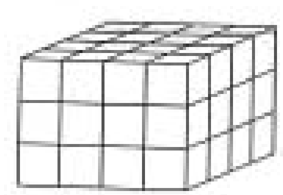
Figure 4.17. Jose's building on Problem 4 in the postinstruction Clinical Interview 1.

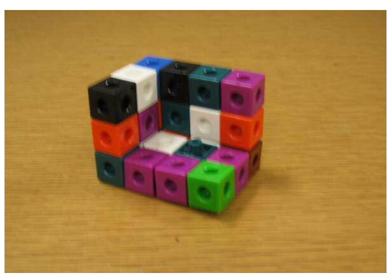
On Clinical Interview Task 2, the two struggling students employed the strategy of mentally matching one of the sides to find 3-D solids that would make a specific silhouette. At baseline interviews, the 2 struggling students used the same procedures of directly placing the object on the card, as the students with MD did. However, after instruction, they attempted to match one of sides of the solids to the silhouettes in their mind. Once they found one answer for one silhouette, they did not explore another answer for the problem.

Typically achieving student. On Clinical Interview Task 1, the typically achieving student used the procedures of considering the number of cubes and the positions of the parts of a building when she was making the sections and putting them together to make a building. Compared to the other groups of students, she put more attention to the number of cubes and positioning the sections of a building to match her buildings to the figures on the cards. Instead of attempting to directly match her building to the 3-D picture on the card, she tried to see if each section of a building had the same number of cubes as in the picture and if she positioned all sections in right directions. She also used the familiar-shape strategy that was discussed in her class.

> Amy: (on Problem 1) Okay, I get the two cubes on the bottom…then, I have to add two on this side. And I have to position this on this side.
>
> Amy: (on Problem 2) There's three on the top. Add this, I'll kind of position it like I need it. Flip it over. Okay. I thought I had to have it in front, but it's on the side.
> Tester: How did you know how to make the building?
> Amy: I make my building, three on the top, three on the bottom, I was kind of confused when there was one, I didn't know where to put it on the side, so that kind of confused me. …Whenever I was counting earlier, I saw the L with this.
> Tester: And how do you know your building is correct?
> Amy: Well I knew my building is correct because when I was counting there was six cubes and this time there are six cubes, I could see how it looks like the same thing, like a pelican.

Amy had difficulties in making 3-D configurations with multiple layers. Compared to her baseline performance, she showed improvement in making the building with a simple multiple layer (Problem 3). However, she still struggled with identifying the volume of the building with complicated multilayers (Problem 4) as well as making the building using cubes (see Figure 4.18).





Original Task　　　　　　Amy's Building

Figure 4.18. Amy's building on Problem 4 in the postinstruction Clinical Interview 1.

On Clinical Interview Task 2, the typically achieving student employed the procedure of mentally matching one of sides to the card after instruction. She said the procedures were automatically implemented when she saw a silhouette:

> Amy: Like I said, whenever I was looking at the bottom, I saw a rectangle and it's my automatic instinct to grab a rectangle prism, so that's how I knew this would automatically be one of my choices. I looked at the bottom and right here, and there's rectangles on each face of the triangular something. It also makes it 3-D, but I just saw there was a rectangle at the bottom.
> Tester: How do you know you found the correct solid?
> Amy: I know I found the correct solid when I put the triangular something down there was all this excess shadow, so I knew that wasn't going to be my answer. I automatically knew that this was going to be one of my choices. It covers the shadow.

Compared to the other groups of students, Amy provided more answers to each problem. Even after she provided a correct answer to a problem, she attempted to explore another answer to the problem. To Problem 1, she presented three answers (wide cylinder, narrow cylinder, and cone). To Problem 3, she provided two answers (cone and square pyramid).

Transfer of Problem Solutions to Problems With Different Similarity to the Original Problem

So far, learning of geometry and spatial reasoning skills by students with differing ability in standards-based mathematics general education classroom has been described in terms of changes in their prerequisite skills, accuracy of problem solutions, and concepts or procedures that they used for problem solutions after receiving instruction on the targeted skills in this instructional environment. This section presents the findings on transfer of the targeted skills after receiving standards-based mathematics instruction on the skills, according to the variations of similarity of new problems to the problems taught in class. Transfer refers to the application of prior knowledge and skills acquired in one situation to new situations (Singley & Anderson, 1989). Students' transfer of problem solutions to new problems in geometry and spatial reasoning were examined by analyzing the students' performance according to transfer problems with different similarity to the original problems used for class instruction. This study aimed to examine student knowledge transfer in terms of (a) general aspects of problem solving on curriculum-based problems and (b) transfer of problem solving according to the similarity of the new problems to the original problem taught in class.

To investigate the second aspect of the knowledge transfer in geometry and spatial reasoning, Clinical Interview Task 1 was designed to include three types of problems, which had different similarity to the original problems used for class instruction for the analysis of transfer of geometry knowledge and skills by the students with different ability. Base problems (Clinical Interview Task 1, Problems 1 and 2) were exactly the same as the original problem used for class instruction in terms of the problem structure (e.g., problem schema and problem components influencing on problem solutions, such as the number of layers of a building) and the surface features of problems (e.g., the number of cubes). Baseline problems were used to examine the students' mastery of the skills of making 3-D buildings shown in 2-D drawings, which they learned in their mathematics class. The other two types of problems (near-transfer and far-transfer problems) were target problems that the students needed to apply the skills taught in class to complete. The near-transfer problem (Problem 3) was the same as the original problems in terms of the problem structure or problem solutions (making single-layered parts of a cube building and putting them together to match the building to the drawing on the card) but different from the original problem in terms of the surface features (the number of cubes or directions). The far-transfer problem (Problem 4) was different from the original problems used to teach the solving procedures in both the problem structure (the number of layers of a cube building) and the surface features (the number of cubes and directions of the sections of the building). To solve the far-transfer problem, the students needed to modify or transform the problem-solving procedures (the procedures related to making 3-D buildings) learned from their class.

Research on transfer of knowledge and skills has procedures of ensuring students' acquisition of knowledge and skills in one situation before investigating transfer of the knowledge and skills to different situations (Bassok, 1997). Thus, for the analysis of transfer of geometry knowledge and skills of the students with different ability, this study included only the students who correctly applied the problem-solving procedures taught in class to solve both base problems in Clinical Interview Task 1. Tina (MD student), Jose (struggling student), and Amy (typically achieving student) were selected and compared in terms of their problem-solving performances according to problems with different similarity to the original problems. Because these 3 students were incorrect on Problems 3 and 4 at baseline interviews, it was expected that their performances on the both problems at postinstruction interviews would reflect transfer of their knowledge acquired during mathematics instruction. The remaining students not selected for this analysis also did not provide correct solutions to both Problems 3 and Problem 4. Table 4.16 presents applications of the problem-solving procedures acquired during their mathematics class to solve problems with different similarity to the original problems.

Table 4.16

Applications of the Problem Solutions Acquired in Class to New Geometry and Spatial Reasoning Problems With Different Types of Similarity to the Original Problems

Transfer problem	Possible problem solutions	Student with mathematics disabilities	Struggling student	Typically achieving student
Near transfer	The same procedures acquired in class	Success Precise use of the same procedures acquired in class	Success Precise use of the same procedures acquired in class	Success Precise use of the same procedures acquired in class
Far transfer	Need for transformation of the procedures acquired in class	Failure Errors in the use of the procedures acquired in class No attempts to transform the procedures acquired	Failure Errors in use of the procedures acquired in class Attempts to transform the procedures acquired	Failure Precise use of the procedures acquired in class Attempts to transform the procedures acquired

Near Transfer

The near-transfer problem (Problem 3) was the same as the original base problems used for teaching the skills of making 3-D buildings with cubes shown in 2-D drawings in terms of the problem structure or problem solutions but were different in the surface features of the problem (the number of cubes and the directions of sections of a building). In other words, Problem 3 was expected to be completed by applying the same procedures used to complete the base problems that were taught in mathematics class. The building on Problem 3 could be made by the procedures of creating single-layered sections of the building and combining them in the way that the sections of the 2-D building on the card were positioned.

On the near-transfer problem, the 3 students of differing ability successfully made the building by using the same procedures taught in class. Commonly, they started from making sections (front, middle, or bottom) according to the number of cubes in each section. Then, they matched their building to the picture to verify their answers.

> Tina: First, I make a square. And then I add one like that. Then, I add three more on the bottom. Once I am done, first I count the blocks and then I looked at the picture then I make the picture. …I match my building to the picture.
>
> Jose: I add three right here. Put them on the bottom, then add four in the middle. And add one on top. …I just counted three right here and four right here and one right here. And I match them to the picture.
>
> Amy: Like I said, I wanted to start from the middle. That kind of confused me down there. …I know how to make the building because well, I saw the picture and like I said what stands out to me the most was the cubes. I knew how to make it by just building from the

> cube building down. I counted earlier the number of cubes that I need to make this building. I look at my building and the picture and now I know they match each other.

In summary, the students, including a student with MD, seemed to be able to successfully transfer their solutions acquired in their class to the new problem, even though the problem was different in the surface features (the number of cubes and directions), once they mastered the solutions. Considering that the 3 students not selected for this analysis because they showed incomplete understanding of the solutions taught in class (did not pass all the base problems) failed on the near-transfer problem, transfer of geometry knowledge and skills to a near-transfer problem on Clinical Interview Task 3 did not seem to be a function of student ability (MD, struggling, and typically achieving) but rather a function of the level of mastery of base learning.

Far Transfer

The far-transfer problem (Problem 4) differed from the original base problems used for teaching the skills of making 3-D buildings with cubes shown in 2-D drawings in both the problem structure and the surface features of the problem. In other words, to produce a complete answer to Problem 4, students should modify or transform the problem-solving procedures that they had learned in their mathematics class at some extent. In fact, the building presented in the Problem 4 could not be made only by making the single-layered sections of a building using the number of cubes shown on the card and combining them in the way shown on the card. After making the visible sections, students should be able to count invisible sections hidden by the visible sections and make the invisible sections to fill the visible sections. In addition, Problems 1–3 consisted mainly of one-layer sections. So, on these problems, students could make and combine the one-layered sections. However, the building on Problem 4 consisted of multilayered sections. So, students should combine multilayered sections to make the whole 3-D building, after making and combining one-layered sections on Problem 4.

Jose seemed to attempt to extend or modify the procedures acquired in class to fit to the structure of the new problem. He made and combined one-layered sections to make one multilayered side of the building, and then made the other sides and combined them together to produce a building matching to the picture on the card. However, he made mistakes when counting the number of cubes in each row when he made one-layered sections. As a result, he produced a $5 \times 4 \times 3$ building instead of the $4 \times 4 \times 3$ building as his output (refer back to Figure 4.17).

Amy seemed to be able to extend or transform the procedures acquired in class to solve the new problem. She made and combined one-layered sections to make one side of the building (the front side in a 4 x 3 array). Then, she made the other two sides (the top side in a 4 x 4 array and the right side of a 4 x 3 array); she put them together to make her building "exactly the same" as the picture on the card. She stated that there would be other sides behind the visible sides, but she would make a building exactly match the picture on the card. Even though her building was rated as incorrect, it should be noted that she purposively excluded the invisible sides when she was making her building; she wanted to follow what was shown in the picture. Also, it should be mentioned that she would be able to transfer the procedures acquired in her class to the nonisomorphic problem with transforming the procedures if she had more flexible interpretations of the building shown in the 2-D drawing. The following shows what brought her to make the building she produced (refer back to Figure 4.18):

> Amy: I'm kind of thinking of this as columns. You know rows, so I'll build 3 of each. That won't work.
> Tester: Please think out loud.
> Amy: Um 3… another column.
> Tester: Are you making the front side?
> Amy: Yeah, the front side. …I have 4 more columns. [Mumbles.] Yeah, just the same thing but positioned differently. Turning it makes it easier. I guess I have to make the top that had 4, 8, 12, 16 cubes. Four in a column. Okay, wait…and then I have to add that. [Mumbles.] Now last column on the top…basically it all fell apart.
> Tester: Beautiful, okay. How did you know how to make the building?
> Amy: I knew how to make the top part because I was multiplying the top to see if that was the correct number. First I counted the sides, what really tricked me was if I was supposed to make the sides or not and the middle part or back.
> Tester: So you basically made to build a side and decided not to do this, why?
> Amy: Not because I didn't want to work as hard, just I didn't see the other sides or anything, so just follow what you see I guess.

In summary, all the students showed difficulties in completing the building in Problem 4. However, student difficulties were different across the students with different ability. Tina (MD student) had difficulty in modifying or transforming the procedures after acquiring them. She had a one-track mind in using the procedures taught in class. Jose (struggling student) had difficulty with matching the number of cubes in his building to the number in the picture, rather than in transforming the procedures to fit to the new problem. Finally, Amy (typically achieving student) had difficulty related to her perceptions of the 3-D building in the 2-D drawing, not her ability to transfer her procedures to the nonisomorphic problem.

Probability and Statistics

Probability and statistics are essential mathematics skills for living in the information-rich contemporary society (NCTM 2000). Accordingly, standards for mathematics education nation-wide, such as the NCTM (2000) *Principles and Standards for School Mathematics*, recommend that all students, including students with MD, should be able to acquire a variety of knowledge and skills in probability and statistics from kindergarten through Grade 12. In particular, students in Grades 3–5 in Texas are expected to learn (a) the skills of using organizational structure such as tables and charts to represent and communicate relationships, make predictions, and solve problems and (b) the skills of organizing data, choosing an appropriate method to display the data, and interpreting the data to make decisions and predications and solve problems (TEA, 2006).

During the observational period of this study, two specific probability and statistics skills were taught to the students in Ashley's class and examined using two clinical interview tasks (see Appendix F for more information on clinical interview tasks): (a) organizing and displaying data in a bar graph (Clinical Interview Task 3) and (b) interpreting bar graphs and comparing two graphs (Clinical Interview Task 4). On Clinical Interview Task 3, the students were asked to organize and display data shown in a T-chart using a bar graph. On Task 4, the students were asked to compare two graphs. As on the tasks in geometry and spatial reasoning, each task consisted of four problems on the skills. Jose was not tested on Clinical Interview Task 3 because he was absent when students were interviewed using Clinical Interview Task 3 before learning the targeted skills. Thus, the group of struggling students on Clinical Interview Task 3 included only Laura.

Prerequisite Skills

Individual students' baseline and postinstruction performance on prerequisite skills on each clinical interview task were analyzed, summarized, and synthesized within each group of student ability. This analysis produced comparisons of the performances across these three groups and comparisons of the performances of these three groups between two interview periods, pre- and postinstruction.

Baseline prerequisite skills. Two prerequisite skills were examined during the baseline clinical interviews: (a) interpreting data shown in a T-chart (Clinical Interview Task 3, Prerequisite Skill 3) and (b) finding a typical number shown in a bar graph (Clinical Interview Task 4, Prerequisite Skill 4). Each task included four different problems on one prerequisite skill required for learning the knowledge and skills in probability and statistics being examined in this study. On the problems for Prerequisite Skill 3, the students were asked to interpret and summarize data shown in the table (e.g., the number of students having four brothers and sisters shown on the graph). On the problems for Prerequisite Skill 4, the students were asked to find a typical value on a bar graph (e.g., typical height in Mr. A's class or typical fruit chosen in Mr. A's class).

Once an individual student's answer to each problem on a prerequisite skill was scored for its correctness (1 point for a complete correct answer, 0.5 point for a partially correct answer, and 0 for an incorrect answer), the individual student's total score on the prerequisite skill was calculated. The total score was later divided by 4 (maximum total score) and multiplied by 100% to calculate the percentage of correct answers (accuracy) on the prerequisite skill task. The accuracy of solving problems on each prerequisite skill was compared across students with different ability (MD, struggling, and typically achieving students). In addition, the process of answering to questions about the prerequisite skills (e.g., the concept of typical for Task 4) was summarized for individual students, synthesized within a group, and then compared across groups of students with different ability. Table 4.17 provides a summary of comparisons on baseline and postinstruction prerequisite skills.

Table 4.17

Pre- and Postinstruction Percentage Accuracy in Solving Problems on Statistics Prerequisite Skills Across Groups of Students With Different Ability

Prerequisite skill	Students with mathematics disabilities	Struggling students	Typically achieving students	Mean
3. Interpreting data in a table involving either numerical or nominal variables				
Baseline	95.8	62.5	100.0	86.1
Postinstruction	95.8	100.0	100.0	98.6
4. Understanding the concept of typical value and finding typical values from a bar graph involving numerical or nominal variables				
Baseline	75.0	25.0	50.0	50.0
Postinstruction	100.0	75.0	25.0	66.7

Overall, baseline accuracy in solving problems on Prerequisite Skill 3 was 95.8%, 62.5%, and 100.0% for the MD student group, the struggling student group, and the typically achieving student, respectively. Accuracy in solving problems on Prerequisite Skill 4 was 75.0%, 25.0%, and 50.0% for the three groups of students in order. Unlike the patterns of performances on the prerequisite skills in geometry and spatial reasoning across three groups of students with different ability, the group of students with MD was not behind in terms of prerequisite skills in probability and statistics. Rather, they performed on average better than the group of struggling students on the problems regarding both Prerequisite Skill 3(95.3% vs. 62.5%) and Prerequisite Skill 4 (75.0% vs.

25.0%). They also performed better than the typically achieving student on the problems regarding Prerequisite Skill 4 (75.0% vs. 50.0%).

Within a group, the variations were not negligible, particularly in the group of struggling students and the group of students with MD. On the problems for Prerequisite Skill 4, Laura did not provide a correct answer at all, whereas Jose was correct on two problems out of four (50.0%). Laura' large deviation on Prerequisite Skill 4 made the average performance of the group of struggling students the lowest among the groups of students with different ability. Similarly, the performances by the 3 students with MD were not homogeneous on Prerequisite Skill 4. On Prerequisite Skill 4, Tina was 100.0% correct, Kevin was 75.0% correct, and Lee was 50.0% correct. The typically achieving student, Amy, provided 100.0% correct answers to the problems on Prerequisite Skill 3 and 50.0% correct answers to the problems on Prerequisite Skill 4.

Even though the baseline scores of the group of students with MD were higher than those of the other groups, it should be noted that the way that they answered to each problem was neither systematic nor based on the better understanding of the concepts related to the prerequisite skills. For example, when they were asked to find the number of students having three brothers and sisters in Mr. A's class as shown in a table on the problem for Prerequisite Skill 3, their answers (student names) were neither sequenced as shown in the table nor reflected their understanding of information shown in the table. They just looked for the number 4 on the right column and found the student names corresponding to the number 3 (e.g., "I am seeing the number 4 beside Fiona. So, I am choosing Fiona."). When asked what the numbers on the table meant, none of students with MD answered correctly, whereas the typically achieving student showed understanding of the information included in the table.

On the problems about Prerequisite Skill 4, the students with MD did not show the understanding of the concept of typical. Tina and Kevin chose specific numbers or values as answers because they were higher or bigger than the others. Even though their answers were based on the concept of typical as mode, they did not know why they were looking for bigger or higher bars to find typical values. Lee's answers were even worse than the others. Her answers were based on her own experiences, not on the data. Compared to the students with MD, the typically achieving student, Amy, showed knowledge about the concept of typical. The following provide a comparison of Lee and Amy's understanding of the concept of typical:

> Lee: Typical number would be two…because not that many people have brothers and sisters, like I have a godsister. But I am not counting her. I have two. And I also have two brothers, no three, but one is also my godbrother. So typical number would really be two. Some could be an only child or something like that.
>
> Amy: The typical number was the most common number of students with the brothers and sisters. There's one who has two, one who has three, one who has six, one who has three. …Two people have three brothers and sisters. So, three brothers and sisters is the most common number in Mr. A's class.

Postinstruction prerequisite skills. Each student's performances on the questions about prerequisite skills during postinstruction clinical interviews were scored in the same way as baseline clinical interviews (e.g., score 1 for correct answer and score 0 for incorrect answer). The number of correct answers of each student was divided by the total number of problems in each task (4) and multiplied by 100% to produce the percentage of correct answers to each task. The percentage of correct answers to prerequisite problems on each task was compared across students with different ability (MD, struggling, and typically achieving students). In addition, the procedures or the concepts related to the prerequisite skills (e.g., procedures of reading data on Task 3 and the concept of typical value on Task 4) were summarized for individual students, compared with their baseline performances, and compared across students with different ability. Table 4.17 provides a summary of comparisons on postinstruction prerequisite skills.

Comparisons of prerequisite skills between baseline and postinstruction interviews. Overall, as shown in Table 4.17, compared to the percentages of correct answers at baseline interviews, the percentages of correct answers at postinstruction interviews were increased on both Prerequisite Skill 3 (86.1% vs. 98.6%) and Prerequisite Skill 4 (50.0% vs. 66.7%). On Prerequisite Skill 3, all the three groups (all students except Lee) got full credit on all four problems (mean = 98.6%) after instruction. The percentages of correct answers on Prerequisite Skill 3 problems were 95.8%, 100.0%, and 100.0% for the group of MD students, the group of struggling students, and the group of a typically achieving student, respectively.

The difference between the mean baseline accuracy (86.1%) and the mean postinstruction accuracy (98.6%) in solving problems on Prerequisite Skill 3 was 12.5%. This gain was from Laura's improvement (62.5% to 100.0%). Like Lee, Laura did not present 100.0% of correct answers to problems on Prerequisite Skill 3. Her performance on the Prerequisite Skill 3 was improved by 37.5%, whereas Lee stayed at the same level of performance (87.5% in both tests). Similarly, the mean postinstruction accuracy in solving problems on Prerequisite Skill 4 was 16.7% higher than the mean baseline accuracy in solving problems on the task.

Comparisons of performances of groups with differing abilities. Comparisons of group performances in prerequisite skill problems on probability and statistics revealed that (a) the postinstruction accuracy in solving prerequisite skill problems on Clinical Interview Task 4 was not associated with the ability of groups of students, and (b) the postinstruction accuracy in solving prerequisite skill problems was related to students' understanding of the concept of typical. First, at postinstruction interviews, the group of MD performed at the level of 100.0% correct, the group of struggling students performed at the level of 75.0% correct, and the typically achieving student performed at 25.0% correct. The group of struggling students was the group having the highest improvement on Prerequisite Skill 4 (25.0% to 75.0%). The MD student group performed 25%

higher in postinstruction interviews than in baseline interviews on Prerequisite Skill 4. However, the typically achieving student's performance on Prerequisite Skill 4 in postinstruction interviews degraded in comparison to baseline performances (50.0% vs. 25.0%).

The performance of the groups of students with different ability on Prerequisite Skill 4 was related to their understanding of the concept of typical. All the students with MD understood the meaning of typical as a mode of X values (e.g., an X value most frequently or commonly appeared on a graph) in both baseline and postinstruction interviews. Using this concept of typical, Kevin, Lee, and Tina got 100.0% correct answers on the problems on Prerequisite Skill 4. Jose and Laura, struggling students, showed a very similar understanding of the concept of typical. Based on the concept of typical as a mode, Jose was correct on 100.0% of the problems on Prerequisite Skill 4, and Laura was correct on 50.0% of problems on that skill, especially problems with nominal variables (Problems 3 and 4). However, Amy showed confusion about the concept of typical. She misunderstood the concept of typical as a mode of Y values (Y value most frequently or commonly appearing in a graph), not as a mode of X values. Accordingly, she attempted to find one of Y values that most commonly appeared in a graph instead of the X value that had the highest bar in a graph. The following shows her understanding of the concept of typical:

> Amy: Um, well, I see that there is two equals, I just looked at the bars, and there's the number of students is, there's three if it's a typical number who has it? There's three that have two brothers and sisters, and three that have five brothers and sisters. So those are the typical number so they're the ones that are the same high and either higher or lower.

Accuracy in Problem Solutions

The knowledge and skills in probability and statistics investigated in this study were (a) organizing and displaying data in a bar graph (Clinical Interview Task 3) and (b) interpreting bar graphs and comparing two graphs (Clinical Interview Task 4). Each student's problem-solving performances on a problem in each task (Clinical Interview Tasks 3 and 4) were scored for a completely correct problem solution (1 point), partially correct problem solution (0.5 point), or no correct problem solution (0 point). On the problems in Clinical Interview Task 3, students got full credit when they presented a complete correct bar graph representing the data shown in the table. Otherwise, their performance was scored as 0. On the problems in Clinical Interview Task 4, the student answer on each problem was scored as 1 only when it was correct and was based on comparisons using at least two of the following criteria: (a) range, (b) pattern, (c) spread or clumping, (d) typical values, (e) bar-by-bar comparison, and (f) relationships or variables embedded in the data set. When the student answer included comparisons based on only one criterion, the answer received a score for a partially correct problem solution (0.5 point). When the student answer did not include comparisons based on any of the above criteria, the answer was scored as 0. Comparisons based on physical features such as color or size were not counted as correct responses. The scores of four problems were summed and used to calculate accuracy, the percentage of correct answers to the problems on Clinical Interview Task 3 or 4.

Baseline accuracy in problem solutions. Table 4.18 shows accuracy in solving problems on each clinical interview task regarding probability and statistics across three groups of students with differing ability. The table shows baseline as well as postinstruction scores.

Table 4.18

Pre-and Postinstruction Percentage Accuracy in Solving Problems on Two Probability and Statistics Clinical Interview Tasks Across Groups of Students With Differing Ability

Clinical interview task	Students with mathematics disabilities	Struggling students	Typically achieving students	Mean
Task 3				
Baseline	16.7	50.0	75.0	47.2
Postinstruction	16.7	50.0	100.0	55.6
Task 4				
Baseline	50.0	68.8	87.5	68.8
Postinstruction	58.3	62.5	100.0	73.6

On average, 47.2% of answers provided by the three groups of students were correct to the problems on Clinical Interview Task 3, and 68.3% of answers were correct to the problems on Clinical Interview Task 4. On both clinical interview tasks, accuracy in problem solutions depended on student ability. For example, the percentages of successful solutions (accuracy) on Clinical Interview Task 3 were 16.7% for the MD students, 50.0% for the struggling students, and 75.0% for the typically achieving student. Likewise, the percentages of successful solutions on Clinical

Interview Task 4 were 50.0% for the MD students, 68.8% for the struggling students, and 87.5% for the typically achieving student.

The group of students with MD performed lower than the other groups of students on both tasks. On Clinical Interview Task 3, Kevin provided correct answers to two of four problems, but Lee and Tina did not present the correct answer to any problem. Kevin provided correct answers to Problems 3 and 4, which have nominal variables instead of numerical variables. On Clinical Interview Task 4, all 3 MD students provided 50.0% of correct answers to four problems. They provided answers using at least one comparison point to all problems.

The group of struggling students performed with accuracy that fell between the accuracy rating of students with MD and that of the typically achieving student, before they were taught about the probability and statistic skills being examined. On Clinical Interview Task 3, Laura provided correct answers to two of four problems. The problems she got correct were involved in nominal variables (Problems 3 and 4) instead of numerical variables (Problems 1 and 2). On Clinical Interview Task 4, 62.5% of Jose’s answers and 75.0% of Laura’s answers were correct. They also used at least one significant point of comparison when they answered each question.

The typically achieving student, Amy, performed better than the other groups of students on both tasks. On Clinical Interview Task 3, she provided correct answers to three out of four problems (75.0%). She was correct on both problems involving nominal data and correct on one of two problems involving numerical data. On Clinical Interview Task 4, she provided perfectly correct answers (using at least two different points of comparison considered in this study) to three out of four problems and a partially correct answer (using one point of comparison) to the remaining problem.

Postinstruction accuracy in problem solutions. Table 4.18 shows the accuracy in solving problems on each probability and statistic skill task at baseline and postinstruction interviews across three groups of students with differing ability. These data were used to determine improvement across groups of students, postinstruction.

Comparisons of accuracy in problem solutions between baseline and postinstruction interviews. The results from the comparisons of baseline and postinstruction problem-solving accuracy indicated that (a) the postinstruction mean accuracy of all three groups in solving problems on two probability and statistics tasks increased from the baseline mean accuracy, and (b) the greatest mean difference between baseline and postinstruction was from the typically achieving student. Compared to baseline performances, the mean postinstruction performance of students in all three groups increased by 8.3% on Clinical Interview Task 3 and by 4.9% on Clinical Interview Task 4. The three groups of students on average produced correct problem solutions for 55.6% of the problems on Clinical Interview Task 3 (16.7%, 50.0%, and 100.0% for the MD group, the struggling student group, and the typically achieving group, respectively). On average, the three groups achieved 73.6% accuracy on the problems on Clinical Interview Task 4 (58.3%, 62.5%, and 100.0% for the MD group, the struggling student group, and the typically achieving group, respectively) after they received instruction on the skills in standards-based general education classroom. Except the typically achieving student, the other groups of students did not show improvement from baseline to postinstruction interviews in problem-solving accuracy on either Clinical Interview Task 3 or 4.

Comparisons of performances of groups with differing ability. The comparisons of group performances on solving problems on Clinical Interview Tasks 3 and 4 showed two results. First, the improvement was higher for the typically achieving student than for the other two groups of students on both tasks. Second, large variations existed within the postinstruction performance of the students with MD and the struggling students.

On Clinical Interview Task 3, the group of students with MD and the group of struggling students did not show improvement at all, whereas the typically achieving student showed 25% improvement in prerequisite skills. On Clinical Interview Task 4, the group of struggling students showed a 6.3% decrease, whereas the typically achieving student (12.5%) and the group of students with MD (8.3%) showed improvement after they received instruction on the skills.

On Clinical Interview Task 3, no students with MD showed improvement between baseline interviews and postinstruction. In particular, Lee did not show any improvement on either Clinical

Interview Task 3 or Clinical Interview Task 4. She did not solve any problem correctly on Clinical Interview Task 3 postinstruction as well as at baseline interviews. Tina also did not solve any problem correctly on Clinical Interview Task 3 at both baseline and postinstruction interviews. Kevin's answers were 50.0% correct across the problems at both baseline interviews and postinstruction interviews.

Similarly, the struggling student, Laura, did not show improvement in Clinical Interview Task 3 between baseline interviews and postinstruction interviews. The typically achieving student, Amy, showed a 25.0% increase in the percentage of correct answers on Clinical Interview Task 3 between baseline and postinstruction interviews.

On Clinical Interview Task 4, nonnegligible within-group variations were found in both the group of students with MD and the group of struggling students. At baseline interviews, all 3 students with MD performed at the level of 50.0% correct. Lee's performance on the task degraded from 50.0% correct to 37.5% correct between the two interview periods, whereas Tina and Kevin showed improvements between the periods (50.0% to 62.5% for Kevin and 50.0% to 75.0% for Tina). Meanwhile, the 2 struggling students, Jose and Laura, started at the levels of 62.5% correct and 75.0% correct, respectively, on Clinical Interview Task 4. However, after receiving instruction in their classroom, Laura's percentage of correct answers on the task decreased by 25.0%, whereas Jose's percentage of correct answers increased by 12.5%. Like Jose, the typically achieving student, Amy, showed improvement in the percentage of correct answers by 12.5% from baseline to postinstruction interviews.

Concepts or Procedures Used for Problem Solutions

The baseline and the postinstruction concepts or procedures that individual students with different ability used for solving problems on Clinical Interview Tasks 3 and 4 were summarized and synthesized by ability group and by interview period. These data were compared to produce information about differential problem solutions among the three groups of students and by the three groups of students between the interview periods.

Baseline Concepts or Procedures

Students with MD. On Clinical Interview Task 3, students with MD seemed not to have stable knowledge about what the bar graph was, how each datum should be summarized to appear on the graph, or what should be decided to make a bar graph (e.g., X and Y variables). Tina employed the procedure of copying a table to produce her outputs, whereas Kevin and Lee produced their graph outputs by using procedures including counting the numbers (Problems 1 and 2) or the names (Problems 3 and 4). The following were the responses by the students with MD to the question about their strategies used to make a bar graph:

> Kevin: (Problem 2) Looking at the card I put the numbers on the side and I look at all of the numbers and I look at the name and I colored it in.
> Tester: Okay, How do you know if your graph is correct?
> Kevin: By looking at the numbers on the side of the paper.

Lee: (Problem 2) Cathy, Erin, and Kelly have 56; Noah has 58; and Isabella has 54. So, I count up 56, 58, and 54 to make a bar graph. …(Problem 3) I looked at the table and saw how many people have an apple, how many have a peach.

Tina: I look at this page and I just copy it down.
Tester: Okay. How do you know if your graph is correct?
Tina: Well after I put the students I look at the students and same with all of them.

On Clinical Interview Task 3, Tina had difficulty with the concept of bar graphs as well as the procedures of making a bar graph. As answers to the problems asking the student to draw a bar graph showing the information on the table, Tina just copied the data tables on her papers. When she was asked about if her outputs were bar graphs, she answered they were bar graphs. Figure 4.19 is the table provided as a problem, and Figure 4.20 shows Tina's attempt to draw a bar graph.

Student Name	Height	Student Name	Height
Angela	56	Heike	51
Bob	54	Isabella	54
Cathy	56	Jade	52
David	51	Kelly	56
Erin	56	Luke	52
Fiona	54	Mary Ann	53
George	50	Noah	58

Figure 4.19. Data shown in Problem 2 of Clinical Interview Task 3.

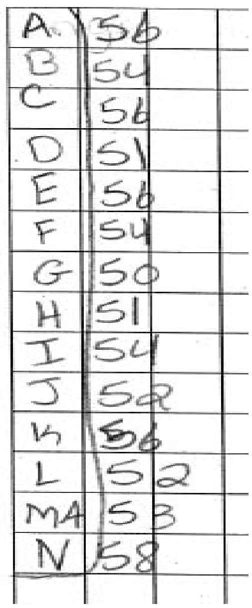


Figure 4.20. Tina's bar graph to Problem 2 in the baseline Clinical Interview 3.

Meanwhile, Lee and Kevin seemed to know the meaning of a bar graph at least in terms of its shape. They tried to make a graph including bar shapes, even though they were not correct all the time. Rather, their problems were related to the procedures of making a bar graph, which included getting the frequency of each category (numbers or names) and graphing the frequency of each category embedded in the data set, instead of graphing the raw data points themselves.

Lee attempted to make a bar for each raw data set, but she did not try to organize or summarize raw data according to the structure embedded in the data (e.g., the number of brothers and sisters on X and the frequency of each number shown in the table on Y). For example, on Problem 1 (see Figure 4.21), Lee drew a horizontal line on the paper, drew four boxes in the first column for the data of "Fiona—four brothers and sisters," wrote "Fiona" on the top of the boxes, drew four boxes in the third column for the data of “Heike—four brothers and sisters,” and wrote “Heike” on the top of the boxes (see Figure 4.22). Likewise, on Problem 2 (see Figure 4.23), Lee drew 56 boxes in the first column to represent the raw data that Cathy, Erin, and Kelly were 56 inches tall and put their names on the top of the boxes (see Figure 4.24).

Student Name	Number of Brothers & Sisters	Student Name	Number of Brothers & Sisters
Angela	2	Heike	4
Bob	2	Isabella	3
Cathy	1	Jade	5
David	1	Kelly	2
Erin	0	Luke	0
Fiona	4	Mary Ann	1
George	3	Noah	1

Figure 4.21. Data shown on Problem 1 of the Clinical Interview Task 3.

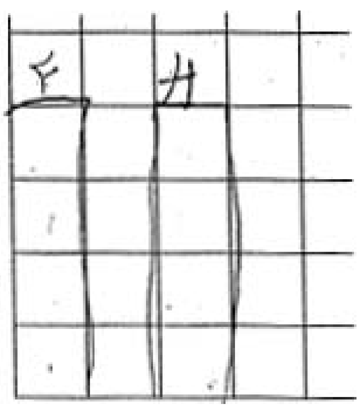


Figure 4.22. Lee’s bar graph to Problem 1 in the Clinical Interview 3.

Student Name	Height	Student Name	Height
Angela	56	Heike	51
Bob	54	Isabella	54
Cathy	56	Jade	52
David	51	Kelly	56
Erin	56	Luke	52
Fiona	54	Mary Ann	53
George	50	Noah	58

Figure 4.23. Data shown in Problem 2 of Clinical Interview Task 3.

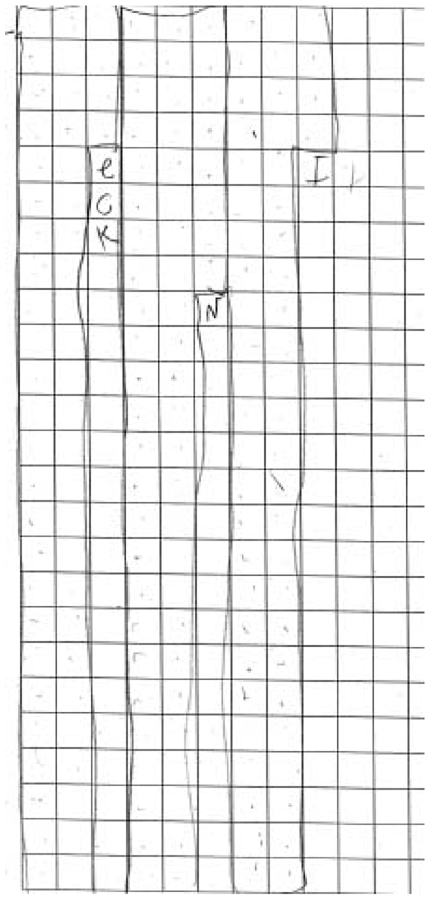


Figure 24. Lee's bar graph to Problem 2 in the baseline Clinical Interview 3.

Kevin's difficulty was related to the procedures of making a bar graph and depended on the types of variables involved in problems (e.g., numerical variables or nominal variables). To make a bar graph on Clinical Interview Task 3, Kevin used the procedures of counting the numbers (Problems 1 and 2) or the names (Problems 3 and 4) and putting them on the side. He seemed to know what to draw to make a bar graph, because he mentioned the procedures of graphing the frequency of each number or name. Although he was successful in making a bar graph of nominal data (Problems 3 and 4) by applying these procedures, he was not able to apply the procedures correctly to make a bar graph of numerical data (Problems 1 and 2). As with Lee, Kevin also set forth each raw datum on the graph by using bars as many as the number of students on Problems 1 and 2 (numerical variables), resulting in incorrect graphs (see Figure 4.25). On Problems 3 and 4 (nominal variables), he attempted to summarize the raw data according to the variables, including the number of students choosing each fruit (Y) and the names of fruits (X), and drew bar graphs of the data correctly.

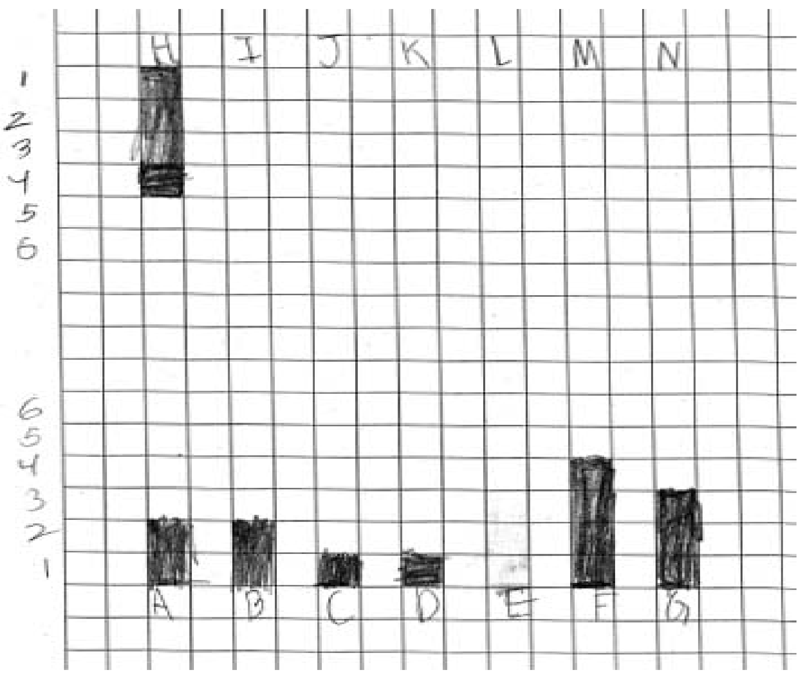


Figure 4.25. Kevin's bar graph to Problem 1 in the baseline Clinical Interview 3.

On Clinical Interview Task 4, the 3 students with MD used the procedures of "looking at and comparing them" to find differences and similarity between two bar graphs. Even though they did not talk about how to compare the graphs specifically, data from their thinking aloud indicated that they made comparisons based on the bar-by-bar range of two graphs.

Lee: (Problem 2) Mr. B's class don't have too many tall people and Mr. A's class do. They go up to the 60, and B's class only goes up to the 50.
Tester: Goes up to what?
Lee: The 58, like the 50ish. They go up to 60.
Tester: What else?
Lee: That's it.
Tester: What about similarities?
Lee: I don't.
Tester: How did you know how to find the differences and the similarities in the two graphs?
Lee: I just looked.

Kevin: (Problem 1) This one is bigger than this one, and this one has eight people, number of brothers and sister, this has six brothers and sister. And this goes to six and this goes to seven.
Tester: Okay…it goes up to seven, B goes up to six. What else?
Kevin: One is taller than the one in A's class. Five in Mr. A's class is bigger than Ms. B's class.
Tester: Okay, what else? Do you see any other differences?
Kevin: And two in Mr. A's class is bigger than Ms. B's class.
Tester: Do you see any similarities between them?
Kevin: Three in both are bigger than their classes.
Tester: What else do you notice?
Kevin: The six, they only have one.
Tester: Do you see any other similarities? Okay, then how did you know how to find the differences and the similarities in the two graphs?
Kevin: I looked at both of the graphs. …I knew that there were more people down here then up there and four have been…that's it.

Tina: Um, Ms. B's class has a lot that do not have the heights, and Mr. A's class only has…six people that doesn't have the heights.
Tester: Great job, and what else? Do you see any other differences?
Tina: On the side it only goes up to 6 on Ms. B's class and on A's class it goes up to 7.
Tester: Do you see any similarity between them?
Tina: Well on the bottom on both of them it goes up to 65.

Tester: Can you tell me how did you know to find the difference and similarities between the two graphs?
Tina: Well I look at A's class and I see if, then I think, then I look on B's class and then I look at both of them and see if it's different or if it's the same.

As in the citation above, Kevin made bar-by-bar comparisons of two graphs on all four problems. He did not consider the other points in comparisons, except when he made comparisons of two graphs in terms of the ranges of the variables on Problem 1.

Lee also mainly used bar-by-bar comparisons (Problems 2, 3, and 4). When she used another point for comparisons, she did not provide correct answers to the problems. For example, when she attempted to compare two graphs of Problem 3 in terms of typical value, she failed to provide a correct statement about similarity between the two graphs. She said that apples were the most popular fruit in both classes, which was true for the Mr. A's class but not for Ms. B's class.

Tina attempted to compare two graphs in terms of the ranges of variables, spread and clumping, and variables as well as each bar. She used two to three different points when she made comparisons of two graphs on each problem. However, her comparisons did not always yield correct answers. No relationship was observed between comparison points and incorrect answers. For example, she got an incorrect statement when she compared two graphs bar by bar on Problem 1 ("Mr. A's class has none on 4, but Ms. B's class has one on 4." A correct statement would have been that Mr. A's class has none on 4, but Ms. B's class has 2 on 4). Meanwhile, she presented an incorrect statement when she compared two graphs in terms of the ranges of variables on Problem 3 (e.g., “B goes up to 6, but A goes up to 7.” A correct statement would have been that Ms. B’s class goes up to 5, but Mr. A’s class goes up to 6).

Struggling students. On Clinical Interview Task 3, the struggling student, Laura, made a graph by employing the procedures of counting the numbers (Problems 1 and 2) or the names (Problems 3 and 4) and putting them on the side, which was the same as the procedures that Kevin and Lee used for their solutions. She could apply the procedures without confusion to the problems involving nominal variables (Problems 3 and 4) but not to the problems involving numerical variables (Problems 1 and 2). On Problems 1 and 2 on Clinical Interview Task 3, she placed each student name on the X axis, placed the number of brothers and sisters on the Y axis, and drew a bar for each student according to the number of brothers and sisters that the student had, instead of getting the frequency of each number of brothers and sisters and drawing the relationships between the number of brothers and sisters and its frequency. Using these procedures, Laura produced outputs very similar to those of Kevin or Lee on Problem 1. On Problem 2, she used an additional procedure as well as the procedures used on the Problem 1. She converted each student height to a certain smaller number by using an inconsistent and unreasonable formula and drew the small number on the graph. The following illustrates her way to draw a bar graph of numerical data having larger numbers and her outputs on the problems involving numerical data (see Figure 4.26):

> Tester: Can you make a bar graph of this data?
> Laura: That’s too hard. How can we do it? How could I do this? It’s too big to draw. I need to change these numbers to others. …

Tester: Why do you color 9?

Laura: Because 7 times 8 is 56. I'm doing multiplication.

Tester: This is for Angela? You used 7 times 8? For Bob, you used 9 times 6 so you colored 9? And this is Cathy, you used what?

Laura: Cathy I used 10 times 5, because 50 is close to 51.

Tester: Okay, and what is this? Erin?

Laura: Erin is 7 times 8, so I color 8.

Tester:…How did you know how to make the bar graph of the data from the table?

Laura: Well I can't color 56, so I multiplied, the easier way.

Tester: How do you know if your graph is correct?

Laura: I could count.

Tester: And how did your teacher. …No, she didn't teach you. Do you know another way to display the data?

Laura: You could divide.

Tester: Divide? How?

Laura: Because 56 divided by 8 is going to be 7.

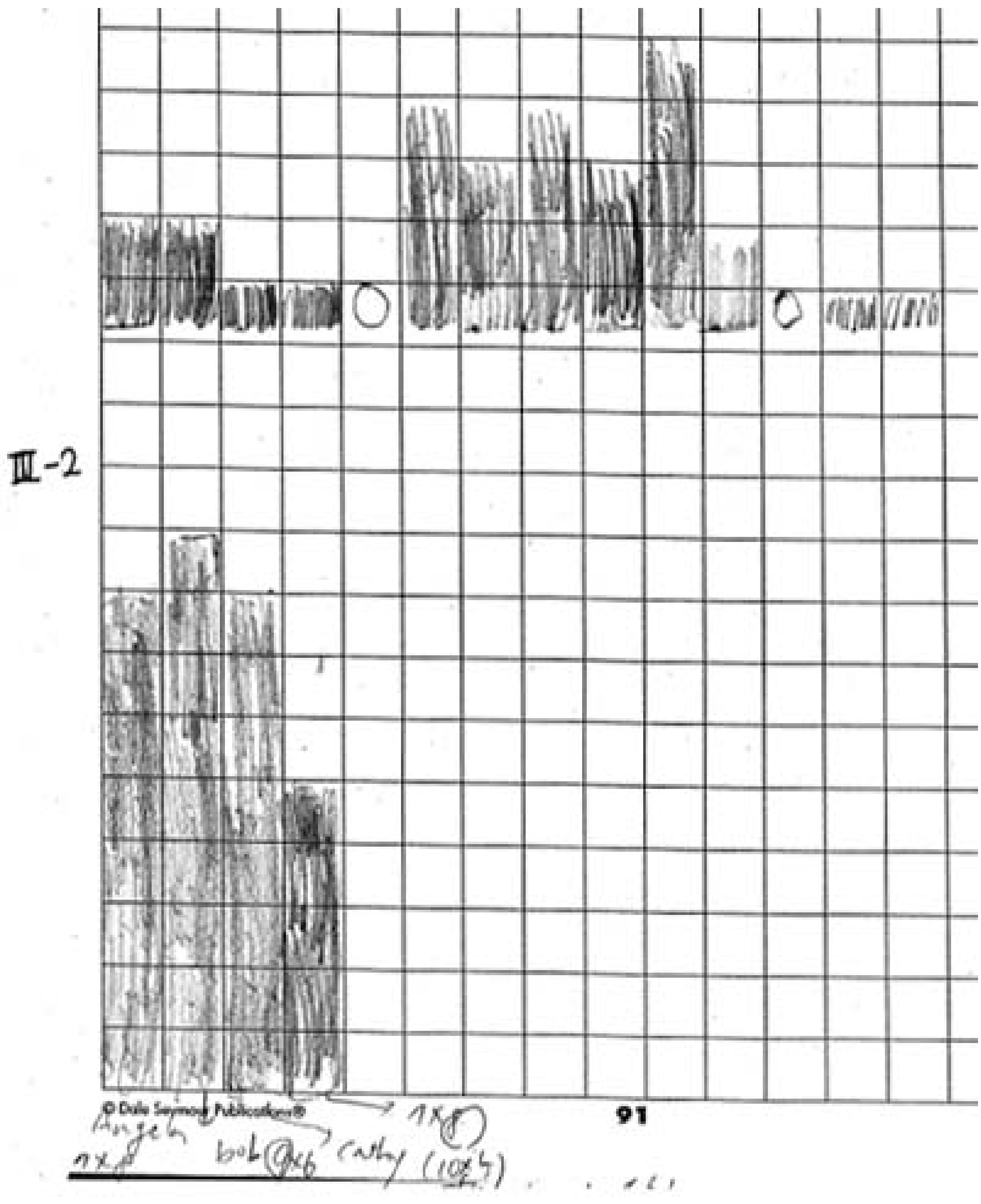


Figure 4.26. Laura's bar graph to Problems 1 and 2 in the baseline Clinical Interview 3.

On Clinical Interview Task 4, the struggling students, Laura and Jose, used the procedures of looking at the graphs and comparing the same parts. Jose made literal, bar-by-bar comparisons on all four problems, whereas Laura used mixed ways of bar-by-bar and pattern comparisons. Jose was observed making multiple comparisons of two graphs by using the physical features of graphs for his comparisons (e.g., bar color, bar size, graph size, etc.). The following provide examples of Laura and Jose's performance on Clinical Interview Task 4:

> Laura: (Problem 2) Ms. B's class, they have short, short, tall, tall, short, short; it goes in a pattern. In Mr. A's class they have short, medium, then tall, then taller, so they go in sizes, or talls.
> Tester: And Mr. A's class is…?
> Laura: It's like short, tall, then tall, and taller.
> Tester: So is it another pattern?
> Laura: Yes, but it's not the same.
> Tester: But those two patterns are not the same. What else do you notice?
> Laura: 57 has 5 in Mr. A's class, but they don't have no height.
> Tester: Okay, and what else?
> Laura: 59 has 2, so I think that person is short, and Mr. A's class 59 has 4, so in Mr. A's, 59 is taller than Ms. B's person of 57, er, 59, too.
> Tester: Okay great job, do you notice any similarity between them.
> Laura: 63 has none, and in Mr. A's class has none too. In Ms. B's class 52, and 53, they have 1 height.
> Tester: Differences or similarities?
> Laura: Similarities.
> Tester: Okay, so 52 has 1. And what about 53?
> Laura: Oh this is different, 53 has 1 and 53 has 2.
> Tester: So are they different or similar? Different? So the similar is 2 is 1? How did you know how to find differences and similarities between the graphs?
> Laura: I'm trying to compare these two, I try and look for it for my eyes, and I could like draw a line where it goes from the number of student.
> Tester: After drawing the line what did you see?
> Laura: I see numbers I know that's the number of the students.

> Tester: (Problem 2) And tell me how the two graphs are different and similar.
> Jose: What's similar is this has two same size.
> Tester: What do you mean by size?

Jose: Both of them are skinny.
Tester: What else?
Jose: They all have numbers. Both of these say students, students' heights, students.
Tester: What else?
Jose: They have the same, they have the same shape.
Tester: And can you tell me the differences between them?
Jose: This one is light and this one is dark. B's class isn't light enough and A's class is darker. A's is dark lines and B's is light.
Tester: Okay, any other differences?
Jose: There's different letters, As and Bs.
Tester: How did you know to find the difference and similarities between the two graphs?
Jose: I knew how to find it, I look in my head I look at these two because all they have same size or shape both of these, these lines have same lines.
Tester: So basically you look at the graphs, then you compare, right? How did you know how to find differences?
Jose: This is darker, this is lighter, this has Bs and As, and they have different letters. This has more…and this only has like 7 the other one has 10.
Tester: Do you know another way to find difference and similarities between them? Another way to find differences? If you don't know, it's okay, just say you don't.
Jose: It's…you just add all this and add all this and this one all, you just add it. Add this and that and the totals right here and the totals right here and it's different. And B add the number of bar too that's less it only has 7.

Typically achieving student. On Clinical Interview Task 3, Amy used the procedure of putting names on the bottom and finding new numbers for each name to draw a graph. These procedures worked very well on the problems involving nominal data but were not always successful on the problems involving numerical data. With the table including nominal data, she put all names (e.g., sports, fruits), found "new numbers" by counting "how many per each name" (the frequency), and drew a bar graph of the frequency coordinating each name (Problems 3 and 4). These procedures made her draw graphs correctly on Problems 3 and 4. However, with tables including numerical data (Problems 1 and 2), she seemed to be confused between the numbers in data and new numbers she needed to figure out as well as confused about which should go on the X axis. For example, on Problem 1, she was supposed to put the numbers in data (the number of brothers and sisters) on the X axis, new numbers (the frequency of each number of brothers and sisters) on the Y axis, and draw bars representing the coordinates of the number of brothers and sisters and its frequency. However, she placed the number of brothers and sisters on both X and Y axes, drew a bar for each student, and put the student name on each bar. Figure 4.27 shows her result. The following demonstrates how she thought out loud during Clinical Interview Task 3:

> Amy: What I like to do is put numbers at the bottom, so Fiona had 4 so I'll put her name. Who else…Heike. And I can see what I like to do is I like to color mine in, and I put 1, 2, 3, 4 and so basically I would color this way I would color 8 in but I'm not. Then we make all the rest? Okay, those are for the 4, now I'm going to do the ones that have 2 brothers and sisters. Angelina and Bob bother have 2 sisters, and looking over here Kelly does too. I'm going to make for the 3, and I'll put the first letter of their name on the box. Angela, Bob, Kelly. Then I have to make another number. Since I don't have any room, I'll just put the numbers down here.
>
> Now I think I'm going to do it with 3 brothers and sisters. And I see George has 3 brothers and sisters, Isabella has 3 brothers and sisters. So I put a G. and an I. And I put 3 at the bottom of both, making my squares, now I will do 1 brother or sister. Let's do 5. Jade is the only one that has 5 brothers and sisters. 1, 2, 3, 4, 5. Then I'll put 5 under that bar. Now I'll color, what else did I do? I see Cathy, David, Mary Anne and Noah with 1 brother and sister, and I'll put 1 under each square. Then with zero brothers and sisters, Erin and Luke, so I'll put L and E for that. So they have zero. And let me check. Everything is correct.
>
> Tester: How did you know how to make the bar graph?
>
> Amy: I knew how to make my bar graph whenever I was counting the numbers and then I saw the names…
>
> Tester: How do you know if your graph is correct?

Amy: Well my graph is correct because like I said I was looking at the numbers and then I saw the names beside it, what guided me also is putting the first letter of their name on top of their bar graph.

Figure 4.27. Amy's bar graph to Problem 1 in the baseline Clinical Interview 3.

On Clinical Interview Task 4, Amy used more diverse references than the other groups of students to make comparisons of two graphs. She compared two graphs in terms of range, relationships between variables embedded in the graphs, spread or clumping, and their bars.

> Amy: (Problem 1) Differences in these, going the number of students in both, one this side it goes up to 7 and this side it goes up to 5, and at the bottom on this one it only goes up to 6, and the bottom of this one goes up to 8. So this would have more brothers and sisters, but less people, this would have less brothers and sisters but more people. That's some of the differences. …
>
> Similarities they both have the number of brothers and sisters at the bottom and the number of students at the top. They're both bar graphs, they're both about the number of brothers and sisters not about dogs or anything. …
>
> Basically the first thing you do is you look at the graph, and the way I found it just these numbers help you on the side, number of students, and on the bottom it was easier for me. I looked at the side so it would get the number of students going that way. So if you look at the line above it…
>
> Amy: (Problem 3) The difference between them, this one has 6 and this one has 7, the difference because these, the second one, Ms. B's class, has 2 groups that don't have any

children filled and Mr. A has all of his class filled. And I see a whole lot more people like apple and banana then, well only banana, in Ms. B's class, and grapes. Then banana and grape…Um, and both names are at the bottom, number of students on the left side. They're both a classroom. That's all I can think of…

I knew how to find it because I used the lines this times, I didn't really just guess, I was counting the name of fruits and the number of people who chose those fruits and that's how I know to do Mr. A's class and same thing with Ms. B's class. Yeah, I did the same thing on both graphs.

Postinstruction Concepts or Procedures

A summary of the features of concepts or procedures used by the three groups of students with differing ability in solving problems on Clinical Interview Tasks 3 and 4 is presented in Table 4.19. As shown in Table 4.19, changes in concepts or procedures used for problem solving on Clinical Interview Tasks 3 and 4 were not noted much in the MD group. Especially on Clinical Interview Task 3, the group of students with MD did not show improvements or changes in their concepts or procedures that they used for problem solutions. For example, as at baseline performances, Tina showed difficulties in understanding the concept of a bar graph and the procedures to make a bar graph. When she was given the data table and asked to draw a bar graph, she just copied the table on the paper, at both baseline and postinstruction interviews. Likewise, changes were not noted between baseline and postinstruction interviews in Lee's concepts or procedures to draw a bar graph from a data table. She drew a bar of each data point by regarding the numbers shown on the table as the frequencies to draw. Kevin also showed the same concepts and the procedures in solving problems on Clinical Interview Task 3 at baseline and postinstruction interviews. At both interview periods, he showed the skills of successfully drawing bar graphs to the problems involving nominal variables but difficulty with problems involving numerical variables.

Table 4.19

Features of Concepts or Procedures Used by Three Groups of Students With Differing Ability to Solve Two Probability and Statistics Tasks

Task and time	Mathematics disabilities (Lee, Kevin, & Tina)	Struggling (Laura)	Typically achieving (Amy)
Task 3			
Baseline	Copying the data table shown to her (Tina) Counting the frequency of each category name and drawing the relationship between the categories and their frequency on nominal variable (Kevin) Taking the numerical information shown on the table as frequencies to draw on both types of problems (Lee) or problems involving numerical variables (Kevin)	Counting the frequency of each category name and drawing the relationship between the categories and their frequencies on the problems having nominal variables Changing the numerical information on the data set using an unreasonable and unsystematic formula on the problems having numerical variables	Counting the frequency of each category name and drawing the relationship between the categories and their frequencies on the problems having nominal variables Taking the numerical information shown on the table as frequencies to draw
Post	Copying the data table shown to her (Tina) Counting the frequency of each category name and drawing the relationship between the categories and their frequencies on the problems involving nominal data (Kevin) Taking the numerical information shown on the table as frequencies to draw on both types of problems (Lee) or on problems involving numerical variables (Kevin)	Counting the frequency of each category name and drawing the relationship between the two Counting mistakes	Counting the frequency of each category name and drawing the relationship between the categories and their frequencies on both problem types.

Task and time	Mathematics disabilities (Lee, Kevin, & Tina)	Struggling (Laura)	Typically achieving (Amy)
Task 4			
Baseline	Comparisons made based on one or two aspects of the graphs Mainly bar-by-bar comparisons made	Comparisons made based on one or two aspects of the graphs Mainly bar-by-by comparisons made Physical features of bars (e.g., color, length of bars) used for comparisons (Jose)	Comparisons made based on diverse aspects of the graphs Some comparisons made based on the relationships between variables embedded in the graphs
Post	Comparisons made based on one or two aspects of the graphs Mainly bar-by-bar comparisons made	Comparisons made based on one or two aspects of the graphs Mainly bar-by-bar and ranges comparisons Physical features of bars used for comparisons Relationships between variables used for comparisons	Comparisons made based on diverse aspects of the graphs (at least three per problem) Relationships between variables used for comparisons of two graphs

On Clinical Interview Task 4, changes were rarely found in the concepts or procedures of the three groups of students with differing abilities from baseline to postinstruction interviews. Even though the targeted skills had been taught in their class, students did not demonstrate changes in comparison criteria from baseline to postinstruction interviews. Like the typically achieving student, 1 MD student and 1 struggling student showed the skills of using a relationship between variables embedded in each graph for comparing two graphs at postinstruction interviews. However, it was not consistently observed throughout problem-solving by those 2 students on Clinical Interview

Task 4, whereas the typically achieving students consistently used the criteria for comparisons to all the problems postinstruction.

Students with MD. On Clinical Interview Task 3, the students with MD did not show the mastery of knowledge and skills of drawing a bar graph of data shown in a table. To draw a bar graph, Lee employed procedures including looking at the names, finding the numbers associated with the names, and drawing a bar graph of the coordinates of the names and the numbers (e.g., Kelly, X axis, and 56 inches, Y axis), which were the same procedures that she used at baseline interviews. She seemed not to understand that the names shown on the tables were not important variables and that she should make a bar graph of the coordinates of the frequency of each number (Problems 1 and 2) or each category (Problems 3 and 4). Using the procedures, she did not provide a correct answer to any of the problems on Clinical Interview Task 3 (see Figure 4.28).

> Lee: (Problem 1) First I'm going to count out 4 squares, then I'm going to draw a line.
> Tester: Why did you count the 4 squares?
> Lee: Because if they have 4 you need to count up 4. I'm going to put a F for Fiona and then skip and a line and put an H for Heike above my 4.
> Tester: What about the other students?
> Lee: They don't have 4.
> Tester: It's okay, display the whole data set. You just draw Fiona and Heike, I want you to draw every student.
> Lee: I'm going to draw 2 for Angela, 2 for Bobby, 1 for Cathy, 1 for Erin, 1 for David, none for Erin, 3 for George, 3 for Isabella, and 5 for Jade, and then 2 for Kelly, and then 1 for Luke, 0 for Luke, 1 for Maryanne, then 1 for Noah. Now I'm done.
> Tester: What are you going to put here and this side? Nothing? What about this side?
> Lee: Nothing.
> Tester: Okay, How did you know how to make the bar graph?
> Lee: I just looked at it, the name, and tell how many they have.

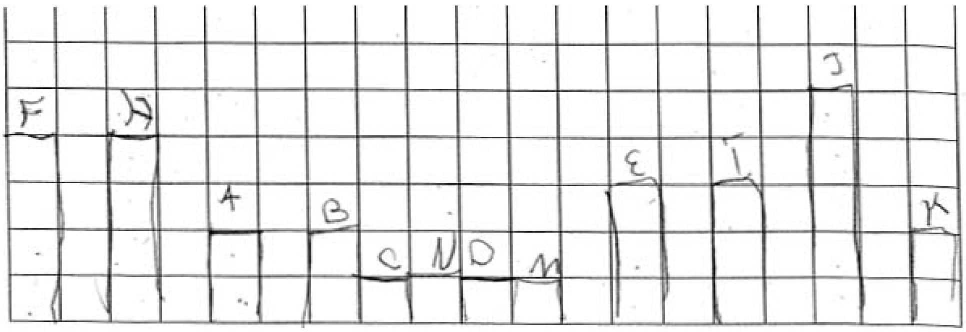


Figure 4.28. Lee's bar graph to Problem 1 in the postinstruction Clinical Interview 3.

Kevin employed different procedures according to the types of problems. On the problems involving numerical variables (Problems 1 and 2), he used the same procedures as Lee did to make bar graphs (e.g., Kelly on the X axis and 56 inches on the Y axis). Using these procedures, Kevin failed to provide correct graphs on both problems. However, on the problems involving nominal variables (Problems 3 and 4), he corrected his procedures so that his graphs represented the relationships between each categorical variable (e.g., fruit names or sports names) and its frequency. Using these procedures, Kevin produced correct graphs to Problems 3 and 4. Figure 4.29 illustrates how he drew a graph of numerical data (Problem 1), and Figure 4.30 shows how he drew a graph of nominal data (Problem 3).

> Kevin: (Problem 3.) (Mumbling letters] B have 2. C have 1. F have none. B have 1. G have…6. Eight K (More mumbled letters and numbers.]
> Tester: Basically you are putting student name at the bottom and drawing the number of their brothers and sisters?
> Kevin: (Mumbling]
> Tester: Why did you make your bar graph like this?

Kevin: Because this is how many brothers and sisters they have so I did it how the order it is in on the card I put it on the graph paper. …I looked at the people and the numbers and put them on the card.

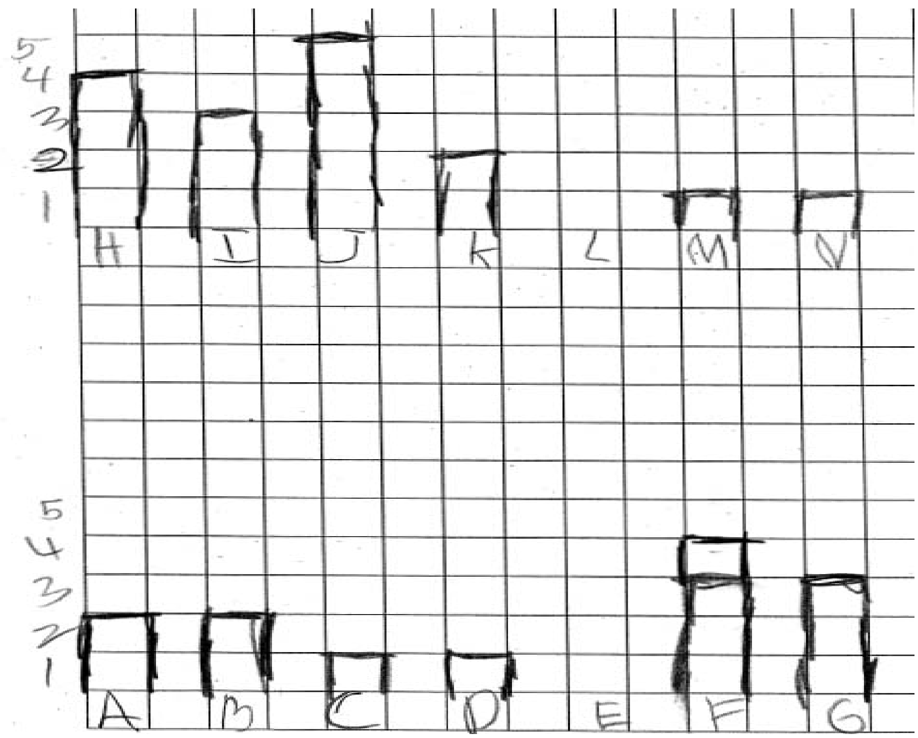


Figure 4.29. Kevin's bar graph to Problem 1 in the postinstruction Clinical Interview 3.

Kevin: (Problem 3) Apples, bananas, apples, oranges, peaches, grapes.
Tester: Why did you change it? First time you put student names, but why did you put fruits?
Kevin: Because I was going to write the numbers up to 5, then most of them are 3 counting how many apples on the first.
Tester: Why up to 5?
Kevin: Because there's no more than 3…apples 3, bananas 2, oranges 1, no, grape? (Mumbling]
Tester: Great job. How did you know how to make the bar graph of the data?
Kevin: By looking at the card and matching those with the number.
Tester: What numbers?
Kevin: The numbers of the side.
Tester: Where?

Kevin: …Three people had apples, 3 people like apples. But 2 people like bananas, 1 person like oranges, 1 person like peaches and nobody like grape. On the other set, 3 persons like apples, no people like bananas, 2 people like oranges, nobody like peaches, and 2 people like grapes.
Tester: How do you know if your graph is correct?
Kevin: By looking at the fruit and counting how many apples equal up with the numbers on the side.

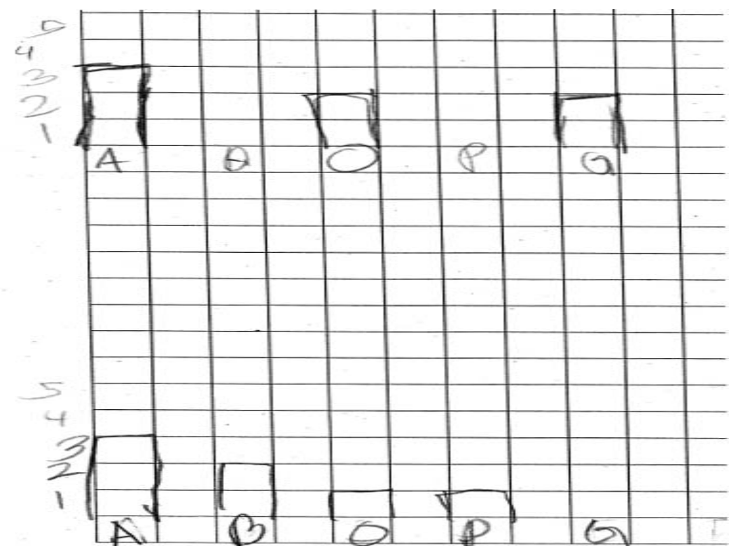


Figure 4.30. Kevin's bar graph to Problem 3 in the postinstruction Clinical Interview 3.

As at baseline interviews, Tina seemed not to understand what a bar graph was and what she should do to draw a bar graph, even after she received instruction on the skills being examined. When she was asked to draw a bar graph of data shown in a table, she looked at the table and copied it exactly (see Figure 4.31).

Tester: (Problem 2) How did you know how to make the bar graph?
Tina: I looked at the table.
Tester: And then?
Tina: Then I copied it on the paper.
Tester: How do you know if your graph is correct?
Tina: I match the table and the one that I did.

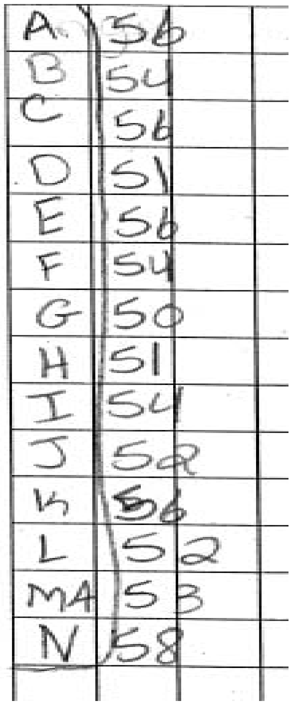


Figure 4.31. Tina's bar graph to Problem 2 in the postinstruction Clinical Interview 3.

On Clinical Interview Task 4, as at baseline interviews, all 3 students with MD tended to use one (Kevin and Lee) or two aspects (Tina) to compare two bar graphs. The 3 students with MD most frequently used bar-by-bar comparisons for finding differences and similarities between two graphs. Kevin made mainly bar-by-bar comparisons for most problems at postinstruction interviews as well as at baseline interviews. On Problem 1, however, he tried to compare two graphs in other aspects, including range and typical values.

> Kevin: (Problem 2) Fifty-three in Ms. B's class it had 1, and 53 in Mr. A class has 2. Fifty-four in A class has none and B class has 3. And 55 in B class has 3 and 55 in A class has 4. And 56 in B class has 2 and 56 in A class has 4. Seven in A class has none and 7 in B class has 5. And 58 in B class has none and A class has 1…Uh…52 and 51 is the same, they both have 1. That's it.

> Kevin: (Problem 4) Baseball and basketball. Baseball in A class has 5 and in B class has 1. Basketball in A class has 2 and B class have 4. Swimming in B class has 5 and swimming in A class has 2. Soccer in B class have 4, soccer in A class have 2. Skiing in B class has 5 and skiing in A class has 1. Gymnast in B class has none and gymnast in A class has 1.

Even though Tina made comparisons of two graphs in some different aspects including ranges, spread or clumping, bar-by-bar comparisons, and relationships or variables embedded in the graphs at baseline interviews, her comparisons at postinstruction interviews were mostly based on range comparisons and bar-by-bar comparisons of two graphs.

> Tina: (Problem 1) A's class there's no 4 and B's class there is a 4. Um, A's class on the side it only goes up to 7 and on bottom it goes up to 6, B's class the side goes up to 5 and the bottom goes up to 3.
> Tester: Do you see any similarities between them? No? How did you know how to find the differences and similarities between the 2 graphs?
> Tina: Because I looked at A's class and then I look at B's class
> Tester: Then? When you look at A's and B's class, what did you look at specifically?
> Tina: I look how many brothers and sisters they have.
> Tester: Okay, and how do you know if you found the differences and similarities correctly?
> Tina: Well I looked at one, then the other and look both again.
>
> Tina: (Problem 3) A's class has some that chose orange and peach, but B's class nobody chose orange or peach. …A's class the side only goes up to 7, and B's class goes up to 6.

At baseline interviews, Lee made bar-by-bar comparisons for most problems on Clinical Interview Task 4 and presented incorrect answers when she used other aspects as criteria. However, after receiving instruction on the skills, she used only spread or clumping as the criterion of comparing two graphs. She made only one comparison as an answer to each problem. The following were Lee's responses to the problems on Clinical Interview Task 4: For Problem 2, she said, “Mr. A’s class has a lot of tall children, but Ms. B’s class does not have a lot of tall children.” For Problem 3, she said, “Mr. A’s class has one larger bar, and Ms. B’s class has two larger bars.” For Problem 4, she said, “Mr. A’s class has more bars than Ms.B’s class.”

Interestingly, all the students with MD did not remember what they were taught about in their classroom. They did not even think that they were taught about the skills during their mathematics classes.

Struggling students. On Clinical Interview Task 3, Laura, the struggling student, showed improvements in drawing bar graphs of data involving numerical variables or nominal variables from the baseline to the postinstruction interview. At the baseline interview, she was not able to draw a bar graph of data involving numerical variables on Problems 1 and 2, but at the postinstruction interview, she showed the skills of drawing a bar graph of data involving numerical variables on Problem 2. At the baseline interview, she applied an unreasonable and unsystematic method to transfer the big numbers shown on the table to smaller numbers to draw a bar graph on Problem 2. However, when she was drawing a bar graph of the number of students having specific heights on Problem 2 at the postinstruction interview, she put heights on the X axis, put the number of students on the Y axis, counted the number of students who were 56 inches tall (three students), colored three boxes on the column of 56 inches tall, and so on. She showed the skills of drawing a bar graph of data involving numerical variables by following these correct procedures at the postinstruction interview.

> Laura: (Problem 2) I see there's one 56, two 56, and three 56. I could color three. I'm looking at the height not students' names. …I see 54, Fiona has 54, Isabella…that's three people who have 54 as their height. Then I do 51 there's only two 51 so I shade two. I see only one 50. I see only one 58.
> Tester: How did you know how to make the bar graph of the data?
> Laura: I look at the height I started from the top going down, but sometimes they have 56 right here and right here, I already shaded them so I skip them and do the next number.
> Tester: How do you know if your graph is correct?
> Laura: I could look over it and I could try to write their name and then I could do their height copy what they do on their paper and do it again.

Counting mistakes resulted in her failure in Problem 1 involving numerical variables at the postinstruction interview. When she was drawing a bar graph of the number of students having two brothers and sisters, she counted the total number of brothers and sisters (2 each × 3 students = 6) instead of counting the number of students (3 students) having two brothers and sisters (see Figure 4.32).

> Laura: (Problem 1) I'm looking at the graph and I see 3 people have 2 brothers and sister, so I'm going to add 2 plus 2 is 4. Four plus 2 is 8.
> Tester: Can you say that again?
> Laura: Kelly has 2, so 4, 5, 6, so I'm going to color 6. That's 6. Next I'm going to look at Cathy, she has 1, and I see 2 that has 1 brother and sister so there's 4 people that have 1 brother and sister, so I'm going to shade it in 4. Next I'm going to do zero and I saw 2 zero so I'm just going to shade a 2. So then I see 4, the next color, Heike, Fiona, that's 8 because 4 plus 4 is 8. Then I see 3, Isabella and George has 3, so 3 plus 2, add them and then that's it. I see only 1 5, it's Jake. And that's it.
> Tester: Who was 5? Jake? How did you know how to make the bar graph of the data?
> Laura: Uh, I just add. Then I equaled it.
> Tester: What did you add?

Laura: One plus 1 is 2.

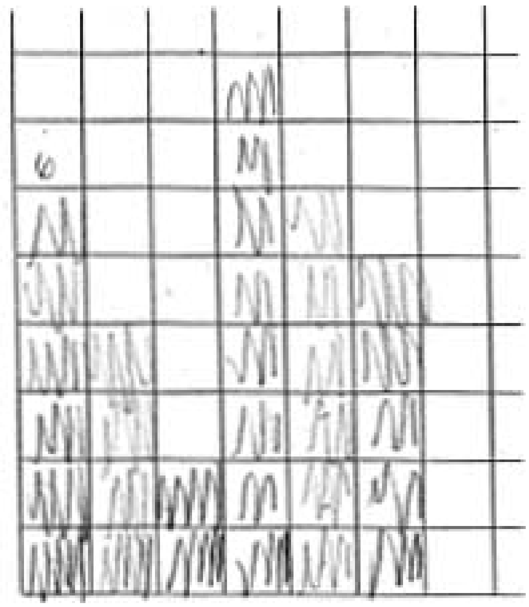


Figure 4.32. Laura's bar graph to Problem 1 in the postinstruction Clinical Interview 3.

However, as on the problems involving numerical variables, Laura showed the skills of drawing a bar graph of data involving nominal variables at the postinstruction interview. Her failure on Problem 3 was caused by counting mistakes (e.g., she counted the number of people who chose an apple as 5 instead of 6). On Problem 4, she applied the procedures correctly and produced a correct graph.

> Laura: (Problem 4) I'm going to start from the top. Baseball is 1, 2, 3. Three people who like baseball. Only 2 soccer, so now I do soccer because there's 2 people who like it. Basketball, there's 2, 3, 4, only 4. One, 2, 3, 4… swimming I see 2. There's only 2. So I'm skipping baseball and doing skating. Isabella and…2 people who like skating. Then now I'm on, I'm skipping Heike, Isabella, and Jade because I already did them. I'm on Kelly gymnastics.
> Tester: How did you know how to make the bar graph of the data?
> Linda: I look at the column. The favorite sport column, then I start from the top go down and I see 2, I see 3, so I shade 3.

On Clinical Interview Task 4, the 2 struggling students performed very differently. At baseline interviews, Jose used two aspects to compare two graphs: ranges and bar-by-bar features. After receiving instruction on the skills, he used the relationships or variables shown in the graphs as well as ranges and bar-by-bar features to make comparisons of two graphs. He used more diverse criteria for comparisons than Laura and all 3 students with MD. It was also notable that he used physical features of graphs for comparisons on all problems at both interview periods, and he was the student who most frequently used the physical features for comparisons.

> Jose: (Problem 1) The bar graph is skinner than the other graph, and this has more bars than this one, this graph have like only 5 and this one have like 8. The second graph they're talking about B's class, the first, they're talking about A's class.
> Tester: Any similarities?
> Jose: They both have bars. Talks about the number of brothers and sisters and more than 2 it's talking about the same number of brothers and sisters.
> Tester: What else?
> Jose: Um they have…they're talking about classes they're saying classes.
> Tester: How did you know to find the difference and similarities between the two graphs?
> Jose: I look at these and I compare them its like smallest to biggest compare them look at two of them.
>
> Jose: (Problem 3) A's class has more bars than B's class.
> Tester: What else?
> Jose: Peach and orange doesn't have any fruits that they like the most but peach and orange in A's class has more than in B's class. Seven, they start with the number 7 in A's class and B's they start with only 6. The graph of B's class is lighter than the graph of A's class.
> Tester: Can you tell me the similarities?
> Jose: Both talking about name of fruits and number of students. Bars of A's and B's has the same size. Both are talking about choices of fruits. They're talking about classes.
> Tester: How did you know to find the difference and similarities between the two graphs?
> Jose: I look at this one and I look at the other one. I compare it; I look at this one first second that one.

Compared to her performance on baseline interviews, Laura's postinstruction performance on Clinical Interview Task 4 was different in two aspects: (a) the number of criteria for comparisons and (b) the number of comparisons made on each problem. Laura used patterns and bar-by-bar features for comparisons of two graphs at baseline interviews. However, she made only bar-by-bar comparisons on all problems at postinstruction interviews. Also, she made more comparisons at postinstruction interviews than at baseline interviews, even though they all were based on bar-by-bar comparisons. The following illustrates her answers to the problems on Clinical Interview Task 4.

> Laura: (Problem 1) Well, in Ms. B's class is 4 and 3 in Mr. A's class goes to 6. Three in A class they have more students and 3 in B class has only a little students. I see 5 is short, it's small in B's class, in A's class 5 they have 3 so it's more than the 5 in B's class.
> Tester: What else? Any similarities between them?
> Laura: I see 5, 6, and 7 they have 1 in B's class and 6 in A's class has 1 too. And B's class 2 and 4 have 2 and that's the same. That's what I see.
>
> Laura: (Problem 3) Peach in B's class has zero and A's class has 1, so there are students that like peach. Grape has 5 in B's class and A's class it only has 2. So there are lots of people who like grapes. Banana has 5 in A's class, A's class banana has 2. Orange has zero in B's class and has only 3 in A's class.
> Tester: Can you tell me the similarities?
> Laura: Apple banana in B's class has 5. And grape and banana has 2.
> Tester: How did you know how to find the differences and similarities between the two graphs?
> Laura: I just look at the number of students right here and just follow it and I could see the difference and the similarities.

Typically achieving student. On Clinical Interview Task 3, the typically achieving student, Amy, showed improved skills of drawing a bar graph of data shown in tables between baseline interviews and postinstruction interviews. At baseline, she was able to correctly draw a bar graph of only nominal data. However, she was able to correctly draw a bar graph of numerical as well as nominal data at postinstruction interviews. She presented correct graphs for all four problems on Clinical Interview Task 3. It should be noted that Amy was the only student who provided a correct graph to Problem 1, which was exactly the same as the teacher's example during her instruction. At baseline interviews on Problem 1, she showed confusion about the "numbers" that should be on the Y axis, because the data already had a variable named as "the number of brothers and sisters." For example, even though she was supposed to put "the numbers of brothers and sisters" in data on the X axis and the frequency of students who have specific brothers and sisters on the Y axis, she placed "the numbers of brothers and sisters" on both X and Y axes at baseline interviews. However, after receiving instruction on the skills in class, she was able to make the bar graph of Problem 1 correctly (Figure 4.33). The following illustrates her thinking out loud while she was working on the Problems 1 and Problem 2. Figures 4.33 and 4.34 illustrate her outputs to the problems at postinstruction interviews.

> Amy: (Problem 1) So the highest number is 5. Then on this side I guess I'll put whatever is on that side, I put the number of students. So how many people have 1. Cathy, David, Maryanne, and Noah. And so Angela, Bob, and Luke have 2, 3 George and Isabella, 4 Fiona and Heike, 5 Jade. This is the number of brothers and sisters and classmates.

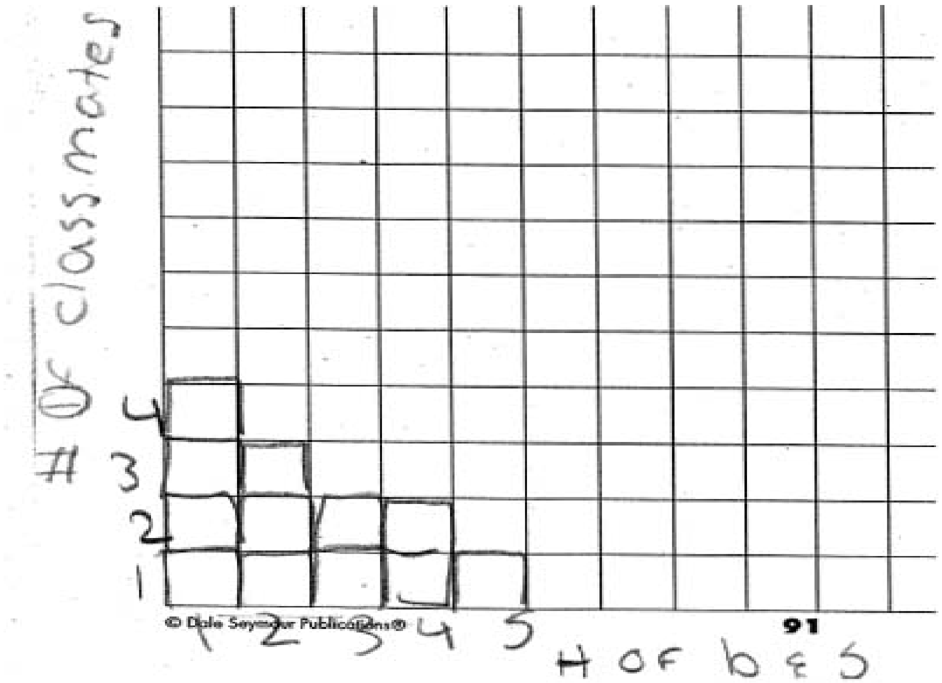


Figure 4.33. Amy's bar graph to Problem 1 in the postinstruction Clinical Interview 3.

Amy: (Problem 2) Let's start from the inches, so the highest number is 50 and the lowest number is 50 (counting) 58. Angelina is 56, Bob is 54, Cathy also 56, David 51, Erin 56, Fiona's 54, and George is 50, Heike is 51, Isabella 54, Jade is 52, Kelly is 56, Luke's 52, Maryanne's 53, and Noah's 57.

Tester: How did you know how to make the bar graph?

Amy: I did it differently this time. I put the numbers at the bottom, I looked at height instead of names first, I knew to put the height at the bottom.

Tester: Tell me what should go here, title or name? What does it represent? What is it?

Amy: These are the number of students and this is the height.

Tester: How do you know if your graph is correct?

Amy: I knew it was correct. I was following by my finger I was looking at the height so I wouldn't think it's a different number. I didn't rethink it or anything, I just didn't mess up.

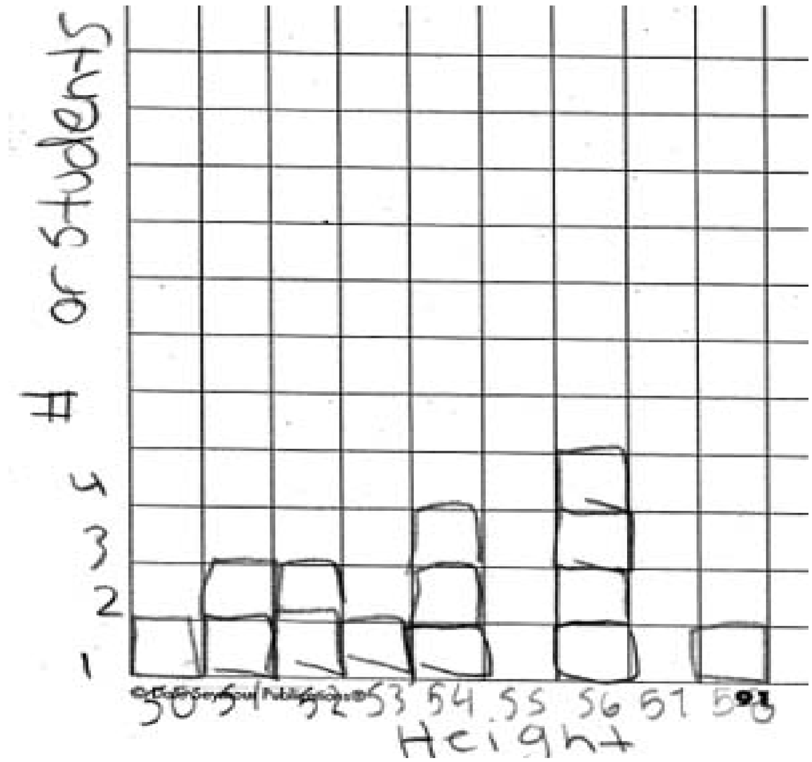


Figure 4.34. Amy's bar graph to Problem 2 in the postinstruction Clinical Interview 3.

On Clinical Interview Task 4, Amy was the only student who used multiple criteria for comparing two graphs at both baseline and postinstruction interviews. Particularly at postinstruction, she used at least three comparison criteria to answer to each problem and used all criteria for her comparisons, except patterns. Whereas the other groups of students made multiple bar-by-bar comparisons to answer most problems, Amy made a single bar-by-bar comparison of two graphs only on the problems involving nominal variables. The following were her answers to Problem 1, involving numerical variables, and Problem 2, involving nominal variables.

> Amy: (Problem 1) Well, outside of number of students, there's 5 in Ms. B's class and 7 in Mr. A's class. In Ms. B's class, number of students goes up to 8 and Mr. A's only 6. So Mr. A's class, well, because you know that there's going to be more bars on Ms. B's class, so

there's more bars on Ms. B's class and less bars on Mr. A's class because there are less numbers on the bottom for brothers and sisters. And they other difference is just instead of being 2 typical numbers there's 4 typical numbers on here.
Tester: What does typical mean?
Amy: One student had 5, 1 student 6, 1 student had 7, 1 student had 8. There's 2 that had 2, but there's more of 1 person.
Tester: What about similarities?
Amy: They both have the number of brothers and sisters at the bottom and students on the side. They both have a typical number of students. Its kind of similar in this was because we have 3 right here and 2…never mind, it kind of has the same number of bars but there's more numbers right here, so it has some similar numbers of students that have those brothers and sisters. And that's it.
Tester: How did you know how to find the differences and the similarities in the two graphs?
Amy: Well I knew how to find them because it's a graph, so you can just look over and there's a number and basically looking with your eyes, because these numbers down here help you, and you also can kind of figure out the number of lines, you can count the lines and figure out which bar you have. Each bar represents 1, basically.

Amy: (Problem 4) There's zero in gymnastics in Ms. B's class and 1 in gymnastics in Mr. A's class, their numbers are different. Yeah, they're just different. Well they are different classes and that's it.
Tester: Then do you see any similarities between them?
Amy: They both have 6 students, and they both have the same sports. They both have name of sport at the bottom and number of students on left side. Both bar graphs. And the more students you have the smaller your lines and get, and they have the same length of lines because of same number of students.
Tester: Okay, can you tell me how do you know if you found the differences and similarities correctly?
Amy: I didn't use the imaginary pencil, I used what the numbers are for and I used the numbers first and then used the bottom, that's my strategy. It would have been helpful to use the imaginary pencil but I didn't want to…I used the numbers. The reason they have these is so you can use them, so I used that and I kind of used the lines too. Basically I followed the lines until I hit that number.

Transfer of Problem Solutions to Problems With Different Similarity to the Original Problems

Transfer refers to the application of prior knowledge and skills acquired in one situation to new situations (Singley & Anderson, 1989). In this study, Clinical Interview Task 3 was designed to include three types of problems that had different similarity to the original problems used for class instruction, in order to analyze the transfer of knowledge and skills on probability and statistics by the students with different ability. The base problem (Problem 1 on the task) was exactly the same as the original problem used for class instruction in terms of the problem structure (e.g., the relationships or variables embedded in data and problem solutions) and the surface features of problems (e.g., context or physical features of data). The base problem on Clinical Interview Task 3 was used to examine the students' mastery of the skills of drawing a bar graph of data involving numerical variables shown in a table, as taught in their class. The other two types of problems were target problems in which the students needed to apply the skills taught in class to complete. The near-transfer problem (Problem 2) was the same as the original problems in terms of the problem structure (e.g., the type of variables and problem solutions) but different from the original problem in terms of the surface features such as context (e.g., the number of brothers and sisters vs. student heights). The far-transfer problems (Problems 3 and 4) were different from the original problem used to teach the solving procedures in both the problem structure and the surface features. To solve the far-transfer problem, the students needed to modify or transform the problem-solving procedures learned from their class. For example, on Clinical Interview Task 3, the students should count the number of students who chose a specific fruit instead of counting the number of students who had a specific number (heights or the number of brothers and sisters).

Research on transfer of knowledge and skills has procedures of ensuring students' acquisition of knowledge and skills in one situation before investigating transfer of the knowledge and skills to different situations (Bassok, 1997). Among all students, it was necessary to filter the students who performed correctly on Problem 1 after receiving instruction on the skill for the analysis of transfer of probability and statistics knowledge and skills of the students with differing ability. However, except the typically achieving student, none of the students passed the criteria to be included in the analysis of knowledge transfer in probability and statistics. Accordingly, it was not possible to make comparisons of knowledge transfer in the area of probability and statistics among the groups of students with different ability.

Emerging Themes

This study examined a fourth-grade teacher's instructional adaptations for 3 students with MD and the mathematics learning of 6 fourth-grade students with differing ability (3 identified MD, 2 struggling, and 1 typically achieving) in a standards-based mathematics, general education classrooms. These topics were examined when two mathematics topics were taught: geometry and spatial reasoning, and probability and statistics. Specifically, the teacher's instructional adaptations were explored in terms of (a) frequency and contexts associated with the occurrence of instructional adaptations, (b) categories of instructional adaptations, (c) use of evidence-based mathematics instructional components, and (d) instructional adaptations addressing student difficulties in prerequisite skills required for learning each mathematics content. The students' mathematics learning was explored in terms of changes in (a) prerequisite skills, (b) accuracy of problem solutions, (c) concepts or procedures used for problem solutions, and (d) application or transfer of knowledge and skills to new problems after receiving mathematics instruction on the targeted skills in the standards-based mathematics, general education classroom. Several overarching themes emerged from case analyses of data on the teacher's instructional adaptations and the mathematics learning of students with differing ability in the standards-based mathematics, general education classroom across two different mathematics topics. The data were reviewed again to confirm the themes.

Theme 1: The Teacher Varied Instructional Adaptations by Student

The first theme was that the teacher made different number of instructional adaptations across 3 individual students with MD during her standards-based mathematics instruction. She made the largest number of adaptations for the student whom she rated as most struggling in the prerequisite skills relating to the mathematics content.

The total frequency of adapted instruction provided to individual students with MD throughout lessons was quite different across students during instruction on both geometry and spatial reasoning, and probability and statistics. The teacher also provided adapted instruction for the student whom she rated as most struggling with the prerequisite skills relating to each mathematics content, more frequently than for the other students with MD. During the lessons on geometry and spatial reasoning, the teacher made 12 instructional adaptations for Lee, the student rated as most struggling, and only 2 or 3 adapted instructions for Kevin or Tina throughout three lessons on this mathematics content.

Likewise, during instruction on probability and statistics, the teacher provided the larger number of instructional adaptations for Kevin, who was rated as most struggling with the prerequisite skills for that content area. She provided 12 instructional adaptations for Kevin, compares to 2 or 3 for Lee and Tina.

Thus, even though the teacher recognized the difficulties of the other 2 students with MD (Kevin and Tina on the prerequisite skills relating to geometry and spatial reasoning, and Lee and Tina on the prerequisite skills relating to probability and statistics) in the prerequisite skills at interviews or on survey questionnaire, it was rarely observed that she provided adapted instruction for those students during instruction on each content. The findings emerged from data of this study indicated that the frequency of instructional adaptations that the teacher provided in her class might be related to the teacher's perceptions on the comparative level of difficulty of a student with the prerequisite skills within the group of students with MD, not to the existence of difficulty of a student in the skills.

Theme 2: More Adaptations Were Made in Delivery of Instruction Than in the Other Categories

More adaptations were made in terms of delivery of instruction than in the other categories of instructional adaptation. The data from observations of the teacher's instructions on both areas (geometry and spatial reasoning as well as probability and statistics) revealed that the teacher made more adaptations in delivery of instruction for the students with MD than adaptations in the other categories, including instructional content, instructional activity, and instructional materials or technology.

For example, during the lessons on geometry and spatial reasoning, the teacher modified her instruction in terms of delivery of instruction approximately seven times (7.3) for each student with MD across three lessons, whereas she changed her instruction none in instructional content, approximately twice (1.7) in instructional activity, and once in instructional materials or technology. Similarly, during the lessons on probability and statistics, the teacher modified her instruction approximately 11 times (11.3), averaged for each individual student in terms of delivery of instruction, whereas she made no changes in instructional content and in instructional materials or technology, and approximately two modifications in instructional activity across five lessons on this topic.

Five types of adaptations were commonly found in instructions on both geometry and spatial reasoning as well as probability and statistics relating to the category of delivery of instruction. These types of adaptations included instruction that (a) explicitly provided teacher examples corresponding directly to the specified learning objective, (b) provided practice opportunities, (c) provided prompting, (d) controlled task difficulties, (e) provided group instruction, and (f) monitored students' understanding or performance.

First, in relation to adaptations that provided the teacher example, the teacher changed instructional examples by using concrete and meaningful examples. For instance, she used Kevin's cube building or multiple examples to show students how to position two buildings so that they could be matched during a lesson on geometry and spatial reasoning. Likewise, during Lesson 5, the teacher provided a teacher example of family size in class to show how to make a bar graph from the T-chart.

The teacher also adapted practice opportunities by changing the number of items, extending the time allotted for each practice, or providing repeated reviews. In addition, the teacher provided corrective feedback to student's incorrect performances or misunderstanding.

Next, the teacher also tried to modify the level of task difficulty by segmenting the tasks into smaller parts or by prompting the students to derive the correct answers. During a lesson on probability and statistics, when Kevin showed difficulty calculating 15 times 4, the teacher prompted him to use segmented procedures (15 times 2, then times 2). Group instruction was also an adaptation that commonly occurred during the teacher's mathematics instruction. During instruction on geometry and spatial reasoning, the teacher paired each student with MD with a high- or average-achieving student in two out of three lessons on this topic. During instruction on probability and statistics, the teacher purposively paired individual students with MD with highly achieving students for four lessons.

Theme 3: The teacher Supplemented Standards-Based Mathematics Instruction With Evidence-Based Mathematics Instructional Components for Students with MD.

The teacher adapted her instruction for the students with MD by supplementing standards-based mathematics instruction with evidence-based mathematics instructional components with more variety; she utilized prompting, direct questioning, and group instruction more frequently than the other components. The teacher observed in this study employed instructional components that are known to be effective for teaching students with MD when she adapted her standards-based mathematics instruction during lessons in both areas (geometry and spatial reasoning, and probability and statistics). In facts, some evidence-based mathematics instructional components were observed in her typical standards-based mathematics instruction for all students as well as in her adapted instruction. For example, instructional components of (a) direct questioning, (b) review of prerequisite skills, (c) group instruction, (d) strategy instruction, (e) progress monitoring, (f) use of manipulatives, (g) use of teaching examples, and (h) practice opportunity were observed during typical standards-based instruction for all students as well as in instructional adaptations for the students with MD during the lessons on geometry and spatial reasoning. In the same line, evidence-based instructional components of (a) group instruction, (b) modeling, (c) teacher examples, (d) direct questioning, and (e) review of prerequisite skills were used for both teaching all students in standards-based mathematics classroom and adapting her instruction for the students with MD during the lessons on probability and statistics.

However, compared to the variety of evidence-based instructional components used in her typical standards-based instruction for all students, the teacher attempted to use more varieties of evidence-based instructional components when she adapted her mathematics instruction for the students with MD. For example, during instruction on geometry and spatial reasoning, the teacher used (a) prompting, (b) control difficulty, and (c) group instruction only for adapting her instruction for the students with MD, not for teaching all students in her class. Similarly, the teacher utilized (a) prompting, (b) explicit explanations, (c) control difficulty, (d) progress monitoring, (e) practice opportunity, and (f) reteaching only to adjust her instruction for the students with MD, not to teach all students in her class during instruction on probability and statistics.

Commonly, six evidence-based mathematics instructional components were observed in the teacher's instructional adaptations across lessons on both geometry and spatial reasoning and on probability and statistics. The teacher employed (a) prompting, (b) control difficulty, (c) direct questioning, (d) review of prerequisite skills, (f) group instruction, and (g) teacher examples to adapt her instruction for the students with MD across the two mathematics content areas.

Of these components, prompting and direct questioning were most frequently observed in her adaptations during instruction on both mathematics content areas. Different from expectations based on interview data, Ashley rarely adjusted her instruction in terms of manipulatives and guided or independent practices for the students with MD. In addition, she was not consistently observed using explicit modeling or explicit explanations of a skill in adapting her instruction for the students with MD. She used these two components in adapting her instruction on probability and statistics but did not use them for adapting instruction on geometry and spatial reasoning.

Theme 4: Instructional Adaptations Were Restrictively Implemented

The teacher's instructional adaptations were restrictively implemented in terms of the number of student whose difficulties were addressed and the number of difficulties in prerequisite skills which were tackled through instructional adaptations. Across lessons on both topics, the teacher was consistently observed addressing student difficulties in the prerequisite skills relating to geometry and spatial reasoning, or probability and statistics, through interactions with students with MD. While she was teaching geometry and spatial reasoning, the teacher provided reviews of geometry vocabulary, which was one of difficulties that the students with MD showed relating to geometry prerequisite skills. Likewise, during instruction on probability and statistics, the teacher provided a review of the prerequisite skill of transferring a probability into fractions, through interactions with a student with MD. Although all students with MD showed difficulties in multiple prerequisite skills, adaptations addressing their difficulties in other prerequisite skills were not implemented while she was teaching.

Theme 5: Students With Differing Ability Showed Improvement on Prerequisite Skills Postinstruction

The 6 students with differing ability showed improvement on prerequisite skills proximal to the knowledge and skills being taught after receiving instruction in standards-based mathematics general education. After receiving instructions on two different geometry and spatial reasoning skills and two probability and statistics skills in the standards-based mathematics, general education classroom, the 6 students, including 3 students with MD, performed better on problems about prerequisite skills proximally relating to the target skills being taught. For example, compared to baseline performances, the 6 students on average showed over 20% of improvement in accuracy in solving the problems on both the prerequisite skills relating geometry and spatial reasoning skills after receiving instruction on the skills in the standards-based mathematics, general education classroom. Likewise, except the typically achieving student, 3 students with MD and 2 struggling students showed more than 10% of improvement in accuracy in solving the problems on both prerequisite skills relating to probability and statistics from baseline to postinstruction interviews.

Also, improvement in the prerequisite skills was more likely to be observed when the prerequisite skills were proximal to or directly related to the targeted skills being taught. Both prerequisite skills on geometry and spatial reasoning (counting the number of cubes in 2-D drawings and recognizing and naming the shapes) were more directly related to the targeted skills being taught (making 3-D cube buildings based on 2-D configuration and finding shapes that would make specific silhouettes). Therefore, these prerequisite skills were more likely to be reviewed through the lessons on the targeted geometry skills than were the other prerequisite skills. So, instruction on the targeted geometry skills resulted in improvement in the prerequisite skills as well as the targeted skills.

Similarly, one of the prerequisite skills on probability and statistics (reading or interpreting information from a T-chart) was directly related to the targeted statistics skills (drawing a bar graph based on a T-chart). So, instruction on the targeted skills brought improvements in the prerequisite skills as well as the targeted skills. However, the other prerequisite skill on probability and statistics (finding the typical number on a bar graph) seemed not to be related as directly to the targeted skills (comparing two bar graphs). Accordingly, the typically achieving student's skills on one of prerequisite skills (finding a typical number on a bar graph) relating to probability and statistics declined from baseline to postinstruction interviews, even though she showed better performances on the targeted skills after receiving instruction.

Theme 6: The Students With MD and The Struggling Students did not Perform as Well as the Typically Achieving Student

The students with MD and the struggling students did not perform as well as the typically achieving student on the tasks about specific mathematics knowledge and skills after receiving instruction in the standards-based mathematics, general education classroom. This theme emerged from an analysis in which the three groups of students with differing ability were compared in terms of changes in problem-solution accuracy and concepts or procedures used in problem solutions after receiving mathematics instruction in the standards-based mathematics, general education classroom. According to the findings on problem-solving accuracy in two geometry and spatial reasoning clinical interview tasks, all three groups of students showed improvement in accuracy using both targeted skills between baseline and postinstruction interviews. Additionally, the group of students with MD showed the largest improvement in accuracy on both tasks.

From the students' performances on the two geometry and spatial reasoning clinical interview tasks, it was also noted that three groups of students changed their concepts or procedures to solve problems on both tasks after receiving instruction on the skills in the standards-based mathematics classroom. For example, on Clinical Interview Task 1, students with MD showed consideration of the number of cubes when they were making a building, which was not found in their performances at baseline, even though those considerations did not lead them to get the correct answers all the time. The group of struggling students considered the positions of sections of a building as well as the number of cubes in each section at postinstruction interviews, whereas they considered only the number of cubes in each section at baseline interviews. As well, a struggling student used a strategy for recognizing and remembering the positions of sections, which was taught in class. The typically achieving student did not show large changes in concepts or procedures because she already knew and applied the desirable concepts or procedures at her baseline performance.

On Clinical Interview Task 2, the group of students with MD considered the sides as well as the bottom of a building to find a solid that would make a specific silhouette, and they explored multiple answers after they found one solution after they received classroom instruction on the skills. This group had considered only the bottom of a solid to find the silhouette and did not explore multiple answers at baseline interviews. The changes in performance of the group of struggling students and that of the typically achieving student between baseline and postinstruction interviews included (a) considering the sides as well as the number of cubes for building creation and (b) using mental matching of their building to the 2-D figure on the card instead of directly matching their building to the card. The group of struggling students did not explore multiple answers at postinstruction, as at baseline interviews. However, the typically achieving student tried to explore multiple answers at both baseline and postinstruction interviews.

However, it should be noted that even though the group of students with MD and the group of struggling students showed improvements in problem-solving accuracy and concepts or procedures after receiving instruction on the skills in fourth-grade geometry and spatial reasoning, their performance was not comparable to that of the typically achieving student. For example, compared to the postinstruction accuracy of the typically achieving student, 75.0% on Clinical Interview Task 1 and 87.5% on Clinical Interview Task 2, the accuracy of the students with MD stayed below 60.0%, even after the instructions on the targeted skills. In addition, they did not even remember that they were taught about the mathematics knowledge and skills in their class a day before. The typically achieving student was the only student who remembered and attempted to use what she was taught in her class, including concepts or procedures, across all clinical interviews.

As in the skills of geometry and spatial reasoning, the group of students with MD and the group of struggling students did not perform as well as the typically achieving student on both probability and statistics clinical interview tasks at postinstruction interviews. For instance, compared to the typically achieving student's postinstruction accuracy on both tasks on probability and statistics (100.0% on both tasks), the students with MD and the struggling students performed at 16.7 % and 50.0% of accuracy on Clinical Interview Task 3, respectively, and at 58.3% and 62.5% of accuracy on Clinical Interview Task 4 after receiving standards-based mathematics instruction on the skills. Moreover, after receiving class instruction, the students with MD and the struggling students did not perform better on both probability and statistics tasks than before they were taught about the skills. Their postinstruction, problem-solving accuracy stayed at the similar level to the baseline accuracy or was degraded (e.g., the struggling students' performances on Clinical Interview Task 4 degraded from 68.5% to 62.5% between baseline and postinstruction interviews).

Likewise, the group of MD students and the group of struggling students did not use concepts or procedures for problem solving similar to what the typically achieving student used. For example, the typically achieving student showed improvements in problem-solution accuracy by applying the concepts or procedures that were taught in class to the problems on probability and statistics clinical interview tasks. For example, at baseline on Problem 1 on Clinical Interview Task 3, Amy showed confusion about the "numbers" that should be on the Y axis, because the data already had a variable named "the number of brothers and sisters." For example, even though she was supposed to put "the numbers of brothers and sisters" in data on the X axis and the "frequency of students who have a specific number of brothers and sisters" on the Y axis, she placed "the numbers of brothers and sisters" on both X and Y axes at baseline interviews. However, after receiving instruction on the skills in class, she was able to make the bar graph of Problem 1 correctly by using the same procedures that her teacher used to draw a bar graph in class. It also should be noted that Amy was the only student who provided a correct graph to the problem on Clinical Interview Task 3, which was exactly the same as the teacher's example during her instruction.

However, the students with MD did not perform as well as the typically achieving student after receiving instruction on the skills. For example, a student with MD (Tina) showed the same difficulties at the postinstruction interview as at baseline in understanding the concept of a bar graph and the procedures to make a bar graph. When given the data table and asked to draw a bar graph, she just copied the table on the paper, as at baseline interviews.

Likewise, changes were not noted in the other 2 students with MD. Lee drew a bar of each data point by regarding the numbers shown on the table as the frequencies to draw. At both interview periods, Kevin showed the skills of successfully drawing bar graphs to the problems involving nominal variables but difficulty in the skills of drawing bar graphs to the problems involving numerical variables. From those findings, a theme emerged that the group of students with MD and the group of struggling students did not perform as well as the typically achieving student on the tasks about specific mathematics knowledge and skills after receiving instruction on the skills in the standards-based mathematics classroom.

Theme 7: Transfer of Mathematics Knowledge and Skills Differed Across Students With Differing Ability

Transfer of mathematics knowledge and skills was different across the three groups of students with differing ability when solving a new problem required their own modifications of the problem-solving procedures taught in class. The findings of this study on transfer of mathematics knowledge and skills by three different groups of students (MD, struggling, and typically achieving) indicated that they were not different in solving new problems that could be solved by using the same procedures taught in their class (near-transfer problem). After the students were taught about how to make 3-D buildings with cubes shown in 2-D drawings during a lesson on geometry and spatial reasoning, all 3 students were able to solve a new problem by using the procedures of creating single-layered sections of the building and combining them in the way that the sections of the 2-D building on the card were positioned, as was taught in their mathematics class. Commonly, they started from making sections (front, middle, or bottom) according to the number of cubes in each section. Then, they matched their building to the picture to verify their answers. Once they mastered the solutions, the students, including a student with MD, seemed to be able to transfer their solutions acquired in class to the new problem, even though the problem was different in surface features (e.g., the number of cubes and directions).

However, on the far-transfer problem, which was solved by modifying or transforming the problem-solving procedures learned in class, all the students showed difficulties in completing the problem. However, student difficulties were different across the students with different ability. Tina (MD student) had difficulty in modifying or transforming the procedures after acquiring them. She had a one-track mind in using the procedures taught in class, although the problem could be solved by using transformed procedures. Jose (struggling student) showed difficulty in matching the number of cubes in his building to those in the picture, rather than in transforming the procedures to fit to the new problem. Finally, Amy (typically achieving student) showed difficulty related to her perceptions of the 3-D building in the 2-D drawing, not her ability to transfer her procedures to the nonisomorphic problem.

CHAPTER 5:

DISCUSSION

IDEA (2004) requires all students, including students with disabilities, to participate in and make progress in the general education curriculum. Under IDEA, students with disabilities, including students with MD, are entitled to be provided with adapted instruction using empirically validated instructional approaches to teaching mathematics, which can help them succeed in the general education classrooms. However, there is limited knowledge about whether and in what ways instruction is adapted for students with MD and the degree to which students with MD have access to the standards-based mathematics, general education curriculum adopted by today's mathematics education. Thus, the purpose of this case study was to examine (a) 1 fourth-grade teacher's instructional adaptations for 3 students with MD in a standards-based mathematics, general education classroom and (b) the mathematics learning of 6 fourth-grade students with differing levels of ability (3 students identified MD, 2 students struggling with mathematics, and 1 student without a disability) in a standards-based mathematics, general education classroom. The following research questions guided this study:

1. How does a fourth-grade general education teacher adapt mathematics instruction within a standards-based mathematics curriculum for students who have an IEP in mathematics and who receive mathematics instruction in the general education classroom?

2. How do 6 fourth-grade students with different ability (3 identified MD, 2 struggling, and 1 without disability) who receive mathematics instruction in a standards-based mathematics, general education classroom use mathematics knowledge and skills taught in class to solve curriculum-based problems after they have received classroom instruction?

Overview of This Study

Characteristics of Participants

This study included 7 participants: 6 student participants and 1 teacher. Six student participants, including 3 students with MD, 2 students whose teacher identified as struggling with mathematics, and 1 typically achieving student, participated in this study. The following provides an overview of their characteristics to nurture the discussion of the findings of this study.

Students With MD

IEP. Three students with MD, Lee, Kevin, and Tina, participated in this study. Lee, Kevin, and Tina were fourth-grade students who had an IEP in mathematics and were receiving mathematics instruction in a standards-based mathematics, general education classroom. These 3 students had IEP goals not only in mathematics but also in the other academic areas. For example, Lee had IEP goals in content mastery as well as in mathematics. Similarly, Kevin and Tina had IEP goals in other areas such as language art and content mastery as well as in mathematics. However, their IEPs in mathematics did not include goals for learning geometry and spatial reasoning or for learning probability and statistics.

Difficulties in prerequisite skills. MD student participants were noted or rated as struggling with most prerequisite skills that were supposed to be mastered before learning fourth-grade geometry from their teacher. In particular, geometry vocabulary and the skills of locating and naming points on a line using fractions such as halves were the most severe struggles of all 3 students. Lee was identified by the teacher as the student struggling most with geometry. Her prerequisite skills related to geometry and spatial reasoning were rated as the lowest among the 3 students with MD. Other than geometry vocabulary and geometry-related fraction skills, the teacher noted that Lee had problems with using whole numbers to locate and name a point on a line.

With regards to the prerequisite skills related to probability and statistics, the teacher noted that the 3 MD student participants struggled with most prerequisite skills that were supposed to be mastered before starting fourth-grade probability and statistics. Particularly, Lee and Tina had difficulties in the skills of (a) remembering orders and steps of probability and (b) understanding probability in fractions and decimals. Kevin had difficulties in the skills of (a) remembering orders and steps of probability and (b) using organized data to construct real object graphs. Among the 3 students, the teacher rated Kevin as struggling most in the prerequisite skills related to probability and statistics.

Struggling Students

Laura and Jose participated in this study as struggling student participants. The teacher ranked them between 25 percentile and 45 percentile on mathematics performances in her class and noted that they passed TAKS mathematics. The teacher described them as students who had not been identified as having MD but usually did not meet the learning expectations of each mathematics lesson.

Typically Achieving Student

Amy was participating in this study as a typically achieving student participant. The teacher, Ashley, nominated her as a typically achieving participant because (a) she perceived that Amy would be ranked between 70 percentile and 80 percentile on mathematics, (b) Amy usually satisfied the learning expectations of each lesson, and (c) Amy did not have any problems or difficulties in any areas.

Teacher Participant

Ashley Hamilton was a fourth-grade general education teacher in a suburban school district in central Texas. She majored in elementary education at a college in eastern Texas. She was certified in teaching reading in Grades 1–8. She had been teaching elementary schools for 4 years. Since she started her career as a teacher, she had always taught at the fourth-grade level. It was her 2nd year to use the Math Investigations program for her mathematics instruction. Before starting to teach Math Investigations, she was trained to teach the program at a daylong workshop by the school district where she is employed. After starting to use the program for her mathematics instruction, she received ongoing, whole-day training at 6-week intervals about how to use the program to teach mathematics.

Mathematics Topics Examined in This Study

The two research topics of this study, the teacher participant's instructional adaptations for her 3 students with MD and the learning of 6 students with different ability in a standards-based mathematics classroom, were examined during instruction on geometry and spatial reasoning and on probability and statistics. These two mathematics content or topics have been considered as important mathematics areas to be attained throughout school in most standards for mathematics education, including the NCTM (2000) *Principles and Standards for School Mathematics*. For example, the NCTM (1989) noted that instruction on geometry and spatial reasoning would play a significant role in developing students' systematic representation of their world and that instruction on probability and statistics would enhance important skills of living and working in the contemporary society with a superfluity of data and information to be analyzed (J. D. Baker & Beisel, 2001; TERC, 1998).

Particularly, in relation to students with MD, the difficulties of learning these two mathematics topics may be just the opposite, but they are common in that instructional adaptations on those two topics should be implemented for students with MD. Geometry and spatial reasoning is a mathematics area in which students with MD can be successful problem solvers using problem-solving strategies proportionate to their abilities (Grobecker & De Lisi, 2000). A small change in mathematics instruction on this mathematics topic may help students with MD succeed in the general education curriculum. By contrast, learning probability and statistics is based on various prerequisite skills (Pearson Education Inc. 2004). For example, figuring out a typical value of a data set involves understanding the concept of typical as an average (e.g., mean, mode, or median) and producing a value for typical based on computation skills. However, research in the field of mathematics disabilities has revealed that students with MD struggle even with basic mathematics skills, including computations, and their difficulties continue to exist throughout their school years (Cawley & Miller, 1989). Consequently, students with MD are likely to struggle with learning statistics and its related skills in the standards-based mathematics, general education classroom. This struggle is not only because they do not understand the concepts or procedures taught at a given grade, but also because they are not equipped with these prerequisite skills for learning the grade-level content. Thus, general education teachers should ensure that students with MD in their classrooms possess the prerequisite skills and deal with the paucity of these skills, if any, before or during instruction on these two mathematics topics.

In the field of students with MD, little study has been conducted on instruction on geometry and spatial reasoning for students with MD (Rivera, 1997) and probability and statistics, let alone instructional adaptations for students with MD in standards-based mathematics classrooms. Thus, it was important to investigate how a general education teacher implemented instructional adaptations for her students with MD in a standards-based mathematics, general education classroom and what learning of students with MD looked like in such a classroom in comparison to their peers with different ability.

Data Collection and Analysis

Data of this study were collected by case study methods, including observations, interviews, survey, and document reviews, and analyzed both qualitatively and quantitatively under the case study design. For exploring possible answers to the first research question, the teacher's instructional adaptations for her students with MD, data from classroom observations, teacher interviews, survey, and document reviews were triangulated to produce the findings of this study. Possible answers to the second research question, the learning of students with different ability in standards-based mathematics instruction, were explored and derived from data from clinical interviews and document reviews.

Themes

Seven primary themes emerged from this study. Four themes emerged from the findings on the teacher's instructional adaptations for 3 students with MD in standards-based mathematics general education classroom, and 3 themes emerged from the findings on the mathematics learning of the 6 students with differing ability in a standards-based mathematics, general education classroom. Each research question is discussed under relevant themes. Research Question 1 is addressed by Themes 1–4, and Research Question 2 is addressed by Themes 5–7.

Theme 1: The Teacher Varied Instructional Adaptations by Student

The Findings of This Case Study

The teacher made a different number of instructional adaptations across 3 individual students with MD during her standards-based mathematics instruction, and she made the largest number of adaptations for the student whom she rated as most struggling in the prerequisite skills relating to the mathematics content. The total number of instructional adaptations provided to individual students with MD throughout lessons was quite different across students during instruction in both geometry and spatial reasoning (12, 2, and 3 for Lee, Kevin, and Tina, respectively) and probability and statistics (9, 13, and 2 for Lee, Kevin, and Tina, respectively). The teacher, Ashley, most often provided adapted instruction for the student whom she rated as struggling the most with the prerequisite skills relating to the current area of mathematics content. For example, Ashley rated Lee as the student who struggled most in the standards-based or teacher-identified prerequisite skills required for learning fourth-grade geometry and spatial reasoning. During the lessons on geometry and spatial reasoning, Ashley made instructional adaptations for Lee more frequently than she did for the other 2 students with MD. She made 12 adaptations for Lee but only 2 instructional adaptations for Kevin throughout the three lessons on this mathematics content.

Likewise, during instruction on probability and statistics, Ashley was observed providing the larger number of instructional adaptations for Kevin, whom she rated as most struggling with the standards-based prerequisite skills for learning fourth-grade skills in the area of probability and statistics. Ashley identified the following four knowledge and skills as minimum prerequisite skills for learning probability and statistics: (a) remembering orders and steps, (b) understanding the meaning of probability related to chance and probability vocabulary, (c) understanding of a total and part of a total, and (d) understanding that probability can be in fractions and how to express it in fractions. She provided 12 instructional adaptations for Kevin but just 2–3 instances of adapted instruction for Lee or Tina.

Relating This Theme to Previous Research

Previous research on general education teachers' instructional adaptations had indicated that they do not readily adapt their instruction for the students with disabilities (J. M. Baker & Zigmond, 1990; Fuchs et al., 1992; Kagan & Tippins, 1991). Thus, compared to such research, it is encouraging that the general education teacher in this case study endeavored to address the difficulties of at least 1 student with MD through adapting her instruction. However, it should be noted that even though Ashley recognized the difficulties of the other students with MD (Kevin and Tina on the prerequisite skills relating to geometry and spatial reasoning, and Lee and Tina on the prerequisite skills relating to probability and statistics) in the prerequisite skills at interviews or on survey questionnaire, she rarely provided adapted instruction for those students during instruction on each content. The results of this study indicated that instructional adaptations that Ashley provided in her class appeared to be implemented mainly with the student whom Ashley rated or noted as struggling most among the students with MD. The finding of this study includes evidence corroborating previous findings that doubted "the typical capacity of general education settings to address multiple, unique learning problem simultaneously" (Fuchs et al., 1995, p. 456).

Two potentially related explanations for Ashley's unequal adaptations for individual students with MD may be made. First, during the observational period of this study, she was responsible for teaching 13–15 students in her mathematics class, including the 3 students with MD and 3–4 students struggling with mathematics. Given the large number of students with difficulties in mathematics she needed to consider, she may not have been capable of evenly implementing a sufficient, appropriate number of instructional adaptations for each individual student, in spite of her recognition of their difficulties or lack of response to her standards-based mathematics, general education instruction. Durkin (1990) discussed three factors that may impede differentiated instruction in classrooms: Teachers' instructional differentiations for individual students are influenced by (a) class size, (b) dependence on curriculum defined by basal reader materials, and (c) questionable testing practices. Among those factors, class size has been discussed as a factor that can explain the lack of instructional adaptations for students with learning disabilities in general education classrooms (Fuchs et al., 1995; McIntosh, Vaughn, Schumm, Hagger, & Lee, 1993; Vaughn, Moody, & Schumm, 1998). Class size may be interpreted simply as the number of students in each class, but class size may include issues that a teacher needs to solve or manage at a time (e.g., behavioral management or the number of students with difficulties in mathematics other than the number of typically achieving students) (Fuchs et al., 1995). Ashley's class was not large, but it included many students who were struggling with mathematics. Given the number of students with difficulties in mathematics, including students with MD in her class, she likely could not take into consideration the needs of all 3 students with MD, and instead focused on the student she recognized as most struggling with each topic. This could be her classroom-management strategy to maximize the benefits to her whole class including students with MD within the limited instructional resources and her own capacity (Fuchs et al., 1995).

Second, Ashley might have attempted to address the other 2 students' needs through core instruction instead of by differentiating core instruction for them. Previous research has shown that general and special education teachers consider integrating evidence-based instructional components into standards-based mathematics instruction to address the difficulties of students with MD in the class and to meet the expectations for all students' performances from the standards (Maccini & Gagnon, 2000). An analysis of Ashley's baseline standards-based mathematics instruction in this case study revealed that her standards-based instructional practices for the whole class involved a considerable number of evidence-based instructional components for teaching students with MD (e.g., the use of advance organizers, explicit vocabulary instruction, and manipulatives) (Cawley, Parmar, Foley, Salmon, & Roy, 2001; Maccini & Gagnon, 2006; Salend & Hofstetter, 1996). Considering that her instruction for the whole class already included effective, evidence-based mathematics instructional components for students with MD, she might have expected the other 2 students' needs or difficulties to be addressed within that standards-based mathematics instruction for the whole class. She also might have provided additional adaptations for only the students who seemed to have the most significant difficulties, in spite of the core instruction, which incorporated multiple evidence-based instructional components.

Based on the findings of this study, being identified as having MD or having difficulties in a learning curriculum may not guarantee instructional adaptations of adequate quality and quantity (Vaughn et al., 1998). Rather, the frequency of Ashley's instructional adaptations for individual students with MD depended on her perceptions or evaluations of the severity of the student's difficulties. The findings of this case study suggest that general education teachers should be trained to accurately identify and evaluate the needs of individual students using formal and informal assessments that would inform them of students' needs and their access to general education curriculum and instruction (Cawley et al., 2001).

Theme 2: More Adaptations Were Made in Delivery of Instruction Than in the Other Categories

The Findings of This Case Study

More adaptations were made in terms of delivery of instruction than in the other categories of instructional adaptations, based on observations of Ashley's instruction on both geometry and spatial reasoning, and probability and statistics. Adaptations in delivery of instruction were more widely implemented for the students with MD by Ashley than adaptations in the other categories, including instructional content, instructional activity, and instructional materials or technology. For example, during the lessons on geometry and spatial reasoning, the teacher modified her instruction in terms of delivery of instruction an average of 7.3 times for each student with MD across three lessons, compared to no changes or adaptations observed in instructional content, 1.7 times in instructional activity, and once in instructional materials or technology. Similarly, during the lessons on probability and statistics, Ashley modified her instruction 11.3 times, averaged for each individual student in terms of delivery of instruction, but she made no changes in instructional content or in instructional materials or technology, and she made approximately 2 modifications in instructional activity across five lessons on this topic.

Relating This Theme to Previous Research

Previous studies also documented that teachers implemented or preferred to implement instructional adaptations for students with LD including MD in terms of delivery of instruction rather than instructional content, activity, and materials (Schumm & Vaughn, 1991; Vaughn et al., 1998). Vaughn and her colleagues (1998) found that even special education teachers did not change instructional materials or content, even though those adaptations would benefit students with varied abilities. In the same context, Schumm and Vaughn (1991) reported that general education teachers from elementary, middle, and high schools considered adapting regular materials using alternative materials as the least feasible based on their resources and time.

Instructional adaptation for students with MD in terms of materials or technology was most frequently noted from the teacher's statements during interviews, but adapting instructional technology or materials for students with MD was rarely observed. Ashley stated,

> Materials, I always try to have manipulatives, cubes, or shapes, you know, or whatever it is, clocks or money. Hand it for them to use and put them in a group usually together.

Maccini and Gagnon (2000) found that the most popular adaptation noted by general education teachers were using calculators, which belongs to the adaptation category of instructional materials or technology, to address student difficulties on their IEPs. Further, Maccini and Gagnon (2006) found that secondary general education mathematics teachers and special education teachers reported using calculators for students with MD as instructional adaptations when they taught basic math facts, computation, and word-problem solving. That is, the secondary teachers noted they adapted their instructions in terms of instructional technology frequently. The difference between the findings of the current study and those of Maccini and Gagnon (2006) may be explained by the differences in student ages and instructional topics examined between two studies (fourth graders vs. secondary students, basic mathematics facts and computations vs. geometry and statistics). Compared to instructions on basic math facts and computations, most geometry instruction or statistics instruction examined in this study did not involve calculations to the extent that use of calculators was recommended or desirable for students with MD. Another possible explanation for differences between the two studies could be due to possible differences between teachers' perceptions on their use of evidence-based effective instruction and actual instructional practices by the teacher participants in Maccini and Gagnon's (2006) study. Their findings were based on the teachers' self-report, rather than direct observations of the teachers' instruction.

It should be noted that the teacher participant used some instructional technology, including an overhead projector and manipulatives including cubes, cans, and diverse objects in her classrooms during her mathematics instruction. However, these were not considered as adapted instruction for students with MD, but standards-based mathematics baseline instruction (core instruction) for all students, because the mathematics curricular program (Math Investigations) suggested the use of these technology or instructional materials for the lessons for the benefit of the whole class, not individual students with MD.

Theme 3: The Teacher Supplemented Standards-Based Mathematics Instruction With Evidence-Based Mathematics Instructional Components for Students with MD

The Findings of This Case Study

The teacher adapted her instruction for the students with MD by supplementing standards-based mathematics instruction with evidence-based mathematics instructional components with more variety; she utilized prompting, direct questioning, and group instruction more frequently than the other components. Ashley employed instructional components known to be effective for teaching students with MD when she adapted her standards-based mathematics instruction for the students with MD during instruction in both mathematics content areas. However, some evidence-based mathematics instructional components were observed in her typical standards-based mathematics instruction for all students as well as in her adapted instruction. For example, instructional components including (a) direct questioning, (b) review of prerequisite skills, (c) group instruction, (d) strategy instruction, (e) progress monitoring, (f) use of manipulatives, (g) use of teaching examples, and (h) practice opportunity (Swanson et al., 1999) were observed during her typical standards-based instruction for all students as well as instructional adaptations for the students with MD while she was teaching geometry and spatial reasoning. In the same line, evidence-based instructional components of (a) group instruction, (b) modeling, (c) teacher examples, (d) direct questioning, and (e) review of prerequisite skills were used for both teaching all students in standards-based mathematics classroom and adapting instruction for the students with MD while Ashley was teaching probability and statistics.

Compared to the variety of evidence-based instructional components used in her typical standards-based instruction for all students on the two content areas, the teacher attempted to use more varied and unique evidence-based instructional components when she adapted her mathematics instruction for the students with MD. For example, during instructions on geometry and spatial reasoning, Ashley (a) used prompting, and (b) controlled difficulty of tasks only for adapting her instruction for the students with MD, not for teaching all students in her class. Similarly, Ashley (a) provided prompting, (b) provided explicit explanations, (c) controlled difficulty of tasks, (d) implemented progress monitoring, (e) provided practice opportunity, and (f) implemented reteaching only to adjust her instruction for the students with MD, not to teach all students in her class, during instruction on probability and statistics.

Relating This Theme to Previous Research

Although standards-based mathematics instruction supports more student-centered and problem-solving-based teaching to all students than teacher-directed, task-analysis-based instruction, the results of this study indicated that Ashley seemed to think that using direct, explicit instructional practices for students with MD in standards-based mathematics instruction would not be against the principles and standards for mathematics instruction. Rather, she thought that these instructional components should be used to meet the standards for student performances and mathematics instruction for all students in the standards-based mathematics classroom (Cawley et al., 2001; Salend & Hofstetter, 1996; Ysseldyke, Kosciolek, Spicuza, & Boys, 2003). The following quote reveals her thoughts about instructional adaptations that could be implemented for students with MD in standards-based mathematics instruction.

> [For students with MD in the class], I am trying to make it sure that all have the access to manipulatives, something hands-on, that they can touch. Umm, also, I encourage them to draw pictures. And, you know, I let them use highlighters to highlight key words in problems. Umm, you know, even turning into real world examples, I can pull the kids up to in front of room and actually demonstrate, you know model, a problem using kids in class. Umm, so just access, plenty of access to hands-on things like cubes like we use pattern blocks and dices just whatever they can, even multiplication times table, just help them remember that fact. Even sheets like a multiplication table, I want them to be able to use that until any assessments. On any assessment I wouldn't, but on daily work, any hands-on materials, they can get access to, let them use dices, cubes, blocks, whatever they need.

This finding may corroborate the findings of previous studies on teachers' practices of standards-based mathematics instruction (Maccini & Gagnon, 2000, 2006). According to Maccini and Gagnon (2000), secondary general education teachers and special education teachers also reported that use of evidence-based mathematics instructional components, use of manipulatives, and real-life application were important to implement the goals of NCTM standards, especially for students with MD in the standards-based mathematics core curriculum and instruction. Particularly, the teacher respondents in Maccini and Gagnon's (2000) study noted the importance of evidence-based instructional components, including explicit teacher modeling, multiple teacher examples, pacing, corrective feedback, use of manipulatives, and group instruction, in implementing standards-based mathematics instruction. In fact, research in general education settings has revealed that implementing evidence-based components of effective instruction in fourth- and fifth-grade general education classrooms using standards-based mathematics curriculum (e.g., Everyday Math Curriculum) can result in greater mathematics achievement gains to students in these instructional environments than implementing only standards-based mathematics curriculum (Ysseldyke et al., 2003).

In this case study, Ashley's use of evidence-based mathematics instructional practices in her typical standards-based mathematics instruction for all students may be explained in two ways. First, while current standards for mathematics education emphasize that teachers should provide more opportunities to engage in and solve meaningful problems in multiple ways, the standards also direct that teachers should ensure that students are provided with practice opportunities to learn the basic knowledge and skills necessary to work on the problems before having them engaging in problem solving using direct, explicit instructional practices (Bottge, 2001). According to Bottge's (2001) theoretical model, providing explicit, direct instruction on these foundation skills in standards-based mathematics classrooms is not against the NCTM standards. He suggested that integrating explicit, direct instructional components into standards-based mathematics instruction to improve students' skills on basic mathematics knowledge and skills may be a good decision based on the equity principle for school mathematics by the NCTM (as cited in Bottge, 2001), emphasizing high expectations and strong supports for all students.

Second, Ashley's adherence to textbooks for her mathematics instruction may explain her use of evidence-based mathematics instructional components in her typical standards-based mathematics instruction for all students. Although teachers are supposed to design their own instruction to meet their students' needs under effective mathematics instructional models aligned with the standards (Bybee, Ferrini-Mundy, & Loucks-Horsley, 1997), in reality, teachers' instruction tends to depend mainly on the textbooks that they employ for their mathematics instruction (Jitendra et al., 2005). According to Garner (as cited in Jitendra et al., 2005), textbooks may "serve as critical vehicles for knowledge acquisition in school" and may "replace teacher talk as the primary source of information" (p. 53). During observational periods of this study, Ashley's instruction was based on the *Investigations in Number, Data, and Space* (Pearson Education Inc., 2004) textbook, which was designed to incorporate with the NCTM standards (Technical Education Research Center, 2004). According to Jitendra et al. (2005), textbooks adopted in today's mathematics classrooms meet some but not all of instructional design criteria, including explicit explanations using teaching examples and corrective feedback. Thus, using evidence-based instructional components in standards-based baseline instruction may be influenced by the textbooks the teacher used.

Another finding of this study related to this theme was that Ashley implemented diverse instructional components as a trial to address the needs of her 3 students with MD. For example, to assist students with MD in learning knowledge and skills in geometry and spatial reasoning, Ashley attempted to use two evidence-based instructional components (prompting, and difficulty control) other than what she used for all students in her typical standards-based geometry instruction. Likewise, to help the students with MD understand knowledge and skills related to probability and statistics, she employed six more evidence-based instructional components (prompting, explicit explanations, control difficulty, progress monitoring, practice opportunity, and reteaching) other than what she used for all students in her typical standards-based statistics instruction. Even though this study did not systematically examine the variety of evidence-based instructional components used across individual students and across situations, observation showed that Ashleys' instructional adaptations occurred in two different situations: (a) when she taught concepts or procedures, and (b) when she responded to students' errors or difficulties. Although the association between the situations and the instructional components used in each situation was not clear in her geometry instruction, the findings from her statistics instruction revealed that she used prompting and reviewed prerequisite skills mainly in order to correct students' errors. This finding supports the premise that she endeavored to effectively address the needs of individual students with MD using multiple and unique evidence-based mathematics components in her standards-based mathematics classroom.

This finding could make a contribution to the findings on general education teacher's instructional adaptations to address the needs of students with MD (Fuchs et al., 1995). The findings of previous research have indicated that some general education teachers rely on the use of the same instructional strategies over time in making adaptations of their mathematics instruction, whereas other teachers make substantially important, individually tailored adjustments to address the needs of their students with MD (Fuchs et al., 1995; Maccini & Gagnon, 2000). Fuchs et al. (1995) found substantial variability in general education teachers' instructional adaptations to address the needs of students with MD. Maccini and Gagnon (2000) noted that secondary general education teachers' typical adaptations of mathematics instruction for students with MD included some uniform instructional practices, even including instructional practices that have not been validated (e.g., assignment modification and use of calculators).

Related to this finding, Ashley, the teacher participant of this study, rarely adjusted her instruction in terms of use of manipulatives, even though she stated that standards-based mathematics instruction could be characterized by multiple hands-on activities, and the use of manipulatives is desirable for teaching Geometry (Cass et al., 2003; Maccini & Gagnon, 2000). This is explained by the fact that the curriculum (Mathematics Investigations) already included many opportunities to use manipulatives, because the NCTM standards emphasize the use of manipulatives for teaching mathematics. In fact, in most lessons she was observed using a variety of manipulatives (e.g., cubes, the overhead projector, and raisins) to teach mathematics knowledge and skills to her students. However, these were not considered instructional adaptations using evidence-based instructional components because they were suggested by the curriculum for all students and were not targeting only the students with MD.

Notably, Ashley used prompting, direct questioning, and group instruction more frequently than the other evidence-based instructional components. Across the three lessons on geometry and spatial reasoning, prompting was used in instructional adaptations for the students with MD more than three times in each lesson (five times for Lesson 1, three for Lesson 2, and five for Lesson 3). Direct questioning was used in instructional adaptations two or three times in each lesson, and group instruction was used in instructional adaptations three times in two lessons (but not in Lesson 2). In comparison, the other components were observed used in instructional adaptations once or less in each lesson (for more information, refer to Table 4.5). Similarly, across the five lessons on probability and statistics, the average frequency of instructional adaptations using prompting, direct questioning, and group instruction in instructional adaptations was two to four times. The average frequency of instructional adaptations using the other evidence-based instructional components was not at all (strategy instruction) to two times (explicit explanations) (for more specific information, refer to Table 4.11).

Prompting, direct questioning, and group instruction with control of difficulty of tasks have been discussed as critical factors of interventions for students with LD (Swanson et al., 1999; Vaughn, Gersten, & Chard, 2000). The finding of the current study differs from the findings of previous studies. Previous studies in general education settings indicated that general education teachers did not integrate these components into their core instruction, even for students with LD, including students with MD (J. M. Baker & Zigmond, 1990; McIntosh et al., 1993; Schumm et al., 1996). This is important because it may indicate a remarkable improvement of general education teachers after reforms in education and recent legal efforts (e.g., No Child Left Behind Act, 2002; IDEA, 2004) in identifying student needs and integrating evidence-based instructional components to teach students with disabilities who are included in general education classrooms.

Theme 4: Instructional Adaptations Were Implemented Restrictively.

The Findings of This Case Study

The teacher's instructional adaptations were limited in addressing student difficulties relating to prerequisite skills required to learn each mathematics content area. Across lessons on both topics, Ashley was consistently observed addressing only one of the many student difficulties in the prerequisite skills relating to geometry and spatial reasoning, or probability and statistics, through interactions with students with MD. While teaching geometry and spatial reasoning, Ashley provided reviews of geometry vocabulary, which was a difficulty that the students with MD showed relating to the geometry prerequisite skills.

Likewise, during instruction on probability and statistics, Ashley provided a review of one of the prerequisite skills, the skills of transforming a probability into a fraction, through her interactions with one of the students with MD. Although all students with MD demonstrated difficulties in multiple prerequisite skills, Ashley did not implement instructional adaptations that addressed their difficulties in other prerequisite skills such as (a) collecting and sorting data; (b) using organized data to construct real object graphs; (c) identifying events as certain or impossible, such as drawing a red crayon from a bag of green crayons; (d) using data to describe events as more likely or less likely, such as drawing a certain color crayon from a bag of seven red crayons and three green crayons; and (e) using data to describe events as more likely, less likely, and equally likely.

Relating This Theme to Previous Research

As discussed in Theme 1, researchers have expressed doubts that most general education teachers have enough time to address individual students' needs (Fuchs et al., 1995) due to the large size of their classes (Durkin, 1990; Fuchs et al., 1995) or lack of general education teachers' skills in identifying and evaluating students' prerequisite skills relating to the target skills being taught (Jitendra et al., 2006). For the benefit of all students, not only students with MD, Ashley might have selected the most efficient way to deal with the difficulties of students with MD in prerequisite skills. She could have focused on the prerequisite skills that the greatest number of students had difficulties with rather than focusing on the other prerequisite skills. Or, Ashley might not have considered the other prerequisite skills as important as the prerequisite skills addressed in her instruction for learning the target skills. In fact, during interviews, Ashley identified different prerequisite skills for learning geometry and spatial reasoning as well as probability and statistics than the prerequisite skills identified by the standards. Moreover, the prerequisite skills addressed by her instructional adaptations were those that she identified as minimum prerequisite skills required for learning fourth-grade skills and that she expected her students to bring from their previous learning.

> A basic understanding of, you know, the geometric vocabulary like polygon. They need to understand or have been introduced about polygons. And, shapes, just to know, what the definition of a shape is.

> Understanding of a total and then single out one, part of a total. A lot of them don't know that when they come to the fourth grade. They may not be able to write them as a fraction, but they might be able to say two out of six socks or oranges. Some may be able to express it poorly. But by the end of fourth grade, they may be able to write it as fractions or as a decimal. Umm, when we talk about probability, I will ask them most fourth graders will know, they recognize the words like probable, probably, and they will be able to relate the word probable to the word, probability.

In addition, Ashley's passive instruction on prerequisite skills the students with MD had difficulties with may be understood by basic assumptions of constructivism, which is the fundamental learning theory for the current principles and standards for mathematics education. Constructivism expostulates that knowledge "evolves as a result of the coordination and differentiations of mental structuring activity as it seeks aliment to overcome conflict" (Grobecker, 1999, p. 48), rather than the sum of specific, hierarchically ordered skills. According to this theory, it is not necessary to teach prerequisite skills or component skills of a target skill, because students' difficulties with grade-level mathematics are not related to the lack of these skills or strategies, but to their mental structuring activity, which is not as well developed as that of their same-aged peers (Grobecker, 1999).

Theme 5: Students With Differing Ability Showed Improvement on Prerequisite Skills Postinstruction

The Findings of This Case Study

The 6 students with differing ability showed improvement on prerequisite skills proximal to the knowledge and skills being taught after receiving instruction in standards-based mathematics general education. Students received instruction on two different geometry and spatial reasoning skills (creating 3-D buildings based on 2-D drawings and finding shapes that would make specific silhouettes) and two probability and statistics skills (drawing bar graphs and comparing two bar graphs) in the standards-based mathematics, general education classroom. Postinstruction, the 6 students, including 3 students with MD, improved from their baseline performance on problems about prerequisite skills that were proximally related to the target skills being taught. For example, compared to baseline performance, the 6 students averaged over 20% improvement in accuracy in solving the problems on both of the prerequisite skills related to geometry and spatial reasoning skills after receiving instruction on these skills in standards-based mathematics, general education classroom. Likewise, the3 students with MD and 2 struggling students (but not the typically achieving student) showed more than 10% improvement in accuracy in solving the problems on both of the prerequisite skills related to probability and statistics from baseline interviews to postinstruction interviews.

In addition, it was noted that improvement in the prerequisite skills was more likely to be observed when the prerequisite skills were proximal to or directly related to the targeted skills being taught. Both prerequisite skills on geometry and spatial reasoning were more directly related to the targeted skills being taught and therefore were more likely to be reviewed through the lessons on the targeted geometry skills than the other prerequisite skills. So, instruction on the targeted geometry skills resulted in improvements in the prerequisite skills as well as improvements in the targeted skills. Similarly, one of the prerequisite skills on probability and statistics (reading or interpreting information from a T-chart) was directly related to the targeted statistics skills (drawing a bar graph based on a T-chart). So, instruction on the targeted skills brought improvements in the prerequisite skills as well as in the targeted skills. However, the other prerequisite skill on probability and statistics (finding the typical number on a bar graph) seemed not to be related to the targeted skills (comparing two bar graphs) as directly. Accordingly, the typically achieving students' skills on one of prerequisite skills (finding the typical number on a bar graph) related to probability and statistics actually declined from baseline interviews to postinstruction interviews, even though she showed better performance on the targeted skills after receiving instruction.

Relating This Theme to Previous Research

The findings of this study may extend support for constructivist beliefs as a foundation for development of mathematical knowledge and skills. Knowledge and skills "evolve as a result of the coordinations and differentiations of mental structuring activity as it seeks aliment to overcome conflict" (Grobecker, 1999, p. 48), which is provoked in problem-solving situations. Although Ashley did not teach specific prerequisite skills for learning fourth-grade geometry or statistics, as discussed in the section of Theme 4, the students in this current study might have benefited from engaging in problem-solving activities that involved the prerequisite skills. In the same line, while research in instruction for students with MD has shown that video-based anchored instruction combined with applied mathematics problems produced positive effects on students' performances in problem-solving as well as computation skills (Bottge, 1999; Bottge & Hasselbring, 1993), students' performances on computations were downsized over the video-anchored intervention when calculators were provided for computations (Bottge, Heinrichs, Chan, & Serlin, 2000). Prerequisite skills proximal to the target skills being taught may be practiced through contextualized problem-solving activities. Accordingly, prerequisite skills may not be practiced through standards-based instruction on target skills, because they are not directly related to the target skills, and thus may not show improvement.

However, this study did not examine the growth of all prerequisite skills identified by the standards (TEKS) or the classroom teacher (for more specific information, refer to Tables 4.1 and 4.7). Each task included only one prerequisite skill. Accordingly, it is not clear that improvement of student prerequisite skills is related to the contiguity of the prerequisite skills to the target skills being taught or practice opportunities through problem-solving activities, or related to mathematics topics. Thus, this finding should be interpreted cautiously.

Theme 6: The Students With MD and The Struggling Students did not Perform as Well as the Typically Achieving Student

The Findings of This Case Study

The students with MD and the struggling students did not perform as well as the typically achieving student on the tasks about specific mathematics knowledge and skills after receiving instruction in the standards-based mathematics, general education classroom. This theme emerged from an analysis in which the three groups of students with differing ability were compared in terms of changes in problem-solution accuracy and concepts or procedures used in problem solutions after receiving mathematics instruction in a standards-based mathematics, general education classroom. According to the findings on problem-solving accuracy in two geometry and spatial reasoning clinical interview tasks, all three groups of students showed improvement in accuracy of problem solving using both targeted skills between baseline and postinstruction interviews. The group of students with MD showed the largest improvement in accuracy of problem solving on Clinical Interview Tasks 1 and 2.

Similarly, three groups of students changed their concepts or procedures to solve problems on both geometry and spatial reasoning clinical interview tasks after receiving instruction on the skills. Tables 4.15 and 4.19 in Chapter IV provide a summary of changes in the features of concepts or procedures used by three groups of differing ability to solve two geometry and spatial reasoning clinical interview tasks and two probability and statistics clinical interview tasks from baseline to postinstruction. For example, on Clinical Interview Task 1, students with MD demonstrated consideration of the number of cubes when they were making a building, which was not found in their performance at baseline, even though this awareness did not lead them to get the correct answers all the time. The group of struggling students considered the positions of sections of a building as well as the number of cubes in each section at postinstruction interviews, whereas they considered only the number of cubes in each section at baseline interviews. As well, one of struggling students used a strategy for recognizing and remembering the positions of sections, which was taught in class. The typically achieving student did not show large changes in concepts or procedures because she already knew and applied the desirable concepts and/or procedures even in her baseline performances.

On Clinical Interview Task 2 on geometry and spatial reasoning, the group of students with MD considered the sides as well as the bottom of a building to find a solid that would make a specific silhouette, and they explored multiple answers after they found one solution or answer at postinstruction. At baseline they considered only the bottom of a solid to find silhouette and did not explore multiple answers. The changes in performance of both the struggling student group and the typically achieving student group between baseline and postinstruction interviews included (a) considering the sides as well as the number of cubes for building creation and (b) using mental matching of their building to the 2-D figure on the card instead of directly matching their building to the card. The group of struggling students did not explore multiple answers at postinstruction, as at baseline interviews. However, the typically achieving student tried to explore multiple answers at both baseline and postinstruction interviews.

However, it should be noted that even though the group of students with MD and the group of struggling students showed improvements in problem-solving accuracy and concepts or procedures after receiving instruction on the skills, their performances were not comparable to those of the typically achieving student. For example, compared to the postinstruction accuracy of the typically achieving student, 75.0% on Clinical Interview Task 1 and 87.5% on Clinical Interview Task 2, the accuracy of the students with MD stayed below 60.0% even after receiving instruction on the targeted skills. In addition, when interviewed, they did not even remember that they were taught about the specific mathematics knowledge and skills in their class the day before. The typically achieving student was the only student who remembered and attempted to use what she was taught in her class, including concepts or procedures across all clinical interviews on both mathematics topics.

As in the skills of geometry and spatial reasoning, the group of students with MD and the group of struggling students did not perform as well as the typically achieving students on both probability and statistics clinical interview tasks at postinstruction interviews. For instance, compared to the typically achieving student's postinstruction accuracy on both tasks on probability and statistics (100.0% on both tasks), the students with MD and the struggling students performed at 16.7% and 50.0% accuracy, respectively, on Clinical Interview Task 3, and 58.3% and 62.5% accuracy on Clinical Interview Task 4 after receiving standards-based mathematics instruction on the skills. Moreover, after receiving class instruction, the students with MD and the struggling students did not perform better on both probability and statistics tasks than before they were taught about the skills. Their postinstruction problem-solving accuracy stayed at the similar level to the baseline accuracy or was degraded (e.g., the struggling students' performance on Clinical Interview Task 4 dropped from 68.5% to 62.5% between baseline and postinstruction interviews).

Likewise, the group of MD students and the group of struggling students did not use concepts or procedures for problem solving similar to those the typically achieving student used. For example, the typically achieving student showed improvements in problem-solution accuracy by applying the concepts or procedures that were taught in class to the problems on probability and statistics. It also should be noted that Amy was the only student who provided a correct graph to the problem on Clinical Interview Task 3, which was exactly the same as the teacher's example during her instruction.

However, the students with MD did not perform as well as the typically achieving student after receiving instruction on the skills. For example, a student with MD (Tina) showed difficulties in understanding the concept of a bar graph and the procedures to make a bar graph after receiving instruction on the skills, which were observed in her baseline performances. When she was given the data table and asked to draw a bar graph, she just copied the table on the paper. Likewise, changes were not noted in the other 2 students with MD. From those findings, a theme emerged that the group of students with MD and the group of struggling students did not perform as well as the typically achieving student on the tasks about specific mathematics knowledge and skills after receiving instruction on the skills in a standards-based mathematics classroom.

Relating This Theme to Previous Research

The students' varied performances and their positive changes could be translated into different levels of geometrical thinking in the Van Hiele (1986) model. According to the Van Hiele model of thinking in geometry (Van Hiele, 1986), students' thinking in geometry passes through five levels, from global (concrete) structures (Level 0), to visual geometric structures (Levels 1–2), to abstract structures (Levels 3–4), in which the students cannot achieve a level of thinking without having passed through the previous levels (Fuys et al., 1988). The growth of students' geometrical thinking derives from appropriate instruction, not from maturation. According to this model, as a result of receiving instruction in standards-based mathematics classrooms, the students with MD could have been able to make the transition from the level of concrete structures, at which students identify, name, compare, and operate on geometric figures according to their appearance (Level 0), to the level of visual geometric structures, at which students analyze figures in terms of their components and relationships among components and discover properties or rules of a class of shapes empirically (Level 1). Yet, it seemed that their transition was not complete, because they started to consider the components (number of cubes) but did not show complete understanding of the relationships among components (positioning of sections or parts of a building). The struggling students possessed incomplete Level 1 geometrical thinking before the instruction but achieved Level 1 of geometric thinking on this content after the instruction. Finally, the typically achieving student was able to consider the number of cubes and positioning of sections or parts of a building before and after instruction. As well, she was able to use the familiar shape strategy to remember the positions of parts of a building. So, it could be said that she showed at least Level 2 understanding even before instruction, in which the student logically interrelates previously discovered properties and rules by giving or following informal arguments (Fuys et al., 1988).

Describing the students' performances and their changes on Clinical Interview Task 2 between baseline and after instruction using the Van Hiele (1986) model of thinking in geometry, it seemed that all three groups of students were at the Level 0 thinking on this content at baseline and transferred to different levels of geometry thinking across groups of students. They looked at the bottom, which is most distinctive part of a solid, and they chose a solid as one that would make a specific silhouette because the figure of the bottom looked the same with the silhouette. However, after receiving geometry instruction in the standards-based general education classroom, the students with MD showed the understanding of geometry at Level 1, whereas the struggling students and the typically achieving students showed the understanding of the skills at least in Level 2.

The finding of this study corroborates the findings of previous research on access of students with disabilities to the standards-based mathematics, general education curriculum (e.g., Wehmeyer, Lattin, Lapp-Rincker, & Agran, 2003; Woodward & Baxter, 1997), indicating that standards-based mathematics instruction benefits students with average and above average academic abilities but is not sufficient to address the needs of students with MD or those at risk for MD. For example, Woodward and Baxter compared the mathematics performance of third-grade students in nine classes, five using standards-based mathematics curriculum and four using a traditional curriculum on the Iowa Test of Basic Skills and a problem-solving measure, the Individual Mathematics Assessment. They found that overall, the majority of students including students with MD and those at risk for MD benefited from standards-based mathematics curriculum and instruction, but the benefits of students with MD and those at risk were marginal in comparison to the benefits of their typically achieving peers. Similarly, the findings of an observational study with students with mental retardation in standards-based mathematics, general education classrooms (Wehmeyer et al., 2003) revealed that many students with mental retardation were engaged in activities in the standards-based general education mathematics curriculum, but student engagements varied considerably by the ability of students. Wehmeyer et al.'s findings suggested that a number of steps, including instructional adaptations, should be taken to ensure the maximum access of students with disabilities to the standards-based general education curriculum. Based on the findings of the current study and previous studies, supplemental support should be provided for students with MD in standards-based mathematics, general education classroom to ensure their access and progress to the general education curriculum.

The relatively lower performances of students with MD on clinical interview tasks in this study, which were based on the standards-based mathematics, general education curriculum, might be due to additional factors, not just educational settings in which they received their mathematics instruction. Previous research has shown that students with MD did not perform as well as same-aged students without MD, even though the effects of IQ were controlled on some tasks on geometry and spatial reasoning (Grobecker & De Lisi, 2000). Consistently, research findings have shown that students with MD have difficulties in (a) retrieving information from long-term memory

(Bryant et al., 2000; Geary, 2004; Gersten & Chard, 1999; Robinson et al., 2002; Swanson & Jerman, 2006), (b) executing multistep procedures or developmentally appropriate procedures for solving a problem (Geary, 2004; Gross-Tsur et al., 1996; Hitch & McAuley, 1991; Jordan, Hanich, & Uberti, 2003; Swanson & Jerman, 2006), (c) representing and proceeding numerical or mathematical information visually or spatially (Geary, 2004; Swanson & Jerman, 2006), and (d) using self-regulative system for their learning (Swanson & Jerman, 2006). Previous studies also showed that these difficulties were related mainly to difficulty in working memory (e.g., Geary et al., 2004; Swanson & Jerman, 2006). These characteristics may provide an explanation for the lower performances of students with MD even though they received the same quality mathematics instruction as their peers in general education classroom. For example, throughout all of the clinical interviews, students with MD did not even remember that they were taught about the problem-solving procedures in their class instruction. Their difficulties in retrieving information from their long-term memory may explain the lower performance of students with MD on clinical interview tasks of this study. In this study, solving clinical interview task problems on geometry and spatial reasoning required visual and spatial processing of mathematical information, especially transferring information presented in a 2-D array to 3-D and mental or physical anticipation of silhouettes of a 3-D shape. Their difficulties in representing visual and spatial mathematics information prevented the students with MD from successfully solving problems in each clinical interview tasks (Geary, 2004). Also, problems in self-regulation might lead the students to skip or not consider parts of the information; they could not provide correct answers to some problems because their outputs were not complete (Swanson & Jerman, 2006).

Likewise, solving clinical interview task problems on probability and statistics required the abilities of applying multistep procedures. For example, problems in Clinical Interview Task 3 included multistep procedures of (a) classification of data according to variations (or ranges) of a variable of interest, (c) counting the frequency per category or range, (c) drawing two lines (X and Y), (d) putting variable names for each axis, (e) marking a dot of X coordinating with Y on the graph, and so on. For the students with MD in this study, coordinating and following these procedures seemed not to be as simple as it was for their typically achieving peer. The students with MD might have been overwhelmed or frustrated by these multistep procedures. For example, a student with MD (Tina) just copied the table on the paper when she was asked to draw a bar graph representing the dataset on the table. Particularly, this finding of the current study indicated that students with MD may not learn as much as their typically achieving peers in standards-based mathematics, general education classes, and they may require special instructional adaptations for dealing with their difficulties to ensure their maximum access and their progress in the general education curriculum.

Theme 7: Transfer of Mathematics Knowledge and Skills Differed Across Students With Differing Ability

The Findings of This Case Study

Transfer of mathematics knowledge and skills was different across the three groups of students with differing ability when solving a new problem required their own modifications of the problem-solving procedures taught in class. The findings of this study on transfer of mathematics knowledge and skills by three different groups of students (MD, struggling, and typically achieving) indicated that they were not different in solving new problems, which could be solved by using the same procedures taught in their class (near-transfer problem). After the students were taught about how to make 3-D buildings with cubes shown in 2-D drawings in a lesson on geometry and spatial reasoning, all 3 students were able to solve a new problem by using the procedures of creating single-layered sections of the building and combining them in the way that the sections of the 2-D building on the card were positioned, which had been taught in their mathematics class. In their class, they were taught to start from making sections (front, middle, or bottom) according to the number of cubes in each section and then to combine the sections of a building as it was positioned in the drawing. After receiving instruction on the skills, the students started from making sections according to the number of cubes in each section. Then, they matched their building to the picture to verify their answers. Once they mastered the solution, the students, including a student with MD, seemed to be able to successfully transfer their solution acquired in their class to the new problem even though the problem was different in the surface features.

Relating This Theme to Previous Research

This finding is consistent with previous research supporting the capability of students with MD to transfer mathematics knowledge and skills (Bottge, 1999). Given the research findings that transfer of knowledge and skills is difficult to achieve, even among general education students (Cooper & Sweller, 1987), it was encouraging to see that the student with MD was able to transfer her knowledge and skills to new problems having the same problem structure. The findings of this study may add additional support to previous research findings on the transfer abilities of students with MD.

There has been considerable research indicating that lower achieving students, including students with MD, do not benefit from the use of instructional methods that may promote problem solving of students without disabilities (e.g., Owen & Fuchs, 2002). However, the findings of this study in standards-based mathematics curriculum and instruction indicate that students with MD as well as students without disabilities are able to apply what they are taught in their classes to solve new transfer problems having the same structure but different surface features, after receiving contextualized instruction using meaningful problems (e.g., Bottge, 1999), which is supported by the NCTM standards. Yet, it should be noted that transfer of mathematics knowledge and skills presumes students' mastery of the knowledge and skills before they are placed in a situation of transfer. So, mastery of knowledge and skills should continue to receive researchers' and educators' attention.

However, all of the students had difficulty on the far-transfer problem, which was solved by modifying or transforming the problem-solving procedures learned in class. Student difficulties were different across the students with different abilities. Tina, the MD student, had difficulty in modifying or transforming the procedures after acquiring them. She was not flexible in changing the procedures taught in class. The struggling student had difficulty in matching the number of concrete cubes in his building to those in the picture, rather than in transforming the procedures to fit to the new problem. Finally, the typically achieving student had difficulty related to her intention not to work as hard. She demonstrated she could make the 3-D buildings with multilayers on other problems and stated that she decided not to work as hard as to make the sides that she could not see. It did not seem to be related to her ability to transfer her procedures to the nonisomorphic problem.

> Amy: I knew how to make the top part because I was multiplying the top to see if that was the correct number. First I counted the sides, what really tricked me was if I was supposed to make the sides or not and the middle part or back.
> Tester: So you decided not to do this, why?
> Amy: Not because I didn't want to work as hard, just I didn't see the other sides or anything, so just follow what you see I guess.

According to Hatano and Inagaki (1993), procedural knowledge does not flexibly change once students reach the mastery level of performing procedures. Students come to know about how the procedures are related when they get to the mastery level of performing the procedures. Based on the understanding of the procedures, students can recognize situations in which the procedures can be applied. Based on the suggestion from Hatano and Inagaki, attaining mastery level of knowledge and skills may not guarantee that the student can modify some of the procedures and create new procedures to solve new problems having different conceptual structures (Bottge, 1999).

Implications for Practice

The findings of this study are supported by previous research that standards-based mathematics textbooks are not sufficient to address the needs of students with MD (e.g., Jitendra et al., 2006) and low-achieving students, including students with MD, who struggle with engagement in classroom activities and learning the curriculum in standards-based mathematics classrooms (e.g., Baxter et al., 2001; Woodward & Baxter, 1997). Therefore, instructional adaptations addressing the needs of students with MD should be considered and implemented in general education teachers' instructional practices.

However, the findings of this study indicate that instructional adaptations in standards-based mathematics general education settings may be implemented very restrictively in terms of the number of students with MD whose difficulties are addressed and the types of difficulties addressed by the adaptations. Ashley's overall recognition of students' difficulties in mathematics (e.g., identifying them as having MD) did not guarantee instructional adaptations for individual students with MD (Vaughn et al., 1998). Ashley's instructional adaptations were concentrated on the individual student with MD whom the teacher recognized as struggling the most in the particular mathematics topic. The types of the difficulties addressed in her adaptations were very restricted. These findings suggested two implications for instructional practices.

First, given that the ability of a general education teacher to deal with a number of students at a time is limited, general education teachers should consider and implement instructional adaptations in the efficient and effective way possible. For example, research has found that explicit, direct instructional components produced large effects for teaching students with MD (e.g., Swanson et al., 1999). In fact, research using both general and special education teachers has reported that they considered integrating evidence-based instructional components into standards-based instruction as a good strategy to meet the NCTM standards on student performance (Maccini & Gagnon, 2000). Thus, a best practice for students with MD in the standards-based mathematics, general education classrooms could be having teachers adapt their instruction by integrating those explicit, direct instructional components into standards-based mathematics instruction (e.g., Bryant et al., 2006; Jitendra et al., 2006; Maccini & Gagnon, 2000; Rivera, 1993).

Second, the occurrence of instructional adaptations for an individual student depended on Ashley's perceptions of the students' difficulties and identification of prerequisite skills required for learning specific grade-level mathematics knowledge and skills. To address this limitation, general education teachers' skills in identifying prerequisite skills for learning specific mathematics skills and assessing student progress accurately should be enhanced (Cawley et al., 2001). Preservice and in-service teachers could benefit from training in these skills.

Third, IDEA requires that students with MD participate in and make progress in the general education curriculum. This study revealed encouraging findings with regard to the learning of students with MD in standards-based mathematics, general education classrooms in terms of prerequisite skills, problem solving, and transfer. However, their learning in the standards-based mathematics, general education classroom was not as good as their typically achieving peers, or even as good as struggling students in most comparisons. This result suggests that while students with MD make progress in standards-based mathematics general education curriculum and instruction, there is still plenty of a room for improving access to the general education curriculum and instruction through instructional adaptations that promote the progress of students with MD.

Correspondingly, the findings on student transfer indicated that once a student with MD reached mastery of a skill, the student was able to transfer the skill to new problems having the same conceptual structures. This finding suggests that instructional practices involving students with MD should focus on the mastery of a skill. Mastery can be achieved by providing a sufficient number of practice opportunities to students with MD as well as providing instruction on strategies to help them solve problems or use the strategies to solve similar problems. Also, findings suggest that instruction should involve training on the identification of problem conceptual structures and exposure to diverse problem structures to promote transfer of mathematics knowledge and skills of students with MD (Bassok, 1997).

Limitations of this Study

Several limitations need to be considered in interpreting the findings of this study. This section addresses six specific limitations.

1. First, this study used a case study methodology to investigate the general education teacher's instructional adaptations for her students with MD in the standards-based general education classroom and the learning of students with differing ability in these environments. Case studies are the preferred strategy when "how" or "why" questions are being posed, when the investigator has little control over events, and when the focus is on a contemporary phenomenon within some real-life context (Yin, 2003). Control or comparison groups and random assignments of participants to each condition were not used in this study. Thus, the findings of this study should be interpreted as contextualized exploratory and instrumental, rather than as generalizable across settings.

2. Observational methodology might have influenced the findings of this study. The researcher's presence in the classroom might have elevated the teacher's attentions on the student participants during her instruction, and the students' engagements in class activities during the

observational period, even though the teacher and the students were not informed of what the researcher was observing.

3. The teacher participant and the student participants in the current study do not represent a random sample of teachers who provide instruction in standards-based mathematics, general education settings and of students with differing ability who receive mathematics instruction in these environments. The teacher was selected by a school district mathematics curriculum coordinator as implementing a standards-based mathematics, general education curriculum and as having students with MD in her class. The students were selected by the teacher participant as students with MD identified by school district, struggling students, and a typically achieving student. Heterogeneity of ability of students with MD, struggling students, and typically achieving students could not be represented by this sample.

4. The general education mathematics curriculum, Math Investigations (TERC, 1998, 2004) may not represent a typical standards-based mathematics curriculum. Jitendra et al. (2006) conducted a curriculum analysis in which programs were selected because mathematics educators, school administrators, and teacher recommended them as representative textbooks typically adopted in the United States. Jitendra et al.'s (2006) study investigating adherence of mathematics curriculum to the NCTM standards did not include the Math Investigations for analysis.

5. The clinical interview tasks used in this study had not been empirically validated through a field study. The NCTM standards support student-centered assessments including portfolio assessment, think-aloud techniques, and student interviews (Salend, 1996). Clinical interviews are a method to explore students' understanding of mathematics knowledge and skills more in depth, as the NCTM standards suggest (Ginsburg, 1997). Previous studies examined students' mathematics knowledge and skills using clinical interview tasks that they had developed for their own studies (e.g., Ginsburg, 1997; Ginsburg & Pappas, 2004). Thus, the researcher of this study developed the clinical interview tasks after reviewing TEKS standards and the textbook. After the clinical interview tasks were created based on the curriculum, the tasks were given to the teacher twice (before and after instruction) to assure that the tasks measured the instructional content she would teach or had taught. The teacher stated that all clinical interview tasks were well aligned with the content that she taught in her class.

> I think that they should be able to do that after my class or homework. They are pretty similar to what they are doing in their class. I think it's good. …Some of these are basically same with what I gave them for homework yesterday.

However, even though the clinical interview tasks were developed based on literature and went along with the suggestions from the NCTM standards for assessment, the fact that the findings of this study were not derived from standardized assessments should be considered in the interpretations of the findings of this study.

6. The findings of this study should be interpreted very cautiously because geometry and spatial reasoning and probability and statistics do not represent typical mathematics topics that students with MD show difficulties in and for which general education teachers' adaptations occur. In addition, there are no appropriate studies to compare the findings of this study with due to the lack of research on these two mathematics content (Rivera, 1997). The instructional adaptations of the teacher during instruction on these topics might be different from the instructional adaptations that occur with other topics such as number and numerical reasoning. Along the same lines, the mathematics learning of students with differing ability in standards-based mathematics general education settings may vary across mathematics topics. Due to the lack of research on these mathematics topics (Rivera, 1997), these limitations should be considered in interpreting the findings of this study.

Implications for Future Research

This exploratory, instrumental study provides several implications for future research. This section presents seen specific implications.

1. This study was designed as an exploratory study of the topics investigated in this study. Future research should be conducted using experimental designs including control or comparison groups, random sampling, and random assignments of participants to each condition, with large sample sizes.

2. The findings of this study were limited to what is happening in standards-based mathematics, general education classrooms during geometry and spatial reasoning instruction as well as probability and statistics instruction. Future research should include other grade-appropriate or developmentally appropriate mathematics topics, including computations, fractions, and word-problem solving.

3. This study did not use standardized assessments, although it included quantitative data as well as qualitative data on student performances. Future research should consider including standardized assessments along with clinical interviews to increase the interpretability of the findings of line of research.

4. This study did not examine the relationships of a teacher's instructional adaptations with the degree of the learning of all students in her standards-based mathematics, general education classroom. Previous research (e.g., Fuchs et al., 1995) noted that general education teachers' instructional adaptations for students with MD resulted in the progress of students without disability as well as that of students with MD. In Fuchs et al.'s (1995) study, students without disability gained more than students with MD from the adapted instruction. Future research may examine how general education teachers' instructional adaptations influence the learning of students, including students with MD, in standards-based mathematics classrooms.

5. The findings of this study on the learning of students with differing ability were based on data from two clinical interview tasks on each mathematics topic. During the period of this study, the teacher taught three to five different knowledge and skills during instructions on each mathematics topic. Because it was not possible to collect data on all the skills taught during the study period, in reality, only two knowledge and skills per each mathematics topic were examined. Future research may include more investigations on student learning, distributed across all fourth-grade lessons on geometry and spatial reasoning as well as probability and statistics.

6. The findings of this study on the teacher's instructional adaptations differ from those of previous studies on the teacher's perceptions of instructional adaptations to meet the performance expectations from the standards (e.g., Maccini & Gagnon, 2000, 2006). The differences were discussed in terms of possible differences between teachers' perceptions and their actual use of evidence-based, effective instructional components to adapt their mathematics instruction. However, no study was found examining potential differences between teachers' perceptions and their actual use of specific instructional components to enrich the discussion. Further study may be conducted to examine this topic.

7. Finally, this study did not include investigations to provide explanations of the underlying processes (e.g., recognizing conceptual structure in a problem, one-to-one correspondence between the problems) that may cause students' failure in transfer of mathematics knowledge and skills. It

would be interesting to examine the processes that underlie the difficulties of students with MD in transfer of their mathematics knowledge and skills and to investigate if students with differing ability show difficulties in the same processes.

Summary

Seven themes emerged from the findings of this study, four on Ashley's instructional adaptations for her students with MD in the standards-based mathematics, general education classroom and three on the learning of students with differing abilities in this environment. The findings of this study indicate that Ashley endeavored to adapt her mathematics instruction for the students with MD using diverse components of effective mathematics instruction, but her adaptation was limited in terms of the number of students targeted for adapted instruction, the difficulties of the students who were targeted for adapted instruction, and the categories of adaptation category. Possible factors inhibiting Ashley's adaptation included the number of students with MD in the classroom.

On the other hand, the findings of this study indicate that the quality and quantity learning of mathematics knowledge and skills was different across students with differing ability in the standards-based mathematics, general education classroom in terms of prerequisite skills, problem-solving accuracy, concept or procedures for problem solutions, and transfer of knowledge and skills. Even though all the students with differing ability benefited to some degree from standards-based mathematics instruction, the benefits of students with MD from this instruction were marginal in comparison to the benefits of their peers without disabilities. In summary, the findings of this study suggest that alternative instructional methods should continue to be explored to maximize the benefits of students with MD in standards-based mathematics, general education classrooms, including more frequent integration of varied types of effective mathematics components into standards-based mathematics instruction and considering the cognitive, behavioral characteristics of students with MD.

APPENDIX A:

OPEN-ENDED INTERVIEW QUESTIONS

1. Standards-based mathematics instruction for all students & Instruction for students with mathematics difficulties (knowledge and skills)

   a. What does standards-based mathematics instruction look like in fourth grade classrooms? Please describe the mathematics instruction you use in your classrooms

   b. Probe:

      i. Is it different for typically achieving students and low achieving students? If so, tell me how?

      ii. Tell me which features of standards-based mathematics instruction you are mostly using in your classrooms?

2. Prerequisite skills for learning fourth grade geometry and probability

   a. What mathematics knowledge and skill do you think a student should have in order to learn fourth grade geometry and probability?

   b. Probe:

      i. Tell me what mathematics knowledge and skills you expect your students will bring from their previous learning so they will be ready for lessons on <u>geometry (Making a 3-D building based on a 2-D configuration & Finding solids that make a silhouette)</u>.

      ii. Tell me what mathematics knowledge and skills you expect your students will bring from their previous learning so they will be ready for lessons on probability (Making a bar graph showing the data set & Comparing two bar graphs).

3. For your students with IEP in mathematics, describe strengths and challenges they have when learning mathematics skills and concepts.

4. Tell more about instruction specifically for students with IEPs. How do you help them learn mathematics in your classroom (adaptations).

Probe for

1. changes in instructional content
2. changes in instructional materials
3. changes in activities
4. changes in pedagogy

5. How do you know if students with IEPs are learning the mathematics skills and concepts (evaluation/PM)?

APPENDIX B:

TEACHER SURVEY QUESTIONNAIRE

Teacher Survey Questionnaire Student name:________________

THE STUDENT IS ABLE TO	Always	Sometimes	Not at all
1. Describe and identify objects in order to sort them according to a given attribute using informal language.			
2. Identify circles, triangles, and rectangles, including squares, and describe the shape of balls, boxes, cans, and cones.			
3. Combine geometry shapes to make new geometry shapes using concrete models.			
4. Identify attributes of any shape or solid.			
5. Use attributes to describe how two shapes or two solids are alike or different.			
6. Cut geometric shapes apart and identify the new shapes made.			
7. Use whole numbers to locate and name points on a line.			
8. Name, describe, and compare shapes and solids using formal geometric vocabulary.			
9. Identify congruent shapes.			
10. Create shapes with lines of symmetry using concrete models and technology.			
11. Identify lines of symmetry in shapes.			
12. Locate and name points on a line using whole numbers and fractions such as halves.			
13. Collect and sort data.			
14. Use organized data to construct real object graphs, picture graphs, and bar-type graphs.			
15. Draw conclusions and answer questions using information organized in real-object graphs, picture graphs, and bar-type graphs.			
16. Identify events as certain or impossible such as drawing a red crayon from a bag of green crayons.			
17. Construct picture graphs and bar-type graphs.			
18. Use data to describe events as more likely or less likely such as drawing a certain color crayon from a bag of seven red crayons and three green crayons.			
19. Collect, organize, record, and display data in pictographs and bar graphs where each picture or cell might represent more than one piece of data.			
20. Interpret information from pictographs and bar graphs.			
21. Use data to describe events as more likely, less likely, and equally likely.			

(Excerpted from Texas Essential Knowledge and Skills (TEKS) for Mathematics (TEA, 1998)

APPENDIX C:

STUDENT DOCUMENT REVIEW FORM

Student ID		Teacher ID	
Date			
1. Source of Document			
2. Summary of Document			
3. Reflective Commentary			

APPENDIX D:

TEACHER DOCUMENT REVIEW FORM

Teacher ID		Student ID	
Date			
1. Topic or skill			
2. Objectives			
3. Prerequisite skills			
4. Instructional routines with time assignment			
5. Activities			
6. Planned (suggested) instructional adaptations			
7. Use of technology			
8. Assessment			
9. Reflective Summary			

APPENDIX E:

START LIST OF CODES

General Property/Individual Code	Code	Research Question
INTENTION	I	RQ.1.
I: PLANNED ADAPTATIONS PRIOR TO INSTRUCTION	I:PAPI	
Student Difficulty Recognized	I:SDR_PAPI	
Special Difficulty Identified by Teacher	I:SDR/SD_PAPI	
Overall Benefit Expected	I:SDR/OB_PAPI	
Difficulty Documented in an IEP	I:SDR/DIEP_PAPI	
Management Issue Recognized	I:MIR_PAPI	
I: UNPLANNED ADAPTATIONS	I:UA	
Response to Student Answers	I:RSA_UA	
Supplemental Instruction Based on Progress-Monitoring	I:SIPM_UA	
SETTING	S	
S: WHOLE GROUP INSTRUCTION	S:WGI	
S: SMALL GROUP INSTRUCTION	S:SGI	
CHANGES	C	
C: USE OF FEATURES OF STANDARDS-BASED MATHEMATICS INSTRUCTION	C:FSMI	
Inquiry-based Instruction	C:IBI_FSMI	
Discourse-driven Instruction	C:DDI_FSMI	
Contextualized Problem-Solving Instruction	C:CPSI_FSMI	
C: USE OF FEATURES OF EFFECTIVE MATHEMATICS INSTRUCTION	C:FEMI	
Explicit Explanation of Concept/Procedure	C:EECP_FEMI	

Verbal Direction	C:EECP/VD_FEMI	
Representation	C:EECP/R_FEMI	
Tricky Part/Possible Errors Discussed	C:EECP/TPPE_FEMI	
Examples Provided	C:EECP/EP_FEMI	
Rephrased Direction in Easy Words	C:EECP/RD_FEMI	
Explicit Modeling	C:EM_FEMI	
Modeling by Teacher	C:EM/MT_FEMI	
Modeling by Student Helper	C:EM/MS_FEMI	
Control Difficulty	C:CD_FEMI	
Sequenced Tasks/Activities	C:CD/STA_FEMI	
Segmented Problems/Tasks	C:CD/SPT_FEMI	
Easier Problems/Tasks	C:CD/EPT/FEMI	
Use of Prompts or Cues	C:CD/PC_FEMI	
Example	C:CD/E_FEMI	
Sample Answers	C:CD/SA_FEMI	
Direct Questioning	C:DQ_FEMI	
Used for Checking for Understanding	C:DQ/CU_FEMI	
Used for Prompting	C:DQ/P_FEMI	
Used for Correcting Answers	C:DQ/CA_FEMI	
Cognitive Memory Level	C:DQ/CM_FEMI	
Convergent Level	C:DQ/CL_FEMI	
Divergent Level	C:DQ/DL_FEMI	
Evaluative Level	C:DQ/EL_FEMI	
Practice	C:P_FEMI	
Guided Practice	C:P/GP_FEMI	
Using Multiple Examples	C:P/GP/UME_FEMI	
Drill & Repeated Practices	C:P/GP/DRP_FEMI	
Reteaching	C:P/GP/RT_FEMI	
Independent Practice	C:P/IP_FEMI	
Teaching Explicit Strategies	C:TES_FEMI	
Cognitive Strategy	C:TES/CS_FEMI	
Meta Cognitive Strategy	C:TES/MCS_FEMI	

Use of Cognitive Strategy	C:TES/MCS/CS_FEMI	
Verification of Answers	C:TES/MCS/VA_FEMI	
Strategy Cue	C:TES/SC_FEMI	
Effective Feedback	C:EF_FEMI	
Corrected Answer by Teacher	C:EF/CAT_FEMI	
Prompts to the Correct Answer	C:EF/PCA_FEMI	
Corrected Answer by Another Student	C:EF/CAAS_FEMI	
Elaborated Student Response by Teacher	C:EF/ESRT_FEMI	
Advanced Organizer	C:AO_FEMI	
Connection to Prior Knowledge	C:AO/CPK_FEMI	
Lesson Purpose	C:AO/LP_FEMI	
Purpose Setting	C:AO/PS_FEMI	
Prompted Review	C:AO/PR_FEMI	
Focused Materials	C:AO/FM_FEMI	
Graphic Organizer	C:AO/GO_FEMI	
Outline/Overview	C:AO/OO_FEMI	
Group Instruction	C:GI_FEMI	
Individualized	C:GI/I_FEMI	
Heterogeneous Small Group Instruction	C:GI/HESG_FEMI	
Homogeneous Small Group Instruction	C:GI/HOSG_FEMI	
Partner Work	C:GI/PW_FEMI	
Multiple Grouping Format	C:GI/MGF_FEMI	
Review of Skills Taught	C:RST_FEMI	
Review of Prerequisite Skills	C:RPS_FEMI	
Vocabulary	C:RPS/V_FEMI	
Component Skills	C:RPS/CS_FEMI	
Use of Manipulatives	C:UM_FEMI	
Teacher Presentation	C:UM/TP_FEMI	
Student Problem-Solving	C:UM/SPS_FEMI	
Progress-Monitoring/Assessment	C:PM_FEMI	
Checking for Understanding Skills Taught	C:PM/CUST_FEMI	
Task/Activity Understanding	C:PM/TAU_FEMI	

Task Completion (Correctness)	C:PM/TC_FEMI	
Prerequisite skills	C:PM/PRS_FEMI	
Use of Technology	C:UT_FEMI	
Teacher Presentation	C:UT/TP_FEMI	
Student Problem-Solving	C:UT/SPS_FEMI	
Vocabulary Instruction	C:VI_FEMI	
Review of Prior Vocabulary	C:VI/RPV_FEMI	
New Vocabulary	C:VI/NV_FEMI	
Peer Tutoring	C:PT	
C: OTHERS	C: O	
Assessment	C:A_O	
Difficulty Change	C:A/DC_O	
Time Change	C:A/TC_O	
Frequency Change	C:A/FC_O	
Mode Change	C:A/MC_O	
Assignment Modification	C:AM_O	
Number of Problems	C:AM/NP_O	
Mode of Products	C:AM/MP_O	
Extended Time	C:ET_O	
Problems Read to Students	C:PRS_O	
Reteaching/ Reexplaining	C:RTRE_O	
Connection to Everyday Life	C:CEL_O	
Peer Tutoring	C:PT_O	
MATHEMATICS KNOWLEDGE & SKILLS OF STUDENTS IN STANDARDS-BASED MATHEMATICS CLASSROOM	MKS	RQ.2.
MKS: PREREQUISITE SKILLS	MKS:PRS	
Algorithm	MKS:A_PRS	
Strategy	MKS:S_PRS	
Key Concepts	MKS:KC_PRS	
Representations	MKS:R_PRS	
Vocabulary	MKS:V_PRS	

MKS:PROBLEM-SOLVING PROCESS	MKS:PSP	
Understanding of the Problem	MKS:UP_PSP	
Use of Skills Taught in Class	MKS:UST_PSP	
Algorithm	MKS:UST/A_PSP	
Strategy	MKS:UST/S_PSP	
Key Concepts	MKS:UST/KC_PSP	
Representation	MKS:UST/R_PSP	
Exploration of Multiple Strategies	MKS:EMS_PSP	
Verification of Solution	MKS:VS_PSP	
MKS:MATHEMATICAL COMMUNICATION	MKS:MC	
Use of Mathematics Vocabulary	MKS:UMV_MC	
Use of Symbols/Notations	MKS:USN_MC	

APPENDIX F:

SCRIPTS FOR CLINICAL INTERVIEW TASK 1

Modeling of Thinking-Aloud

Today, we are going to do some games. In this game, I want you to say out loud what you are thinking in your head about how to solve a problem. I want you to tell me everything you are thinking to answer the problem.

1. Here is a problem we need to answer.

(Show the problem below)

Look at the two letters below and find a letter in symmetry.
P : M

Listen carefully. I am going to show you how I say out loud what I am thinking as I try to answer this problem. I need to find which letter is symmetrical. My teacher told me that when I can divide a shape exactly in half, I can say the shape as symmetrical. Both halves must be the same when I fold a shape in half. I am folding the letter “P” in my mind in a vertical way (*Showing the direction with a hand*) and then in a horizontal way (*Showing the direction with a hand*), but I cannot fold it in halves that are the same in either way. So, the answer is no. “P” is not symmetrical because the halves are not the same. Next, I am trying to fold the letter “M” in halves that are the same. I can fold it to make two halves that are exactly the same size and shape. So, the answer is yes. “M” is symmetrical because the halves are the same.

2. Here is another problem. Let’s solve it together with thinking out loud.

Look at the two figures below and find a figure in symmetry.
 : 

(Provide a time for guided practice).

3. Now, it's your turn to answer a problem, using "think out loud." Here is another problem.

(Show the problem below)

Look at the two figures below and find a figure in symmetry.
☺ : 🗀

Tell me everything you are thinking in your head about how to answer the problem as you solve the question: Is the figure symmetrical-yes or no. Why or why not?

(*Check if the student is actually able to think aloud and encourage the student to do it in 30 seconds; For a student who is not able to think aloud correctly at the first time, give one more guided practice opportunity and then an independent practice opportunity.*)

Instructions for the Task

Now, I am going to introduce another math game. There are many interlocking cubes and a card showing a drawing of a cube building. I am going to ask you to make the building shown in the cards using the cubes.

As you make the building, talk out loud what you are thinking in your head about how to make the building. Tell me everything you are thinking. It's okay to say everything you are thinking. What question do you have?

Task 1_ Problem 1

Student Name:____________________________ Date:______________________________

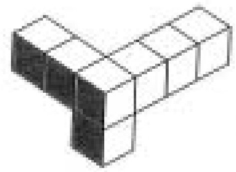
<table>
<tr><th>Prompts</th><th colspan="2">Responses</th></tr>
<tr><td rowspan="3">1. Before making the building, tell me how many cubes it will take to make the building shown in the card.
2. Can you show me how you count cubes?</td><td># of Cubes</td><td></td></tr>
<tr><td>Counting Strategies</td><td></td></tr>
<tr><td>Perception of 3-D in 2-D drawings</td><td></td></tr>
<tr><td rowspan="3">3. Provide each problem on a card. Encourage a student to talk out loud his/her thinking while he/she is trying to answer each problem in 30 seconds.</td><td>Completion Time</td><td></td></tr>
<tr><td>Working Process</td><td></td></tr>
<tr><td>Output</td><td></td></tr>
<tr><td>4. How did you know how to make your building? Why did you make your building like this?</td><td colspan="2"></td></tr>
</table>

5. How do you know if your building is correct? *(If the student says "I don't know" show the drawing of a cube building, have the student compare it to their building and ask him "is your building same with this?" and/or "how are they different?")*	How to know…	
	Differences between their work & drawings	
	Change after the prompt	
6. How did your teacher show how to make the building? How did your classmates make the building?	Teacher	
	Classmates	
7. Do you know another way to make the building?		

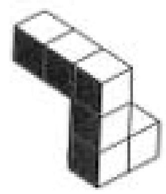
APPENDIX G:

PROBLEMS IN CLINICAL INTERVIEW TASK 1

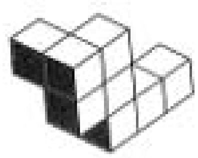
Problem 1



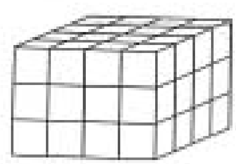
Problem 2



Problem 3



Problem 4



APPENDIX I:

EXAMPLES OF DATA CODED

LESSON	OBSERVATIONS	CODES
1 (86-105)	T: Okay, let's go ahead and someone tell me how many cubes we are going to use for the # 1 building? Lee? Lee: 4. T: Okay, she counted these. Let's point to the one T: How many numbers in the bottom? Lee: One. T: One, How many cubes in the bottom? Lee: One. T: Just one. T: How many on the top? Lee: NR. T: One, two, three.. Lee: four, T & Lee: four, five, six. T: So, there are 6 plus one, so how many? Lee: 7. T: 7 cubes. What did you say, Lee? Lee: 4 (Laugh). T: So, we are going to need 7 cubes.	S:WGI 86-105 I:RSA_UA For all adaptations: C:EF/CAT_FEMI C:DQ_FEMI C:DQ/CM_FEMI C:DQ/P_FEMI C:CD/SPT_FEMI C:RPS/CS_FEMI
(243-266)	(To Lee Group: Lee & S2). T: You started with the bottom 3. Right now, can you try to match this with this so far? L & S2: (Made it). T: There we go. Now, you are going to build the back. That's something you can do to make it sure if you are correct. You already did that. What can you do to check yourself? S_2: Counting the volume of the building. T: It can be, but it would be better to match your work with the building on your sheet. T: Lee, can you point that cube for me? (Pointed at the 2nd cube on the left side on the sheet). Let's put it the way looks first of all. Position it as it looks. Double check it. There we go. Lee: (Made it). T: Good job. Can you point this cube (the second in the middle) for me? Lee: (Made it). T: Very good. (Asked one to S2) Once you finish the number 2, try to label the building. You are going to go ahead to label that.	• S:SGI 243-266 I: RSA_UA For the followings: C:PM/CUST_FEMI C:PM/TC_FEMI C:TES/MCS/VA_FEMI • 243-266 I:SDR/OB_PAPI for C:GI/SG_FEMI

6	PAIRING UP LEE WITH HIGH TO AVERAGE STUDENTS	S:SG I: SDR/OB_PAPI C:GI/HESG_PAPI
(197-214)	T: You may pick one. How many brothers do you think might be a typical in our class? Kevin: 2. T: Why, Kevin? Why 2? Kevin: I see a lot of 2 there. T: You see 2 a lot there? I see 14 has 2, 11 has 2 and that's it. I see a lot of 0. I see number 2, number 7, 8, 9, 10, 12, and 13. These people have no brother. A couple of you have two and a couple of you have 1. What is between 0 and 3? S_some: One. T: Maybe one. Maybe two. I am going to go with one. One brother may be typical in our class. Could zero be a typical number? S_some: Yes. Kevin: No. T: Yes, it could be a typical number. More people in this class have zero brothers.	S:SG 197-214 I:RSA_UA C:DQ/CM_FEMI C:DQ/CU_FEMI C:EF/PCA_FEMI C:EF/CAT_FEMI

APPENDIX J:

SUMMARY OF CODES OF INSTRUCTIONAL ADAPTATIONS FOR STUDENTS WITH MD

<table>
<tr><th>Lesson</th><th>Source of Data</th><th colspan="2">Lee</th><th>Kevin</th><th>Tina</th><th>All Struggling Students</th></tr>
<tr><td rowspan="2">1. Geometry: Building with cubes: Students put interlocking cubes buildings shown in drawings</td><td rowspan="2">Class Observation</td><td>WG</td><td>• 86-105
I: RSA_UA
C:EF/CAT_FEMI
C:DQ_FEMI
C:DQ/CM_FEMI
C:DQ/P_FEMI
C:CD/SPT_FEMI
C:RPS/CS_FEMI
• 177-223
I:SIPM_UA
C:EECP/TPPE_FEMI
C:EF/CAAS_FEMI</td><td>• None</td><td>• 478-485
I:UA
C:EF/PCA_FEMI
C:EF/CAT_FEMI
C:DQ/CU_FEMI
C:DQ/CM_FEMI</td><td>• 18-24
S:WGI
I:SDR/SD_PAPI
C:VI/RPV_FEMI
C:DQ/CM_FEMI
C:DQ/CU_FEMI</td></tr>
<tr><td>SG</td><td>• 243-266
I:RSA_UA
C:PM/CUST_FEMI
C:PM/TC_FEMI
C:TES/MCS/VA_FEMI
• 243-266
I:SDR/OB_PAPI
C:GI/SG_FEMI
• 333-359
I:RSA_UA
S:GI/HESG_FEMI
C:PM/TAU_FEMI
C:EF/PCA_FEMI
C:DQ/CM_FEMI
C:DQ/CU_FEMI

I:SDR/OB_PAPI
C:UM/SPS_FEMI
C:AC_O</td><td>• 143-153
I:UA
C:EF/PCA_FEMI
C:DQ/CU_FEMI
C:DQ/CL_FEMI
• 268-273
I:UA
C:PM/CUST_FEMI
• I:SDR/OB_PAPI
C:GI/HESG_FEMI</td><td>• 415-422
I:UA
C:EF/CAT_FEMI
C:PM/TC_FEMI
• I:SDR/OB_PAPI
C:GI/HESG_FEMI</td><td>• 42-45
S:SG
I:SDR/OB_PAPI</td></tr>
</table>

			• 406-413 I: RSA_UA C:PM/TAU_FEMI C:EF/ESRT_FEMI • 424-432 I:UA C:PM/TAU_FEMI C:DQ/CL_FEMI C:DQ/CU_FEMI			
	Interview	FI	• C:AM/NP_O • C:UM/SPS_FEMI • C:ET_O • C:AC_O • C:A/FC_O • C:A/MC_O • C:GI/HESG_FEMI • C:RTRE_O • C:A/DC_O • C:VI	• C:AM/NP_O • C:UM/SPS_FEMI • C:ET_O • C:AC_O • C:A/FC_O • C:A/MC_O • C:GI/HESG_FEMI • C:RTRE_O • C:A/DC_O • C:VI	• C:AM/NP_O • C:UM/SPS_FEMI • C:ET_O • C:AC_O • C:A/FC_O • C:A/MC_O • C:GI/HESG_FEMI • C:RTRE_O • C:A/DC_O • C:VI	• C:AM/NP_O • C:UM/SPS_FEMI • C:ET_O • C:AC_O • C:A/FC_O • C:A/MC_O • C:GI/HESG_FEMI • C:RTRE_O • C:A/DC_O • C:VI
		IFI	• C:GI/HESG_FEMI • C:AC_O • C:UM/SPS_FEMI	• C:GI/HESG_FEMI	• C:GI/HESG_FEMI	• C:GI/HESG_FEMI
	Lesson Plan	TL	• C:ET_O (e.g., Allow more time for understanding) • C:GI/HESG_FEMI (e.g., Pair her up with high to average students)	• C:ET_O (e.g., Allow more time for understanding) • C:GI/HESG_FEMI (e.g., Pair her up with high to average students)	• C:ET_O (e.g., Allow more time for understanding) • C:GI/HESG_FEMI (e.g., Pair her up with high to average students)	• C:ET_O (e.g., Allow more time for understanding) • C:GI/HESG_FEMI (e.g., Pair her up with high to average students)

APPENDIX K:

EXAMPLE OF A SUMMARY OF AN INDIVIDUAL STUDENT DATA ON RESEARCH QUESTION 2

Skills	Pre	Post	Difference
Kevin			
Prerequisite skills • P1: 2 (Fiona & Heike) • P2: 5(Angela, Cathy, Erin, Kelly, & Noah) • P3: 6(Angela, David, Erin, Isabella, Luke, and Noah) P4: 3(Angela, Jade, & Mary Ann)	• P1:The number of students having 4 brothers and sisters shown in the graph -2(Fiona & Heike) • P2:The number of students being taller than 55 inches -5(Angela, Cathy, Erin, Kelly, & Noah) • P3:The number of students choosing an apple -6(Angela, David, Erin, Isabella, Luke, & Noah) • P4:The number of students choosing baseball -3(Angela, Jade, & Mary Ann)	• P1:The number of students having 4 brothers and sisters shown in the graph -2(Fiona & Heike) • P2:The number of students being taller than 55 inches -5(Angela, Cathy, Erin, Kelly, & Noah) because they have more than 55 • P3:The number of students choosing an apple -6(Angela, David, Erin, Isabella, Luke, & Noah) because they have apples • P4:The number of students choosing baseball -3(Angela, Jade, & MA) because they have baseball on the other side of their names.	• On both pretest and posttest, he showed the skills of reading a table representing data of both continuous variables and categorical variables.
Problem-solving 1. No knowledge about how to draw a graph representing data set 2. Display of data as not summarized a. Attempted but incorrect product b. Attempted and correct product 3. Display of data as summarized a. Attempted but	• P1: -2a(2 stories graph) -Put each student name on the X, put numbers from 0 to 6 on the Y, draw a bar for each student according to the number of their brothers & sisters, and color them. • P2: -2a -Put student names written in the right side of the table on the bottom(did not do any with names in the left side of the table) , put numbers from 51 to 60 on the left side of the paper, draw a bar for each student according to their heights, and color them.	• P1: -2b -Put student names at the bottom, and draw a bar graph for each student according to their brothers and sisters -Showed problems with counting boxes in a bar. -When I asked the meaning of numbers on the Y, he answered "how many brothers & sisters" • P2: -2b -Put student names on the bottom, put numbers from 51 through 58 on the right side, draw a box to show each student height. • P3: -3b ***-Put the student names at the bottom as he did with P1 & P2, erase them, and put fruit names***	• On pretest, he tried to make a graph on P 1 & P2 but they were not correct. On P3 & P3, he produced correct graphs but he did not know semantic relationships on graphs (when I asked "if it is the number of students who chose each fruit or sports", he answered " it is the number of each fruit or each sports). • On posttest, he showed difficulties with counting. He did not try to summarize data before making a graph on both problem 1 and 2. But on problem 3 and 4, he summarized data and drew the graph correctly even though he showed difficulties with counting and unstable understanding of semantic relationship.

incorrect product b. Attempted and correct product(variable & frequency)	• P3:3b -Put fruit names on the X, put numbers from 1 to 6 on the Y, count ***the number of fruits (Not the number of students who chose each fruit),*** and draw a bar for each fruit. (Right answer but incomplete understanding of the data) • P4 -3b -The same way with P3	***at the bottom. When I asked "why did you erase student names?" he answered, "because I want to draw how many peoples pick each fruit up to 5". When I asked "why 5", he answered, "because sometimes there are more than 3 peoples"*** • P4 -3b -Put the sports names at the bottom(because if I do the names of students at the bottom, it will be messed up. So, I am putting sports names here), put the numbers from 1 to 5 on the left side (maybe ***some people (wrong)***go over 3), and draw a bar graph of the number of students choosing specific sports. -Trouble with counting.	
Strategy for problem-solving	• P1: -Look at the numbers and look at the names and color them. -No other strategy -Was not taught • P2: -Look at the numbers on the card, put the numbers on the side, and draw the lines after counting. -No other strategy -Was not taught • P3: -Count the fruits and put the numbers here.(Did not understand semantic relationship shown in the graph) -No other strategy -Was not taught • P4 -count the sports, look for number of sports on the Y, and draw it -No other strategy -Was not taught	• P1: -Look at the card -No other strategy -The teacher did not teach • P2: -Look at the card, and match the numbers on the card with numbers on my graph -No other strategy -She did not teach • P3: -Look at the card, numbers on the card, and count the number of people who pick each fruit. -No other strategy -The teacher did not teach. • P4 -Look at the card, count the number of favorite sports, and match it with number on the side on graph. -No other strategy -She did not teach	• Pretest; Even though they sounded to have used right strategies, they failed because they skipped some information (left side data) or did not summarize them before graphing them. • Posttest: He did not remember he was taught about the skills. He showed difficulties with counting boxes.

Meta-cognitive (Answer verification/ Use of problem-solving strategy)	• P1: Look at the number of the cells of people. Count the cells to check if it is correct. • P2:Look at the numbers on both card and graph • P3:Count the apples, bananas & peaches • P4: Look at and count the sports.	• P1:Look at the card, look at the peoples, look at the numbers, and compare them • P2:Look at people, and match the people with their numbers. • P3:Look at the fruit, count how many apples, and see if it is equal to the number on the side on his graph. • P4: Count ***the favorite sports*** to see if it equals to the numbers on the side on graph	
Lee			
Prerequisite • P1: 2 (Fiona & Heike) • P2: 5(Angela, Cathy, Erin, Kelly, & Noah) • P3: 6(Angela, David, Erin, Isabella, Luke, and Noah) • P4: 3(Angela, Jade, & Mary Ann)	• P1:The number of students having 4 brothers and sisters shown in the graph -2(Fiona & Heike) • P2:The number of students being taller than 55 inches -5(Cathy, Erin, Noah, Isabella (wrong), & Kelly) • P3:The number of students choosing an apple -6 (A, D, E, I, L, & N) • P4:The number of students choosing baseball -3 (A, J, & MA)	• P1:The number of students having 4 brothers and sisters shown in the graph -2(Fiona & Heike) • P2:The number of students being taller than 55 inches -5(A, C, E, K, & N) • P3:The number of students choosing an apple -5 (wrong, A, D, E, I, & N) • P4:The number of students choosing baseball -3(A, J, MA)	
Problem-solving 1. No knowledge about how to draw a graph representing data set 2. Display of data as not summarized a. Attempted but incorrect product b. Attempted and correct product 3. Display of data as summarized a. Attempted but	• P1: Continuous variable -1 -Draw a horizontal line on the grid, draw 4 boxes in the first column, write “Fiona” on the top of it, draw 4 boxes in the 3rd column, and write “Heike” • P2:Continuous variable -1 -Draw a horizontal line on the grid, count 56 boxes in the 3rd column, draw a bar and write “Erin, Cathy, Kelly” on its top; Does the same thing with Noah, and Isabella • P3:Categorical variable	• P1: Continuous variable -2b -Put student names at the bottom, draw the number of boxes as many as the number of brothers and sisters per each student; 2 for Angela, 2 for Bob, One for Cathy, etc. • P2:Continuous variable -2a -Put a horizontal line, count 56 boxes draw a bar, and put “Angela” on the top of it. Count 56 boxes and draw a bar for Cathy. Count 56 boxes and draw a bar for Kelly. Count 58 boxes for Noah and 54 boxes for Bob. • P3:Categorical variable -1 -draw a horizontal line, draw a box in the first	• On pretest: She did not know about how to draw a graph representing neither data including continuous variable nor data including categorical variable. Even though I presented the question several times (make a bar graph representing the whole table), she stuck to the data used for the prerequisite skill question. She did not try to draw a graph for the other data other than ones used for the prerequisite questions. • On posttest: On problem 1 & 2, she showed little improved skills of drawing graphs, but skipped information (data points) and did not try to summarize data before graphing it. On problem 3 & 4, she was totally confused with

incorrect product b. Attempted and correct product(variable & frequency)	-1 -Draw a horizontal line, draw a box in the first column, put "Angela" on its top, draw a box in the second column , put "David" on its top, and does the same thing with Erin, Luke, and Noah • P4:Categorical variable -1 -Made a picto-table including A, J, & MA	column, put "Angela" on top of it, and does the same thing for the other students who picked up apple; When I asked about the other students, she draw a box per each student and put their name on it; She put the line between the bars for students choosing apple and the other bars (because " apple is most important one. So, I put the line between apple and the other). • P4:Categorical variable -1 -Draw the figure of each sports under the line, put a box in each column, and put each student name on each column.	these problems. She just listed the students and their sports on the grid. She did not know which variable should be chosen to be drawn.
Strategy for problem-solving	• P1 -This is how I do it -Another strategy: Put the 4 boxes across two columns instead of one column. -Was not taught • P2 -Cathy, Erin, Kelly have 56, Noah has 58, and Isabella has 54. So, I count up 56, 58, and 54 to make a bar graph -No another strategy -Was not taught • P3 -Look at the table to see how many people have apple -No other strategy -Was not taught • P4 -Look at the table. Each of them has a soccer. -No other strategy -Was not taught	• P1 -Look at the name and see how many they have, and draw it on the graph -No other strategy -The teacher did not teach • P2 -Look at the table, and draw the graph based on the height. -No other strategy -The teacher did not teach • P3 -Look at the table, put how many they have. "This is easier because each has only one." -No other strategy -The teacher did not teach • P4 -One square for each sports and each kid -No other strategy -The teacher did not teach	• On pretest: On the problem 1 & 2, strategies are pretty immature and incorrect, but on the problem 3, she showed little knowledge what to do but did not how to draw it. On the problem 4, she was totally confused with what to do. • On posttest: On the problem 1 & 2, the strategies are better than on pretest, but she did not remember she was taught about the skills. On the problem 3, & 4, her strategies are proper for part of problem-solving but it's not good for the remaining part of problem solving and she did not know what to count and how to summarize data to draw the graph,

Meta-cognitive (Answer verification/ Use of problem-solving strategy)	• P1:Fiona has four, and Heike has four. I found them. • P2:Count them again • P3: DK • P4: They have a soccer.	• P1 -Compare the numbers between the table and the graph. • P2 -Count the number of boxes here on my sheet and double check the numbers are the same with the numbers in the table. • P3 -DK • P4 -Match my graph and the table	• On pretest: Inappropriate • On posttest: She knew what to check or what to review, but because she did not know how to draw a graph, she could not get to the correct product even though she checked numbers. (Answer Verification)

APPENDIX L:

IMPLEMENTATION VALIDITY CHECK FORM OF CLINICAL INTERVIEWS

Date	02-24-06	Interview Task	Task I
Class (Grade)	4^{th}	Interviewer	Sun A Kim

Yes = Followed each step or direction specified below in the same manner as prescribed
No = Not followed each step or direction specified below in the same manner as prescribed

Delivery of Clinical Interview The interviewer….	**Rating**		**Notes**
	Yes	**No**	
1. Reminded (encouraged) the student of using thinking-out loud before presenting the task instruction.			
2. Presented general instruction of the Task as in the protocol. *(Now, I am going to introduce another math game. There are many interlocking cubes and a card showing a drawing of a cube building. I am going to ask you to make the building shown in the cards using the cubes. As you make the building, talk out loud what you are thinking in your head about how to make the building. Tell me everything you are thinking. It's okay to say everything you are thinking.)*			
3. Provided the prompt 1 precisely *(Before making the building, tell me* ***how many cubes*** *it will take to make the building shown in the card.)*			
4. Provided the prompt 2 precisely. *(Can you show me how you count cubes?)*			
5. Presented each problem on a card and provided a brief instruction again.			
6. Prompt the student to talk out loud while he/she was solving the task			

(At least every 20 sec.)			
7. Provided the prompt 4 precisely. *(How did you know how to make your building? Why did you make your building like this?)*			
8. Provided the prompt 5 precisely. *(How do you know if your building is correct?)* *(If the student says "I don't know", show the drawing of a cube building, have the student compare it to their building, and ask him " Is your building same with this?", and/or "how are they different?")*			
9. Provided the prompt 6 precisely. *(How did your teacher show how to make the building? How did your classmates make the building?)*			
10. Provided the prompt 7 precisely. *(Do you know another way to make the building?)*			

GLOSSARY

Algorithms	A procedure for solving a mathematical problem in a finite number of steps that frequently involves repetition of an operation (Mish, 1994).
Assignment Modifications	The teacher reduces classwork problems and homework problems and adjusts the workload via color coding or circles for cuing (Maccini & Gagnon, 2006).
Advance Organizer	The teacher facilitates student learning by linking the current lesson to previous instruction, identifying the daily objective, giving a rationale for learning the skill (Butler et al., 2003), prompting students to review materials before instruction begins, directing students to focus on particular portions of materials that are being presented, providing information to students before engaging in discussion, and using graphic organizers and/or outline/overview organizer (Darch & Gersten, 1986).
Contextualized Instruction	The teacher has students engage with problem first in context, then with mathematical formality, and encourages students to see connections of mathematics to work and life (Forman & Steen, 2000).
Control difficulty (or processing demands) of a task	The teacher controls task difficulty by sequencing tasks from easy to difficult and providing only necessary hints to students, or providing simplified demonstration (Swanson, Hoskyn, & Lee (1999), 1999).
Effective Feedback	"The teacher provides instructive or elaborative feedback specifying the necessary steps, rules, or prompts to help students correctly answer the problem" (Jitendra, Salmento, & Haydt, 1999, p. 73). The teacher responds a student error by providing the correct answer or guide the student to the correct response (Carnine, Silbert, & Kame'enui, 1997).
Delivery of Instruction	The way in which an instructional activity is taught or presented, such as instructional grouping, instructional routines, and instructional language (Bryant & Bryant, 1998)

Differentiated (Adapted) Instruction	Instructional strategies, materials, or goals adapted to meet the abilities and needs of all students, especially struggling learners within a classroom, based on ongoing assessments of student progress (Glaser, 1977; Gunter, Denny, & Venn, 2000) Assignments, materials, and instruction presented in a format that is different for a particular student (Gelzheiser & Meyers, 1991) "Appropriate adjustments, accommodations, and modifications to instruction or supports that allow students to meet academic requirements and conditions of the curriculum, most often the general education curriculum" (Vaughn Gross Center for Reading and Language Arts [VGCRLA], 2001).
Directed Questioning and Responses	The teacher verbally provides process-related and/or content-related questions to students, or has students to ask questions (Swanson, Hoskyn, & Lee (1999), 1999)
Discourse-Driven Instruction	The teacher encourages students to learn mathematics concepts or procedures by engaging in the construction of shared mathematics knowledge in their classrooms, which is usually accomplished by verbal interactions between a teacher and students or among students (Baxter, Woodward, & Olson, 2001).
Elaborative Feedback	The teacher specifies the necessary steps, rules, or prompts to help students correctly answer the problem (Jitendra, Salmento, & Haydt, 1999).
Explicit Explanation of Concept or Procedure	The teacher provides complete, consistent, and logical explanations for concepts, skills, and/or activities through verbal direction and/or representation tools such as manipulatives (Jitendra, Salmento, & Haydt, 1999).
Explicit Instruction	Explicit teaching is instruction in which the teacher serves as the provider of knowledge, presents skills and concepts in a clear and direct fashion that promotes student mastery. In explicit instruction, the teacher provides an explanation or model of a skill or concept, guided practices of a skill or concept in a variety of situations, and multiple opportunities of independent practice to ensure mastery and generalization of the skill or concept taught (Mercer & Mercer, 2005).
Extended time	The teacher provides increased time for students to finish an activity or test (Maccini & Gagnon, 2006).

Group Instruction	The teacher uses grouping formats such as pairs or small groups as an alternative to whole group or independent seat work (Gersten, Schiller, & Vaughn, 2000; Swanson, Hoskyn, & Lee (1999), 1999)
Guided Practice	The teacher prompts students to perform the task and check to see whether they can work successfully without prompts. The teacher consistently monitors student progress and gives corrective feedback until students are ready to work independently. (Mercer & Mercer, 2005, p. 135). "The practice is guided by the teacher through factual or process questions and/or teacher demonstration. During this practice, the teacher provides feedback, evaluates understanding, and provides additional demonstration if necessary" (Rosenshine & Stevens, 1984, p. 759). "The teacher gives prompts and cues as students solve problems together. As students gain independence, the teacher monitors students and assists only as needed" (Butler et al., 2003).
Independent Practice	"Students solve problems independently using the skills that have been taught. The teacher did not provide assistance" (Butler et al., 2003).
Individualized	Each student works on materials unlike other students-may be alone or may be with teacher/aide (Vaughn, Gersten, & Chard, 2000)
Inquiry-Based Instruction	The teacher encourages students to explore and discover a variety of strategies for problem-solutions, and to investigate available data for problem solving (Forman & Steen, 2000).
Instructional Activity	The procedure, lessons, or strategy to teach the skills or concepts that are the focus of teaching and learning (Bryant & Bryant, 1998)
Instructional Content	Skills and concepts that are the focus of teaching and learning (Bryant & Bryant, 1998). It is related to the instructional objective and the state's curriculum.
Instructional Materials/Technology	Textbooks or other manipulatives used for mathematical representation (Bryant & Bryant, 1998) Computer software for drill and practice, calculators, overhead projector, or internet facility used for mathematics activities (Bryant & Bryant, 1998) A computer, structured text, flow charts, pictorial representations, and media to facilitate presentation and feedback (Swanson, Hoskyn, & Lee (1999), 1999)

Level of Questioning	Level of questions is determined by what the teacher is trying to get the student to do in response. Four levels of questions include (a) cognitive memory level, (b) convergent level, (c) divergent level, and (d) evaluative level (Callahan and Clarke, 1988). Questions at the cognitive level have a simple answer that the students are expected to know and are used to determine the student knowledge about factual information. Questions at the convergent level ask the student to explain, interpret, give examples, or summarize concepts in his or her own words and are used to the student's understanding of a subject. Divergent questions have students apply principles in new settings and involve problem solving or decision making. Evaluative questions require students to make a value judgment, to express opinions, to provide a criticism, or to raise their own questions. They require the highest form of thinking there are no right or wrong answers to evaluative questions.
Modeling	The teacher demonstrates the skill, processes or steps to solve a problem, or how to do a task using thinking aloud (Butler et al., 2003, Swanson, Lee, & Hoskyn, 1999). The teacher may solve a problem with students through a question-and-answer format after a short demonstration of the skill, algorithm, or strategy (Butler et al., 2003).
Multiple Grouping Formats	The teacher implement a specific combination of formats systematically (Vaughn, Gersten, & Chard, 2000)
Partners	A grouping format in which students work for sustained periods of time in pairs and take different roles including alternating being the tutor and tutee, and cooperative partnerships (Vaughn, Gersten, & Chard, 2000)
Prerequisite skills	Skills required to accomplish a new task successfully (UTCRLA, 2001).
Progress-Monitoring	The teacher frequently checks students' behavior and academic work and adapt instruction to ensure that an appropriate instructional match is being maintained. The teacher checks to see whether students understand the task requirements and the procedures needed to complete the task correctly. For example, the teacher asks each student to demonstrate how to complete the task. (Mercer & Mercer, 2005).

Prompting	The teacher provides verbal, physical, or written cues to assist students in generating correct response (Rivera & Smith, 1998). The teacher asks leading questions, repeats and rephrases lesson content, points to a specific word or number, provides examples and nonexamples, gives feedback, does tasks partially, does a task with students, provides manual guidance (Mercer & Mercer, 2005).
Purpose setting	The teacher tells students why they are learning or doing something (Gelzheiser & Meyers, 1991).
Questioning	A technique used to tap lower order (e.g., recall, comprehension) and higher order (e.g., analysis, evaluation) thinking; questioning can be used to check for understanding and to generate discussion (Rivera & Smith, 1998).
Representation	The domain heuristic of graphically and/or numerically representing problems based on the relationships among problem components (Jitendra, 1999; Kilpatrick et al., 2001).
Review of Prerequisite Skills	The teacher reviews background knowledge necessary for applying the target skills (Jitendra, Salmento, & Haydt, 1999).
Review of Skills Taught	The teacher reviews the skills taught in subsequent lessons after the initial teaching to enhance retention of a new skill (Jitendra, Salmento, & Haydt, 1999). "For new, unfamiliar information to be remembered effortlessly and accurately, it must be presented frequently and on numerous different occasions (i.e., distributed over time). Moreover, the review must snowball strategically into an integrated form, in which familiar information establishes the groundwork for new information, and both new and old information are melded over time. Finally, it must be applied and practiced in different ways" (Kameenui & Carnine, 2002, p. 15).
Scaffolding	The teacher prompts students to use/learn skills or strategy through modeling, questioning, shaping, correcting, guiding student response to task and gradually gives responsibility for use to the student (Lenz & Hughes, 1990). The teacher provides temporarily supports for students to learn new materials by using concept maps or graphic displays, or placing examples of a particular concept in a specific sequence that makes the introduction of examples early in the sequence easier to understand than examples that come later in the sequence. The scaffolding is faded over time (Kameenui & Carnine, 2002).

Segmenting skills	The teacher breaks down targeted skills into smaller units and then synthesizes the parts into a whole (Swanson, Hoskyn, & Lee (1999), 1999)
Sequencing	The teacher facilitates student learning by breaking down the task, fading prompts or cues, sequencing short activities, or providing step-by-step prompts (Swanson, Hoskyn, & Lee (1999), 1999)
Shaping	Reinforcing successive approximations of the desired behaviors (Rivera & Smith, 1998).
Small Groups	Students work with other students in group sizes of 3–10 (Vaughn, Gersten, & Chard, 2000)
Strategy	A broad range of routines that facilitate both knowledge acquisition and utilization, including various heuristic techniques that allow one to more easily access relevant information during problem-solving as well as general control strategies such as planning, monitoring, checking, and revising (Prawat, 1989).
Strategy Cue	A teacher provides reminders to use strategies or multisteps; verbalizes problem solving or procedures to solve; or presents the benefits of strategy use or procedures (Swanson, Hoskyn, & Lee (1999), 1999).
Systematic Instruction	Systematic instructional approach that includes delivery and design procedures derived from effective schools research and behavior analysis. Components includes: group instruction with high level of teacher and student interactions, emphasis on big ideas, conspicuous strategies, mediated scaffolding, strategic instruction, judicious review, and primed background knowledge (Kameenui & Carnine, 2002).
Teaching Explicit Strategies	The teacher uses visual maps or models to represent or illustrate the strategy, labels or calls attention to the different features of the strategy, or provides full and clear explanation of the strategy (Kameenui & Carnine, 2002).

REFERENCES

American Association for the Advancement of Science. (1989). *Mathematics: Report of the Project 2061 Phase I Mathematics Panel.*

Apthorp, H. S., Dean, C. B., Florian, J. E., Lauer, P. A., Reichardt, R., & Snow-Renner, R. (2001). *Standards in classroom practice research synthesis.* Aurora, CO: Mid-Continent Research for Education and Learning. (ERIC Document Reproduction Service No. ED460110)

Ashcraft, M. H., Yamashita, T. S., & Aram, D. M. (1992). Mathematics performances in left and right brain-lesioned children. *Brain and Cognition, 19*, 208-252.

Badian, N. A. (1983). Arithmetic and nonverbal learning. In H. R. Myklebust (Ed.), *Progress in learning disabilities* (Vol. 5, pp. 235-264). New York: Grunt & Stratton.

Baker, J. D., & Beisel, R. W. (2001). An experiment in three approaches to teaching average to elementary school children. *School Science and Mathematics, 101*(1), 23-32.

Baker, J. M., & Zigmond, N. (1990). Are regular education classes equipped to accommodate students with learning disabilities? *Exceptional Children, 56*, 515-526.

Baker, S., Gersten, R., & Lee, D. (2002). A synthesis of empirical research on teaching mathematics to low-achieving students. *Elementary School Journal, 103*(1), 51-73.

Barley, Z., Lauer, P. A., Arens, S. A., Apthorp, H. S., Englert, K. S., Snow, D., et al. (2002). *Helping at-risk students meet standards: A synthesis of evidence-based classroom practices*. Aurora, CO: Mid-continent Research for Education and Learning. (ERIC Document Reproduction Service No. ED475904)

Bassok, M. (1997). Two types of reliance on correlations between content and structure in reasoning about word problems. In L. D. English (Ed.), *Mathematical reasoning: Analogies, metaphors, and images* (pp. 221-246). Mahwah, NJ: Erlbaum.

Bassok, M. (2001). Semantic alignments in mathematical word problems. In D. Gentner, K. J. Holyoak, & B. N. Kokinov (Eds.) *The analogical mind* (pp. 401-433). Cambridge, MA: The MIT Press.

Bassok, M., & Holyoak, K. (1993). Consequential transitions: A sociocultural expedition beyond transfer in education. In A. Iran-Nejad & P. D. Pearson (Eds.), *Review of research in education* (Vol. 24, pp. 101-139). Washington, DC: American Educational Research Association.

Baxter, J., Woodward, J., & Olson, D. (2001). Effects of reform-based mathematics instruction on low achievers in five third-grade classrooms. *The Elementary School Journal, 101*(5), 529-547.

Baxter, J., Woodward, J., Voorhies, J., & Wong, J. (2002). We talk about it, but do they get it? *Learning Disabilities: Research & Practice, 17*(3), 173-185.

Beatty, A. (Ed.). (1997). *Learning from TIMSS: Results of the third international mathematics and science study.* Washington, DC: National Academy of Sciences.

Bogdan, R., & Biklen, S.(1982). *Qualitative research education: An introduction to theory and methods.* Boston: Allyn and Bacon, Inc.

Bottge, B. A. (1999). Effects of contextualized math instruction on problem solving of average and below average achieving students. *The Journal of Special Education, 33*(2), 81-92.

Bottge, B. A. (2001). Reconceptualizing mathematics problem solving for low-achieving students. *Remedial and Special Education, 22*(2), 102-112.

Bottge, B. A., & Hasselbring, T. S.(1993). A comparison of two approaches for teaching complex, authentic mathematics problems to adolescents in remedial math classes. *Exceptional Children, 59,* 556-566.

Bottge, B. A., Heinrichs, M., Chan, S., & Serlin, R.(2000). *Effects of enhanced anchors on the math performances of low-achieving adolescents.* Madison: Unversity of Wisconsion-Madison.

Braswell, J. S., Daane, M. C., & Grigg, W. S. (2003). *The nation's report card: Mathematics highlights 2003.* Washington, DC: U.S. Department of Education, Institute for Education Sciences.

Bryant, D. P. (2005). Commentary on early identification and intervention for students with mathematics difficulties. *Journal of Learning Disabilities,38* (4), 340-345.

Bryant, D. P., & Bryant, B. R. (1998). Using assistive technology adaptations to include students with learning disabilities in cooperative learning activities. *Journal of Learning Disabilities, 31*(1), 41-54.

Bryant, D. P., Bryant, B. R., & Hammill, D. D. (2000). Characteristic behaviors of students with LD who have teacher-identified math weakness. *Journal of Learning Disabilities, 33*(2), 168-177, 199.

Bryant, D. P., Hartman, P., & Kim, S. (2003). Teaching division skills to students with learning disabilities: Explicit instruction matters. *Exceptionality, 11*(3), 151-164.

Bryant, D. P., Kim, S., Hartman, P., & Bryant, B. (2006). Standards-based mathematics instruction and teaching middle school students with mathematics disabilities. In M. Montague & A. Jitendra (Eds.), *Middle school students with mathematics difficulties.* New York: Guilford.

Butler, F. M., Miller, S. P., & Crehan, K.(2003). Fraction instruction for students with mathematics disabilities: Comparing two teaching sequences. *Learning Disabilities Research and Practice, 18* (2), 99-111.

Bybee, R. W., Ferrini-Mundy, J., & Loucks-Horsley, S. (1997). National standards and school science and mathematics. *School Science and Mathematics, 97*, 325-334.

Carnine, D., Dixon, R., & Silbert, J. (1998). Effective strategies for teaching mathematics. In E. Kameenui & D. Carnine (Eds.), *Effective teaching strategies that accommodate diverse learners* (pp. 92-112). Columbus, OH: Merrill.

Carnine, D., & Jitendra, A. K. (1997). A descriptive analysis of mathematics curricular materials from a pedagogical perspective. *Remedial and Special Education, 18*(2), 66-82.

Carnine, D., Silbert, J., & Kameenui, E. J. (1997). *Direct instruction reading*. Upper Saddle River, N. J.: Merrill.

Cass, M. Cates, D., Smith, M., & Jackson, C. (2003). Effects of manipulative instruction on solving area and perimeter problems by students with learning disabilities. *Learning Disabilities Research & Practice, 18*(2), 112-120.

Cawley, J. F., & Miller, J. H. (1989). Cross-sectional comparisons of the mathematical performance of learning disabled children: Are we on the right track toward comprehensive programming? *Journal of Learning Disabilities, 22*, 250-255.

Cawley, J., Parmar, R., Foley, T. E., Salmon, S., & Roy, S. (2001). Arithmetic performances of students: Implications for standards and programming. *Exceptional Children, 67*(3), 311-328.

Cawley, J. F., Parmar, R. S., Yan, W., & Miller, J. H. (1998). Arithmetic computation performances of students with learning disabilities: Implications for curriculum. *Learning Disabilities Research & Practice, 13,* 68-74.

Chi, M. T. H., & Feltovich, P. J., & Glaser, R. (1981). Categorization and representation of physics problems by experts and novices. *Cognitive Science, 5*, 121-152.

Choi-Koh, S. S. (1999). A student's learning of geometry using the computer. *The Journal of Educational Research, 92*(5), 301-313.

Clements, D. H., & Battista, M. T. (1990). Constructivist learning and teaching. *Arithmetic Teachers, 38*(1), 34-35.

Cobb, P., Wood, T., Yackel, E., & McNeal, B. (1992). Characteristics of classroom mathematics traditions: An interactional analysis. *American Educational Research Journal, 29*, 573-604.

Cole, S., Horvath, B., Chapman, C., Deschenes, C., Ebeling, D. G., & Sprague, J. (2000). *Adapting curriculum and instruction in inclusive classrooms: A teacher's desk reference* (2nd ed.). Bloomington, IN: Center on Education and Lifelong Learning.

Cooper, G., & Sweller, J.(1987). Effects of schema acquisition and rule automation on mathematical problem-solving transfer. *Journal of Educational Psychology, 79*, 347-362.

Darch, C., Carnine, D., & Gersten, R.(1984). Explicit instruction in mathematics problem solving. *Journal of Educational Research, 77*, 350-359.

Davis, R., & Maher, C. (1996). A new view of the goals and means for school mathematics. In M. Pugach, & C. Warger (Eds.), *Curriculum trends, special education, and reform: Refocusing the conversation* (pp. 68-83). New York: Teachers College Press.

Denzin, N. K. (1978). *Sociological methods: A sourcebook.* New York: McGraw Hill.

Dixon, R. (1992). *Research on improving mathematics instruction for low-performing students.* Eugene: University of Oregon, Center for the Study of Improving Math Instruction.

Durkin, D. (1990). Matching classroom instruction with reading abilities: An unmet need. *Remedial and Special Education, 11*(3), 23-28.

Educational Resources Information Center. (1997). Designing structured interviews for educational research. *Practical Assessment, Research & Evaluation, 5*(12). Retrieved April 12, 2005, from http://PAREonline.Net/getvn.asp?v=5&n=12.

Forman, S. L., & Steen, L. A. (2000). Beyond eighth grade: Functional mathematics for life and work. In M. J. Burke & F. R. Curcio (Eds.), *Learning mathematics for a new century* (pp. 127-157). Reston, VA: National Council for Teachers of Mathematics.

Fuchs, L. S., & Fuchs, D. (2001). Principles for the prevention and intervention of mathematics difficulties. *Learning Disabilities Research & Practice, 16*(2), 85-95.

Fuchs, L. S., Fuchs, D., & Bishop, N. (1992). Instructional adaptation for students at risk for academic failure. *Journal of Educational Research, 86,* 70-84.

Fuchs, L. S., Fuchs, D., Hamlett, C. L., Phillips, N. B., & Bentz, J.(1994). Classwide curriculum-based measurement: Helping general educators meet the challenge of student diversity. *Exceptional Children, 60*, 518-537.

Fuchs, L. S., Fuchs, D., Hamlett, C. L., & Stecker, P. M. (1991). Effects of curriculum-based measurement and consultation on teacher planning and student achievement in mathematics operations. *American Educational Research Journal, 28*, 617-641.

Fuchs, L. S., Fuchs, D., & Karns, K. (2001). Enhancing kindergarteners' mathematical development: Effects of peer-assisted learning strategies. *The Elementary School Journal, 101*(5), 495-510.

Fuchs, L. S., Fuchs, D. Phillips, N., & Karns, K. (1995). General educators' specialized adaptation for students with learning disabilities. *Exceptional Children, 61*(5), 440-459.

Funkhouser, C.(1995), Developing number sense and basic computational skills in students with special needs. *School Science and Mathematics, 95*, 236-239.

Fuys, D., Geddes, D., Lovett, C., & Tischler, R. (1988). The Van Hiele model of thinking in geometry among adolescents. *Journal for Research in Mathematics Education (Monograph No. 3).* Reston, VA: National Council of Teachers of Mathematics.

Fuys, D. J., & Liebov, A. K.(1993). Geometry and Spatial Sense. In R. J. Jensen (Eds.), *Research ideas for the classroom (pp. 219)*. New York: Macmillan Publishing Co.

Gall, M. D., Borg, W. R., & Gall, J. P. (1996). *Educational research: An introduction* (6th ed.). White Plains, NY: Longman.

Garner, R. (1992). Learning from school texts. *Educational Psychologist, 27*, 53-63.

Geary, D. C. (2004). Mathematics and learning disabilities. *Journal of Learning Disabilities, 37*(1), 4-15.

Geary, D. C., Brown, S. C., & Samaranayake, V. A. (1991). Cognitive addition: A short longitudinal study of strategy choice and speed-of-processing differences in normal and mathematically disabled children. *Developmental Psychology, 27*, 787-797.

Geary, D.C., Hoard, M. K., Byrd-Craven, J., & DeSoto, M. C. (2004). Strategy choices in simple and complex addition: Contributions of working memory and counting knowledge for children with mathematics disability. *Journal of Experimental Child Psychology, 88*, 121-151.

Geary, D. C., Hoard, M. K., & Hamson, C. O. (1999). Numerical and arithmetical cognition: Patterns of functions and deficits in children at risk for a mathematical disability. *Journal of Experimental Child Psychology, 74,* 213-239.

Gelzheiser, L. M., & Meyers, J.(1991). Reading instruction by classroom, remedial and resource room teachers. *The Journal of Special Education, 24* (4), 512-526.

Gentner, D., Rattermann, M. J., & Forbus, K. D. (1993). The roles of similarity in transfer: Separating retrievability from inferential soundness. *Cognitive Psychology, 25,* 524-575.

Gersten, R., & Baker, S. (1998). Real world use of scientific concepts: Integrating situated cognition with explicit instruction. *Exceptional Children, 65,* 23-36.

Gersten, R., & Chard, D. (1999). Number sense: Rethinking arithmetic instruction for students with mathematical disabilities. *The Journal of Special Education, 33,* 18-28.

Gickling, E. E., & Thompson, V. P. (1985). A personal view of curriculum-based assessment. *Exceptional Children, 52*, 205-218.

Ginsburg, H. P. (1997a). *Entering the child's mind: The clinical interview in psychological research and practice*. New York: Cambridge University Press.

Ginsburg, H. P. (1997b). Mathematics learning disabilities: A view from developmental psychology. *Journal of Learning Disabilities, 30*, 20-33.

Ginsburg, H., & Pappas, S. (2004). SES, ethnic, and gender differences in young children's informal addition and subtraction: A clinical interview investigation. *Applied Developmental Psychology, 25,* 171-192.

Glaser, R. (1977). *Adaptive education: Individual diversity and learning.* New York: Holt, Rinehart, & Winston.

Glaser, B. G., & Strauss, A. L.(1967). *The discovery of grounded theory: Strategies for qualitative research.* Chicago: Aldine.

Greenspan, S., & Greenspan, N. (2003). *The clinical interview of the child.* Washington, DC: American Psychiatric.

Grobecker, B. (1999). Mathematics reform and learning differences. *Learning Disability Quarterly, 22,* 43-58.

Grobecker, B., & De Lisi, R. (2000). An investigation of spatial-geometrical understanding in students with learning disabilities. *Learning Disability Quarterly, 23*(1), 7-22.

Gross-Tsur, V., Manor, O., & Shalev, R. S. (1996). Developmental dyscalculia: Prevalence and demographic features. *Developmental Medicine and Child Neurology, 38*, 25-33.

Gunter, P. L., Denny, K., & Venn, M. L. (2000). Modification of instructional materials and procedures for curricular success of students with emotional and behavioral disorders. *Preventing School Failure, 44*(3), 116-121.

Hanich, L. B., Jordan, N. C., Kaplan, D., & Dick, J. (2001). Performance across different areas of mathematical cognition in children with learning difficulties. *Journal of Educational Psychology, 93,* 615-626.

Harris, K., & Graham, S. (1996). Constructivism and students with special needs: Issues in the classroom. *Learning Disabilities Research & Practice, 11*, 134-137.

Harris, C. A., Miller, S. P., & Mercer, C. D.(1995). Teaching initial multiplication skills to students with disabilities in general education classrooms. *Learning Disabilities Research and Practice, 10*, 180-195.

Hatano, G., & Inagaki, K. (1993). Desituating cognition through the construction of conceptual knowledge. In P. Light & G. Butterworth (Eds.), *Context and cognition: Ways of learning and knowing* (pp. 115-133). Mahwah, NJ: Erlbaum.

Hitch, G. J., & McAuley, E. (1991). Working memory in children with specific arithmetical learning disabilities. *British Journal of Psychology, 82,* 375-386.

Holstein, J. A., & Gubrium, J. F. (1995). *The active interview.* Thousand Oaks, CA: Sage.

Individuals with Disabilities Education Acts (IDEA) of 1997. Pub. L. No. 105-17. U. S. Department of Education, Washington, DC:

Individuals with Disabilities Education Improvement Act, Pub. L. 108-446 (2004).

Jitendra, A. K., Carnine, D., & Silbert, J. (1996). Descriptive analysis of fifth grade division instruction in basal mathematics programs: Violations of pedagogy. *Journal of Behavioral Education, 6*(4), 381-403.

Jitendra, A. K., Griffin, C., Deatline-Buchman, A., Dipipi-Hoy, C., Sczesniak, E., Sokol, N. G., & Xin, Y. P. (2005). Adherence to mathematics professional standards and instructional design criteria for problem-solving in mathematics. *Exceptional Children, 71*(3), 319-337.

Jitendra, A. K., Salmento, M. M., & Haydt, L. A. (1999). A case analysis of fourth-grade subtraction instruction in basal mathematics programs: Adherence to important instructional design criteria. *Learning Disabilities Research & Practice, 14* (2), 69-79.

Jones, E. D., & Krouse, J. P. (1988). The effectiveness of data-based instruction by student teachers in classrooms for pupils with mild handicaps. *Teacher Education and Special Education, 11*(1), 9-19.

Jordan, N. C, & Hanich, L. B. (2003). Characteristics of children with moderate mathematics deficiencies: A longitudinal perspective. *Learning Disabilities Research & Practice, 18*(4), 213-221.

Jordan, N. C, Hanich, L. B., & Kaplan, D. (2003a). Arithmetic fact mastery in young children: A longitudinal investigation. *Journal of Experimental Child Psychology, 85*, 103-119.

Jordan, N. C, Hanich, L.B., & Kaplan, D. (2003b). A longitudinal study of mathematical competencies in children with specific mathematics difficulties versus children with comorbid mathematics and reading difficulties. *Child Development, 74*(3), 834-850.

Jordan, N. C., Hanich, L.B., & Uberti, H. (2003). Mathematical thinking and learning difficulties. In A. J. Baroody & A. Dowker (Eds.), *Development of arithmetic concepts and skills: Constructing adaptive expertise* (pp. 359-383). Mahwah, NJ: Erlbaum.

Jordan, N. C., & Montani, T. O. (1997). Cognitive arithmetic and problem solving: A comparison of children with specific and general mathematics difficulties. *Journal of Learning Disabilities, 30*, 624-634.

Kagan, D. M., & Tippins, D. J. (1991). Help student teachers attend to student cues. *Elementary School Journal, 91*, 343-356.

Kameenui, E. J., & Carnine, D. (2002). *Effective teaching strategies that accommodate diverse learners* (2nd ed.). Upper Saddle River, NJ: Merrill.

Kilpatrick, J., Swafford, J., & Findell, B. (Eds.). (2001). *Adding it up: Helping children learn mathematics.* Washington, DC: National Academy Press.

Kroesbergen, E., & Van Luit, J. E. H. (2003). Mathematics interventions for children with special educational needs. *Remedial and Special Education, 24*(2), 97-114.

Lappan, G. (2000, November). *Transcripts from the press conference announcing the release of "Principles and Standards for School Mathematics."* Available from the National Council of Teachers of Mathematics Web site: http://www.nctm.org/standards/pctranscript.htm.

Lenz, B. K., & Hughes, C. A.(1990). A word identification strategy for adolescents with learning disabilities. *Journal of Learning Disabilities, 23* (3), 149-158.

Lincoln, Y. S., & Guba, E. G. (1985). *Naturalistic inquiry.* Thousand Oaks, CA: Sage.

Maccini, P., & Gagnon, J. C. (2000). Best practices for teaching mathematics to secondary students with special needs. *Focus on Exceptional Children, 32*, 1-22.

Maccini, P., & Gagnon, J. C. (2006). Mathematics instructional practices and assessment accommodations by secondary special and general educators. *Exceptional Children, 72*(2), 217-234.

McIntosh, R., Vaughn, S., Schumm, J. S., Hagger, D., & Lee, O. (1993). Observations of students with learning disabilities in general education classrooms. *Exceptional Children, 60*(3), 249-261.

Mercer, C. D., & Mercer, A. R.(2005). Teaching students with learning problems (7th ed.). Upper Saddle River, NJ: Prentice Hall/ Merrill/ Prentice Hall.

Miles, M. B., & Huberman, A. M. (1994). *Qualitative data analysis* (2nd ed.). Thousand Oaks, CA: Sage.

Miller, S. P., Butler, F. M., & Lee, K. (1998). Validated practices for teaching mathematics to students with learning disabilities: A review of literature. *Focus on Exceptional Children, 31*(1), 24-40.

Mish, F. (Ed.). (1994). *Merriam Webster's collegiate dictionary* (10th ed.).

National Assessment of Educational Progress. (2003). *Mathematics framework for the 1996 and 2000 National Assessment of Educational Progress.* Washington, DC: U.S. Government Printing Office. Available at http://www.nagb.org/ pubs/96-2000math/toc.html

National Center for Education Statistics. (2000). *Pursuing excellence: Comparisons of international eighth-grade mathematics achievement from a U.S. perspective, 1995 and 1999* (NCES 2001-028). Washington, DC: U.S. Government Printing Office.

National Council of Teachers of Mathematics. (1989). *Evaluation standards: Curriculum and evaluation for school mathematics.* Reston, VA: Author.

National Council of Teachers of Mathematics. (2000). *Principles and standards for school mathematics.* Reston, VA: Author.

No Child Left Behind Act of 2001, Pub. L. 107-110, 2102(4) (2002).

Oberdort, C. D., & Taylor-Cox, J. (1999). Shape up: Children's misconception about geometry. *Teaching Children Mathematics, 5* (6), 340-345.

Ostad, S. A. (1997). Developmental differences in addition strategies: A comparison of mathematically disabled and mathematically normal children. *British Journal of Educational Psychology, 67*, 345-357.

Ostad, S. A. (1998). Developmental differences in solving simple arithmetic word problems and simple number-fact problems: A comparison of mathematically normal and mathematically disabled children. *Mathematical Cognition, 4,* 1-19.

Owen, R. L., & Fuchs, L. (2002). Mathematical problem-solving strategy instruction for third-grade students with learning disabilities. *Remedial and Special Education, 23* (5), 268-278.

Patton, M. Q. (2002). *Qualitative research and evaluation methods* (3rd ed.). Thousand Oaks, CA: Sage.

Perie, M., Grigg, W. S., & Dion, G. S. (2005). *The nation's report card: Mathematics 2005.* Washington, DC: U.S. Department of Education, Institute for Education Sciences.

Porter, A. (1989). A curriculum out of balance: The case of elementary school mathematics. *Educational Researcher, 18*(5), 9-15.

Prawat, R. S. (1989). Promoting access to knowledge, strategy, and disposition in students: A research synthesis. *Review of Educational Research, 59*(1), 1-41.

Program for International Assessment in Education. (2003). *International outcomes of learning in mathematics literacy and problem solving: PISA 2003 results from the U.S. perspective.* Retrieved April 10, 2005, from the National Center for Education Statistics Web site: http://nces.ed.gov/pubsearch/pubsinfo.asp?pubid=2005003

Reed, S. K. (1987). A structure-mapping model for word problems. J*ournal of Experimental Psychology: Learning, Memory, and Cognition, 13*(1), 124-139.

Reed, S. K. (1993). A schema-based theory of transfer. In D. K. Detterman & R. J. Sternberg (Eds.), *Transfer on trial: Intelligence, cognition, and instruction* (pp. 39-67). Norwood, NJ: Ablex.

Resnick, L. B., & Ford, W. W.(1981). *The psychology of mathematics for instruction*. Mahwah, NJ: Erlbaum.

Rivera, D. P.(1993). Examining mathematics reform and the implications for students with mathematics disabilities. *Remedial and Special Education, 14* (6), 24-27.

Rivera, D. P. (1997). Mathematics education and students with learning disabilities: Introduction to the special series. *Journal of Learning Disabilities, 30,* 2-19. 68.

Rivera, D. P., & Smith, D. D.(1997). *Teaching students learning and behavior problems (3rd ed.).* Boston, MA: Allyn and Bacon.

Robinson, C., Menchetti, B., & Torgesen, J. (2002). Toward a two-factor theory of one type of mathematics disabilities. *Learning Disabilities Research & Practice, 17*(2), 81-89.

Rosenshine, B., & Stevens, R.(1984). Classroom instruction in reading. In P. D. Pearson, R. Barr, M. L. Kamil & P. B. Mosenthal (Eds.), *Handbook of reading research*, New York: Longman.

Ross, B. H., & Kennedy, P. T. (1990). Generalizing from the use of earlier examples in problem solving. *Journal of Experimental Psychology: Learning, Memory, & Cognition, 16*(1), 42-55.

Rubin, H. J., & Rubin, I. S. (1995). *Qualitative interviewing: The art of hearing data*. Thousand Oaks, CA: Sage.

Russell, R. L., & Ginsburg, H. P. (1984). Cognitive analysis of children's mathematical difficulties. *Cognition and Instruction, 1,* 217-244.

Salend, S. J., & Hofstetter, E. (1996). Adapting a problem-solving approach to teaching mathematics to students with mild disabilities. *Intervention in School and Clinic, 31*(4), 209-217.

Scheurich, J. J. (1997). *Research method in the postmodern*. London: Falmer Press.

Schumm, J, S., & Vaughn, S.(1991). Making adaptations for mainstreamed students: Regular classroom teachers' perspectives. *Remedial and Special Education, 12* (4), 18-27.

Singley, M. K., & Anderson, J. R. (1989). *The transfer of cognitive skill.* Cambridge, MA: Harvard University Press.

Stake, R. (1995). Case studies. In N. K. Denzin & Y. S. Lincoln (Eds.), *Handbook of qualitative research.* Thousand Oaks, CA: Sage.

Strauss, A. L., & Corbin, J. (1998). *Basic of qualitative research: Techniques and procedures for developing grounded theory* (2nd ed.). Thousand Oaks, CA: Sage.

Swanson, H. L., Hoskyn, M., & Lee, C. (1999). *Interventions for students with learning disabilities: A meta-analysis of treatment outcomes.* New York: Guilford Press.

Swanson, H. L., & Jerman, O. (2006). Math disabilities: A selective meta-analysis of the literature. *Review of Educational Research, 76*(2), 249-274.

Technical Education Research Center. (1998). *Investigations in number, data, and space.* New York: Pearson Education.

Technical Education Research Center. (2004). *Investigations.* New York: Pearson Education.

Texas Education Agency. (1998). *Texas Assessment of Knowledge and Skills: Demographic performance summary—mathematics*. Austin, TX: Author.

Texas Education Agency. (2004a). *Public Information Management System*. Austin, TX: Author.

Texas Education Agency. (2004b). *Texas Assessment of Knowledge and Skills: Demographic performance summary—mathematics*. Austin, TX: Author.

Texas Education Agency (2005) 2004-2005 Academic Excellence Indicator System., Austin, TX: Author.

Texas Education Agency. (2006). *Academic Excellence Indicator System*. Austin, TX: Author.

U.S Department of Education. (2002). *Twenty-fourth annual report to Congress on the Implementation of Individuals With Disabilities Education Act.* Available at http://www.ed.gov/about/reports/annual/osep/2002/index.html

U. S. Department of Education. Institute for Education Sciences (2003). National Center for Education Statistics. The Nation's report card mathematics highlights 2003, NCES 2004-451, by J. S. Braswell, M. C. Daane, & W. S. Grigg. Washington, DC

Van Hiele, E. M. (1986). *Structure and insight: A theory of mathematics education*. New York: Academic Press.

Vaughn, S., Gersten, R., & Chard, D. J. (2000). The underlying message in LD intervention research: Findings from research syntheses. *Exceptional Children, 67*(1), 99-114.

Vaughn Gross Center for Reading and Language Arts. (2001). *Effective instruction for struggling readers: Research-based practices.* Austin: Texas Education Agency.

Vaughn, S. Moody, S. W., & Schumm, J. S.(1998). Broken promise: Reading instruction in the resource room. *Exceptional Children, 64* (2), 211-225.

Vygotsky, L. S. (1978). *Mind in society*. Cambridge, MA: Harvard University Press.

Wagner, J. F. (2003). *The construction of similarity: Context sensitivity and the transfer of mathematical knowledge.* Unpublished doctoral dissertation, University of California, Berkeley.

Wehmeyer, M. L., Lattin, D. L., Lapp-Rincker, C., & Agran, M. (2003). Access to the general education curriculum of middle school students with mental retardation: An observational study. *Remedial and Special Education, 24*(5), 262-272.

Wells, G., & Claxton, G. (2002). *Learning for life in the 21st century: Sociocultural perspectives on the future of education*. Oxford, England: Blackwell.

Woodward, J., & Baxter, J. (1997). The effects of an innovative approach to mathematics on academically low-achieving students in inclusive settings. *Exceptional Children, 63*(3), 373-388.

Woodward, J., & Montague, M. (2002). Meeting the challenge of mathematics reform for students with LD-learning disabilities. *Journal of Special Education, 36*(2), 89-101.

Xin, Y. P., & Jitendra, A. K. (1999). The effects of instruction in solving mathematics word problems for students with learning problems: A meta-analysis. *Journal of Special Education, 32,* 207-225.

Yin, R. (2003). *Case study research* (3rd ed.). Thousand Oaks, CA: Sage.

Ysseldyke, J. Kosciolek, S., Spicuzza, R., & Boys, C. (2003). Effects of a learning information system on mathematics achievement and classroom structure. *The Journal of Educational Research, 96*(3), 163-173.

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